

Special Purpose Control Charts in Statgraphics Centurion

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Control Charts

- A control chart:
 - Plots a statistic versus time.
 - Contains control limits to detect unusual values.
- Used for 2 primary purposes:
 - To determine whether a process is in a state of statistical control. (Phase 1 or initial study)
 - To monitor that process to detect deviations from pre-established parameters. (Phase 2 or control to standard)

Common Control Charts

- For variable data:
 - X-bar and R, X-bar and S, X and MR
 - Median and MR
 - Moving average and EWMA
 - Cumulative sum charts
- For attribute data:
 - P or NP for binomial proportions and counts
 - C or U for Poisson rates and counts
 - P-prime and U-prime charts for overdispersed data

Special Purpose Control Charts

1. ARIMA charts
2. Acceptance charts
3. Toolwear charts
4. Charts for rare events
5. Cuscore charts

Usual Model for Variable Data

Let X_t be measurement made at time t . Then

$$X_t = \mu + \varepsilon_t$$

where μ is the process mean and the deviations ε_t are assumed to be $NID(0, \sigma^2)$.

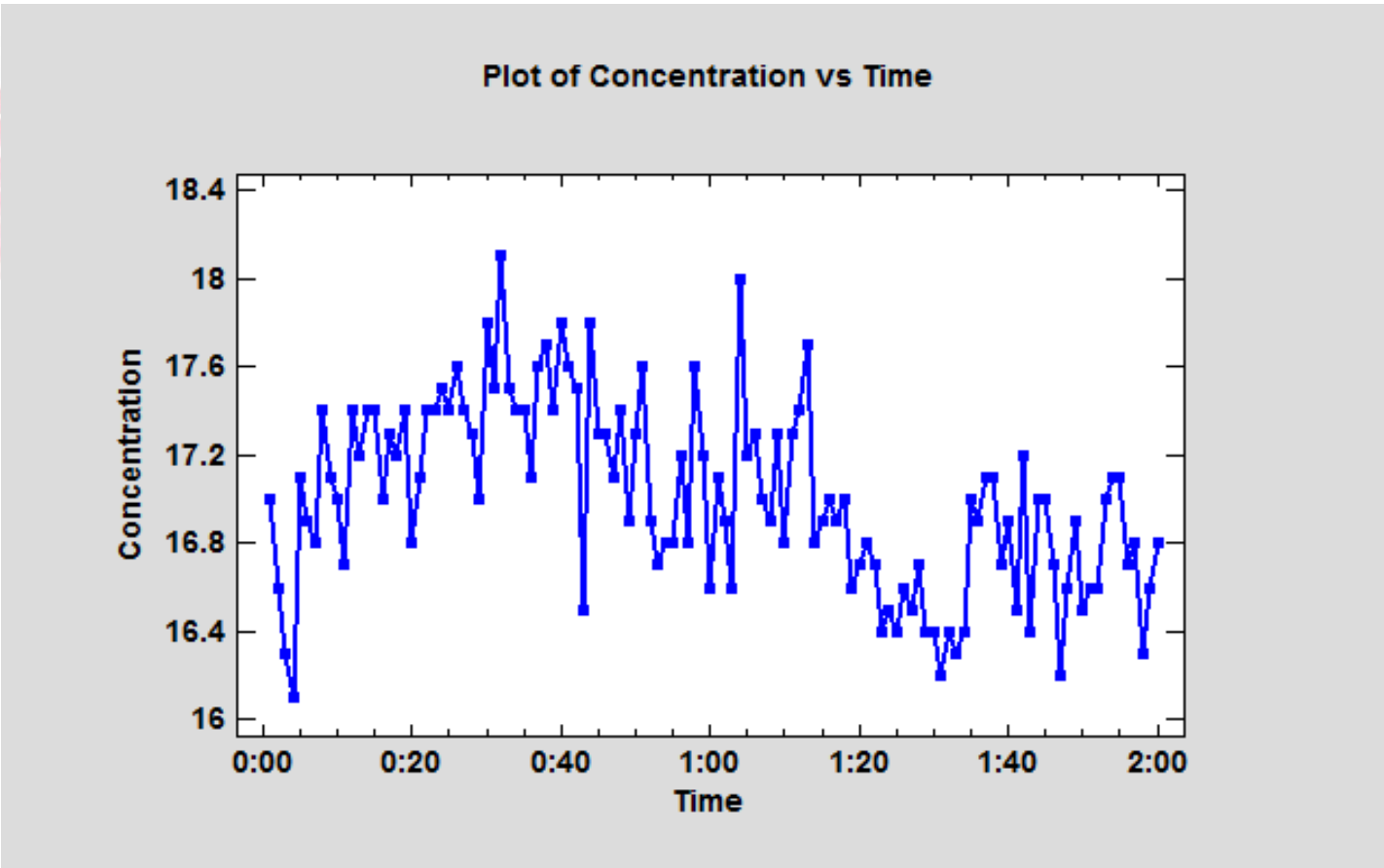
Assumptions:

- Stationarity of mean and variance.
- Independence of successive measurements.
- Normal distribution for deviations from mean.

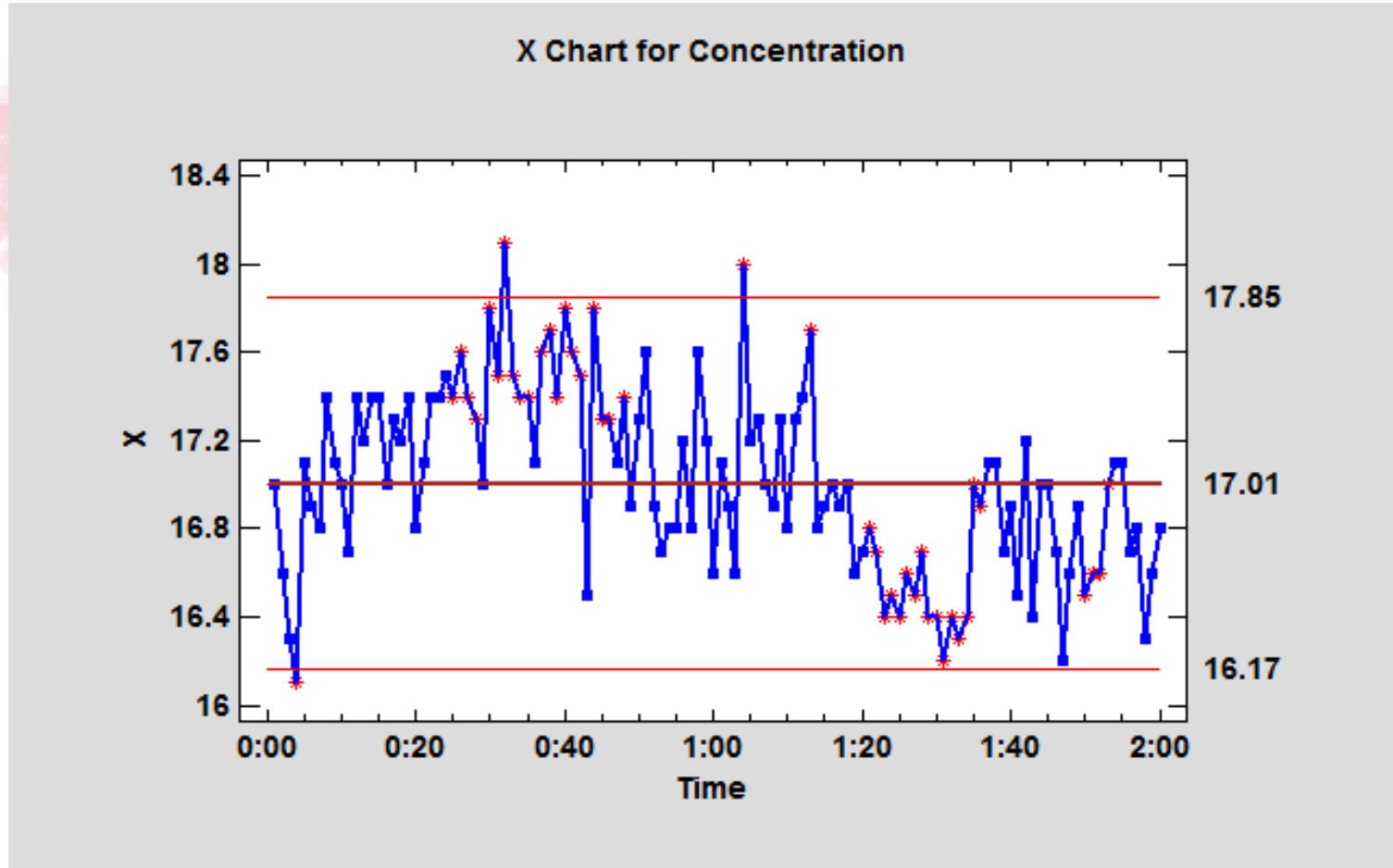
ARIMA Control Charts

- Used for data that are serially correlated (successive observations are not independent).
- Very common when measurements are taken close together in time from a continuous process.

Example: Chemical concentration



Standard Phase 1 X Chart



ARIMA Charts Model

- Relax the assumption that successive observations are independent.
- Instead, assume that the process can be described by an autoregressive moving average model.

Autoregressive Models

- AR(1)

$$X_t = \mu + \phi_1(X_{t-1} - \mu) + \varepsilon_t$$

- AR(2)

$$X_t = \mu + \phi_1(X_{t-1} - \mu) + \phi_2(X_{t-2} - \mu) + \varepsilon_t$$

Moving Average Models

- MA(1)

$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

- MA(2)

$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

ARMA Models

- ARMA(1,1)

$$X_t = \mu + \phi_1 (X_{t-1} - \mu) + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

Selecting the Best ARIMA Model

Automatic Forecasting

Time
Concentration

Sort column names

Data:
▶ Concentration

(Time Indices:)
▶

or

Sampling Interval

Once Every: 1

Year(s) (4-digit) Hour(s)
 Quarter(s) Minute(s)
 Month(s) Second(s)
 Day(s) Other

Starting At:
1.0

(Seasonality:)

(Trading Days Adjustment:)
▶

(Select:)
▶

Number of Forecasts: 60 Withhold for Validation: 0

OK Cancel Delete Transform... Help

Selecting the Best ARIMA Model

Automatic Forecasting Options

Models to Include

<input type="checkbox"/> Random Walk	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Random Walk with Drift	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Mean	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Linear Trend	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Quadratic Trend	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Exponential Trend	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> S-Curve	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Moving Average	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Simple Exp. Smoothing	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Brown's Linear Exp. Smoothing	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Holt's Linear Exp. Smoothing	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Quadratic Exp. Smoothing	<input checked="" type="checkbox"/> Optimize Parameters
<input type="checkbox"/> Winters' Exp. Smoothing	<input checked="" type="checkbox"/> Optimize Parameters
<input checked="" type="checkbox"/> ARIMA: <input checked="" type="checkbox"/> Optimize Model Order	<input checked="" type="checkbox"/> Optimize Parameters

AR Terms (p)

Nonseasonal:

Seasonal:

MA Terms (q)

Nonseasonal:

Seasonal:

Fix q at p-1

Differencing (d)

Nonseasonal:

Seasonal:

Include constant

Method Selection Criterion

Akaike Information Criterion (AIC)

Hannan-Quinn Criterion (HQC)

Schwarz Bayesian Inf. Criterion (SBIC)

Mean Squared Error (MSE)

Mean Absolute Error (MAE)

Mean Abs. Percentage Error (MAPE)

Adjustments...

Parameters...

Estimation...

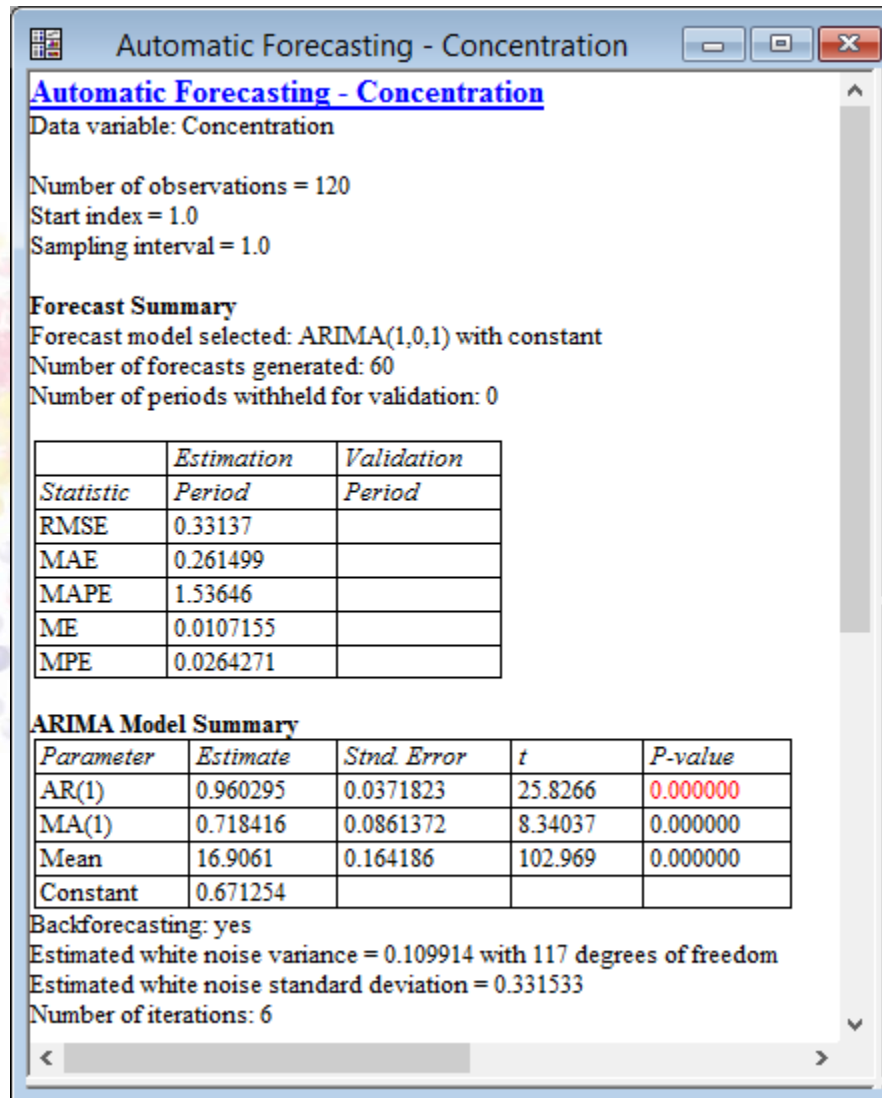
Input series...

OK

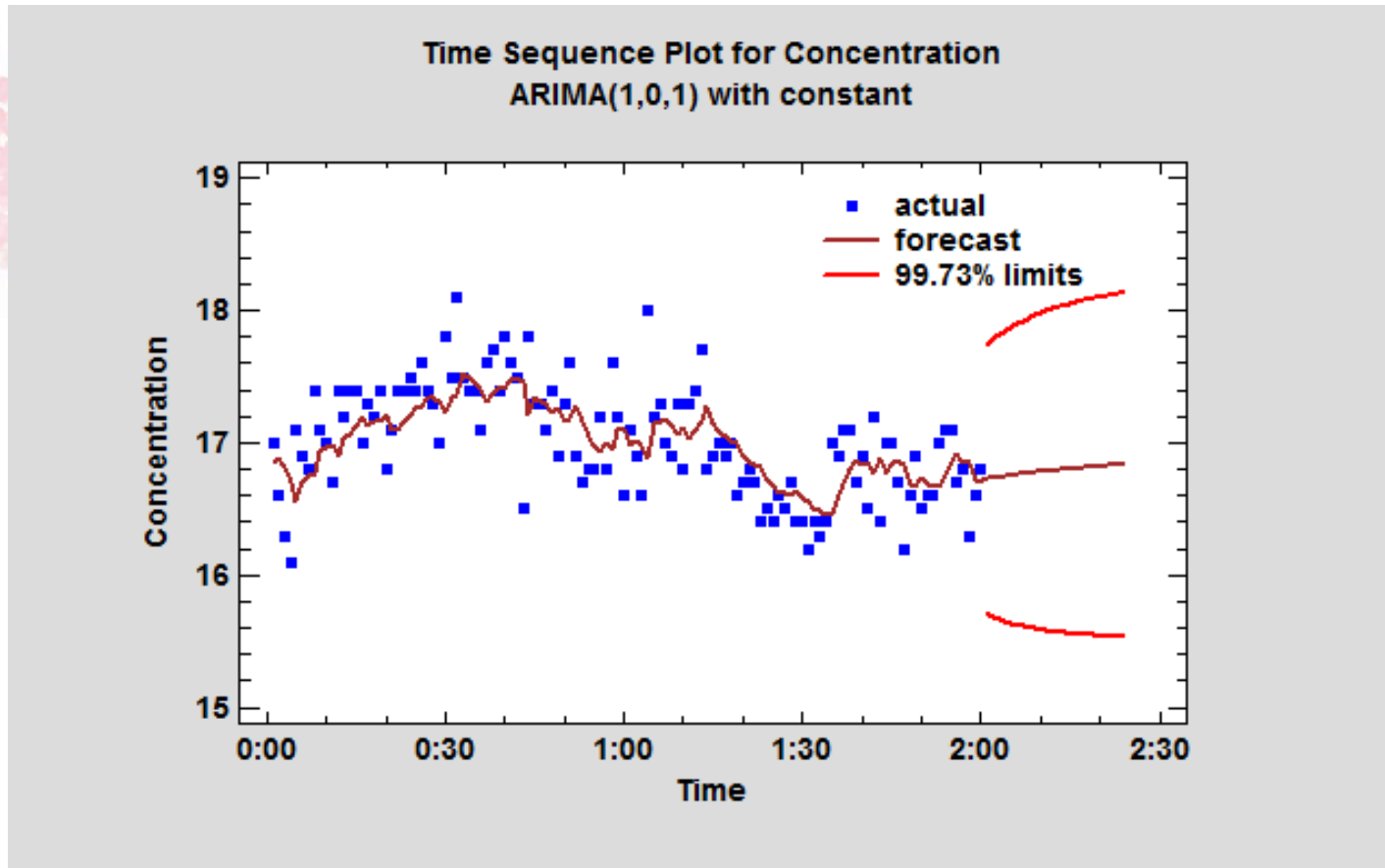
Cancel

Help

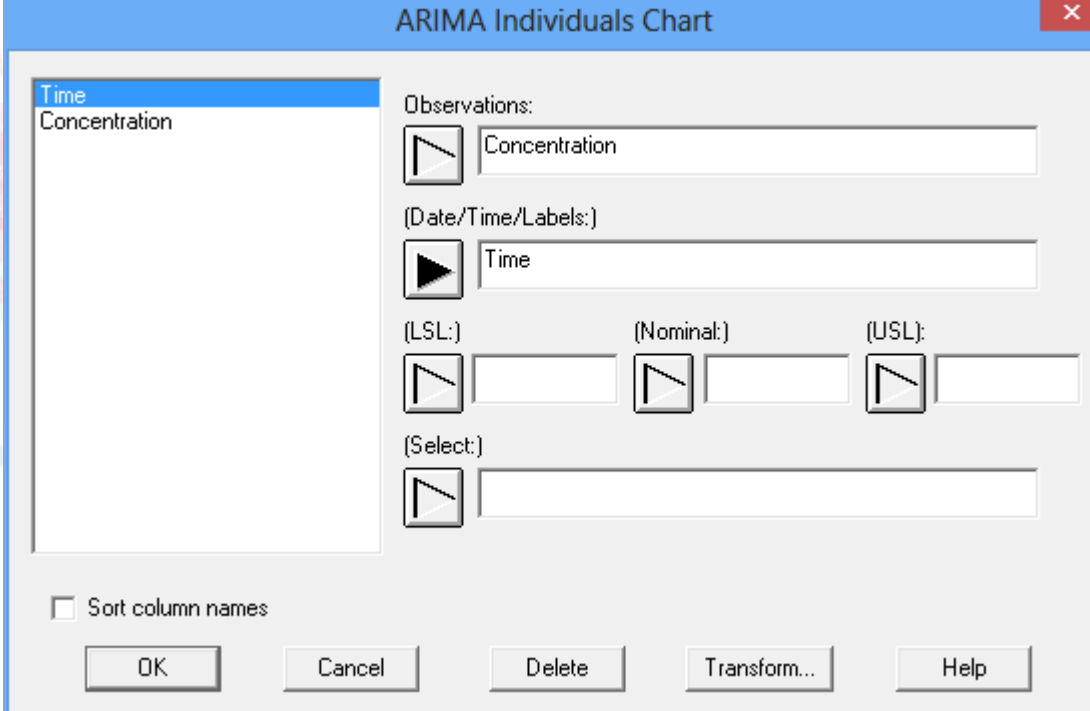
Selecting the Best ARIMA Model



Selected Model



ARIMA Chart – Data Input



The image shows a dialog box titled "ARIMA Individuals Chart" with a close button (X) in the top right corner. On the left, there is a list box containing "Time" (highlighted) and "Concentration". The main area contains several input fields with dropdown arrows:

- Observations: Concentration
- (Date/Time/Labels:): Time
- (LSL:): [empty]
- (Nominal:): [empty]
- (USL:): [empty]
- (Select:): [empty]

At the bottom left, there is a checkbox labeled "Sort column names" which is currently unchecked. At the bottom, there are five buttons: "OK", "Cancel", "Delete", "Transform...", and "Help".

ARIMA Chart – Analysis Options

ARIMA Individuals Charts Options

Type of Study

- Initial Study
- Control to Standard

Control to Standard

Mean:

Sigma:

ARIMA Control Limits

- Upper: Sigma
- Lower: Sigma

MR(2) Control Limits

- Upper: Sigma
- Lower: Sigma

AR(1): MA(1):

AR(2): MA(2):

AR(3): MA(3):

AR(4): MA(4):

Model

AR	I	MA
<input type="radio"/> 0	<input checked="" type="radio"/> 0	<input type="radio"/> 0
<input checked="" type="radio"/> 1	<input type="radio"/> 1	<input checked="" type="radio"/> 1
<input type="radio"/> 2	<input type="radio"/> 2	<input type="radio"/> 2
<input type="radio"/> 3	<input type="checkbox"/> Constant	<input type="radio"/> 3
<input type="radio"/> 4		<input type="radio"/> 4

Estimate sigma from

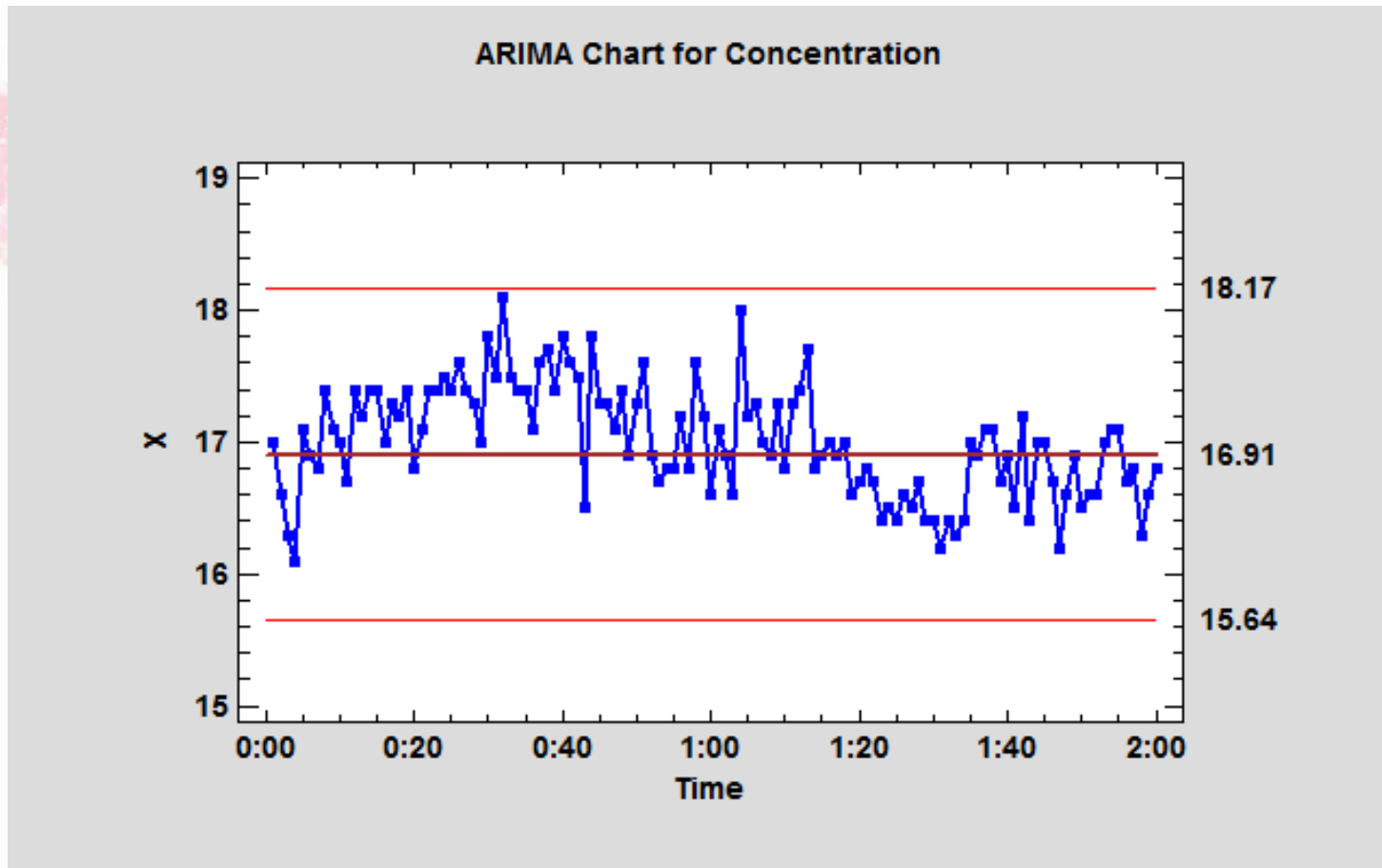
- MR(2) Chart
- Model MSE

Chart type

- Data with long-term limits
- Data with one-step limits
- Residuals
- Normalized Residuals

OK Cancel Transform... Help

Data with Long-Term Limits



Data with One-Step Limits

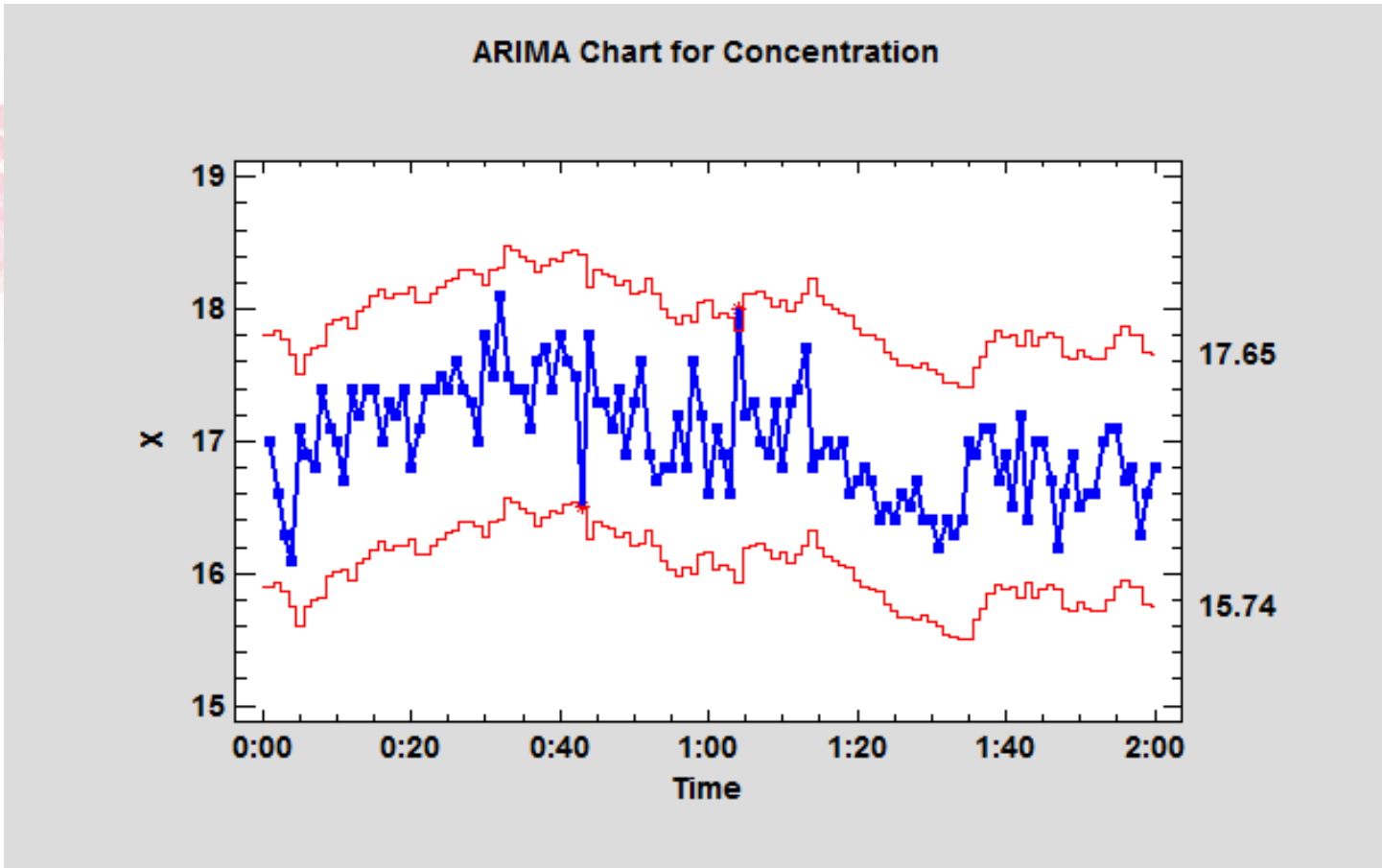
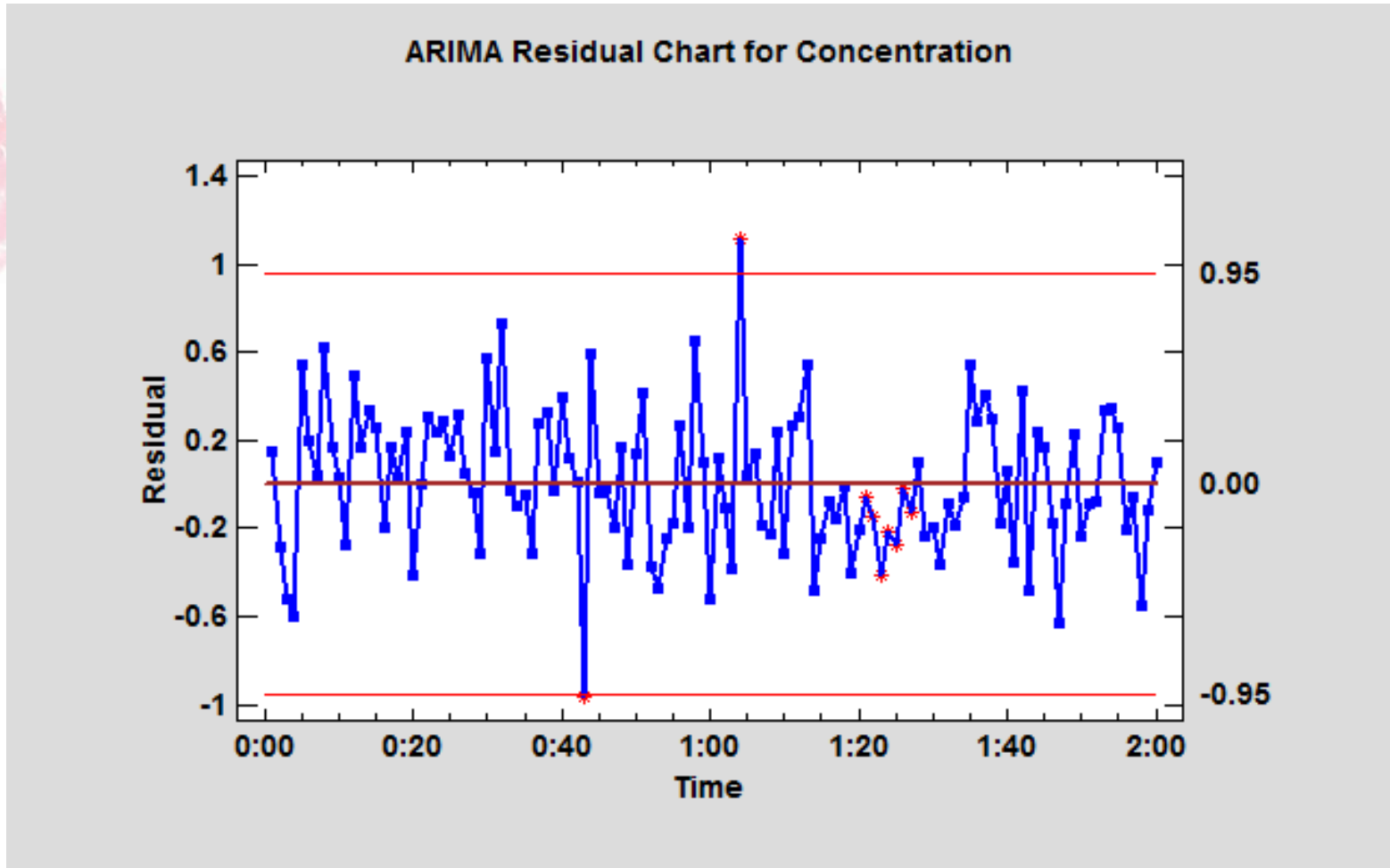
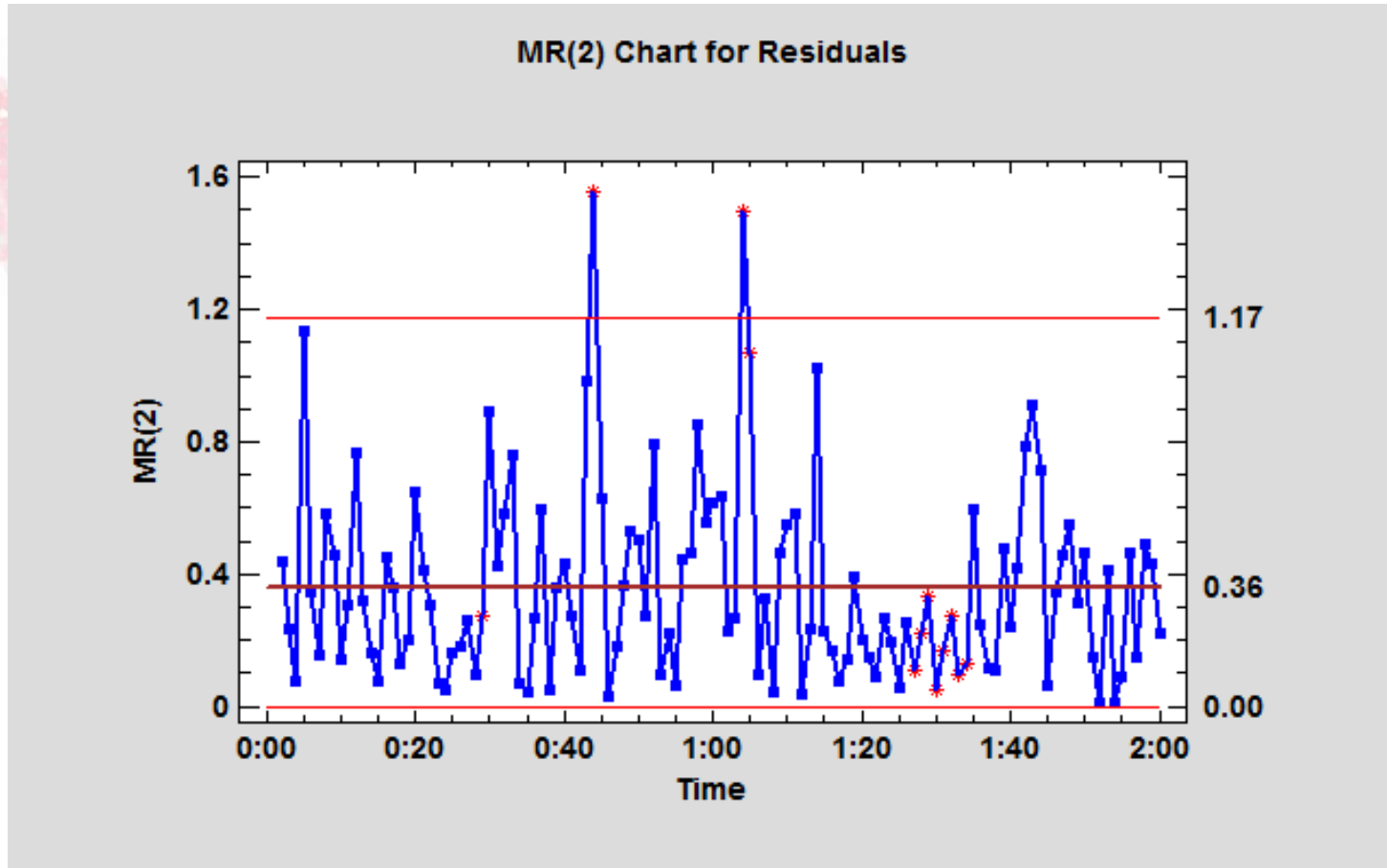


Chart of Residuals



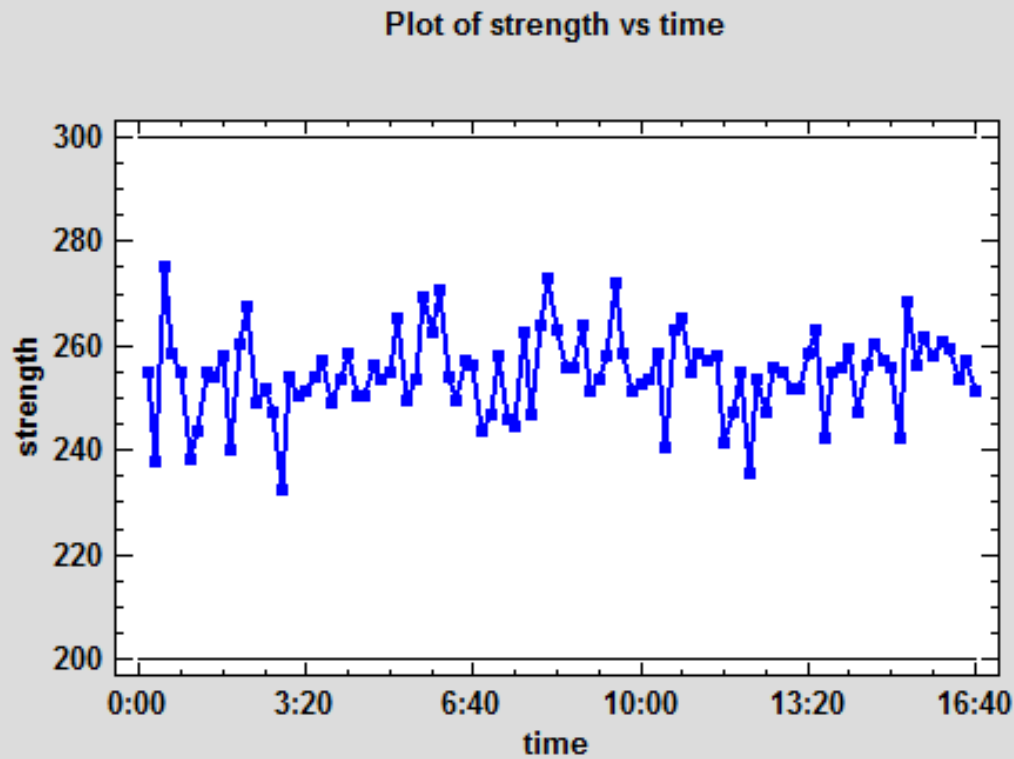
MR(2) Chart for Residuals



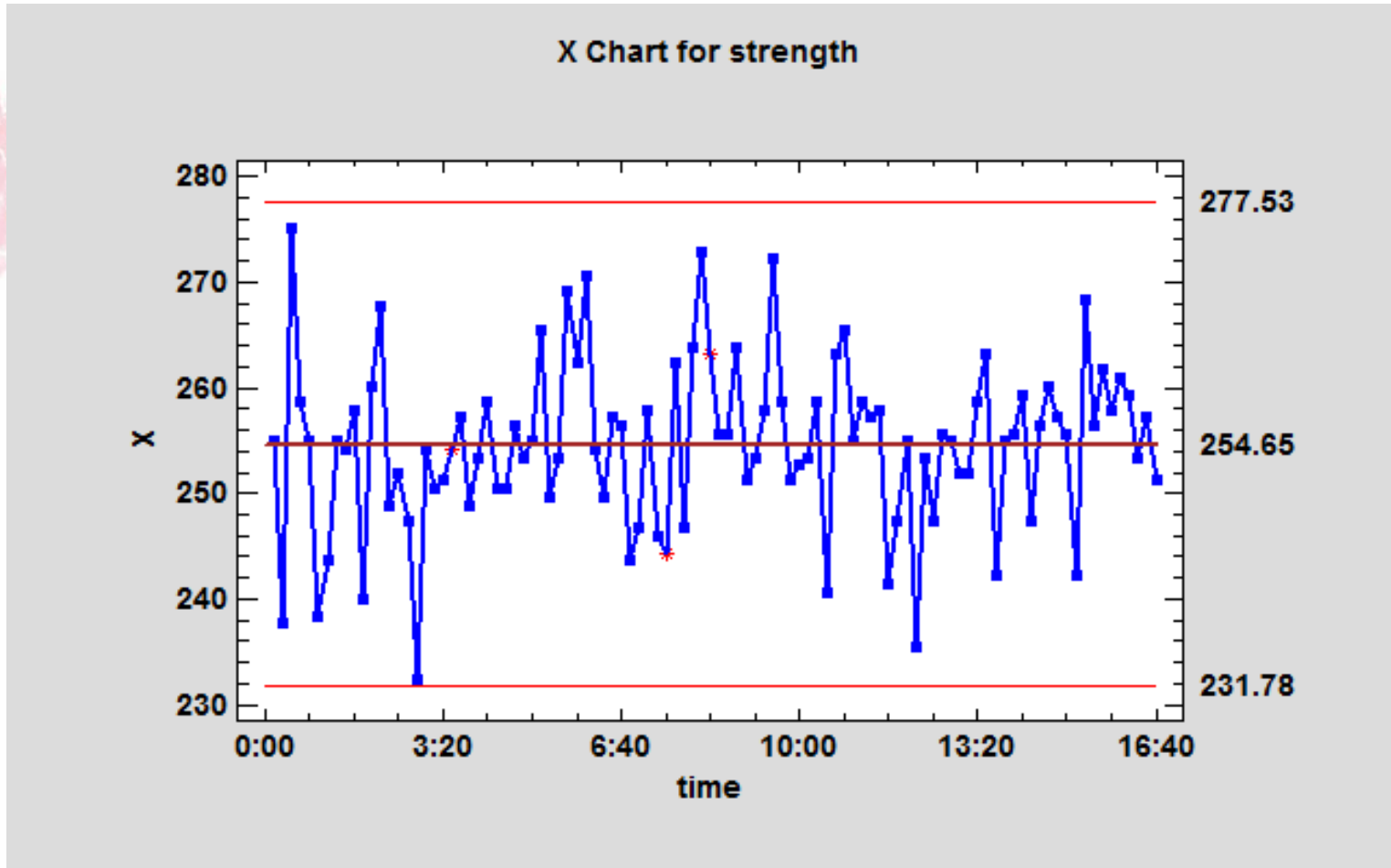
Acceptance Control Charts

- Used to insure that product remains within specification limits.
- Position control limits with respect to the specifications rather than the process mean.
- Useful for high C_{pk} processes which can tolerate some wandering of the process mean.

Sample Data



X Chart



Acceptance Charts

- Begin by specifying f , the maximum allowable fraction of nonconforming items.
- Find most extreme values of process mean that would yield no more than f nonconformities:

Largest allowable mean: $\mu_U = USL - Z_\delta\sigma$

Smallest allowable mean: $\mu_L = LSL + Z_\delta\sigma$

- Position control limits with respect to these extremes.

Positioning Limits

- *Sigma multiple method*: add “3-sigma” to the extreme values for the mean.

$$UCL = \mu_U + 3\sigma$$

$$LCL = \mu_L - 3\sigma$$

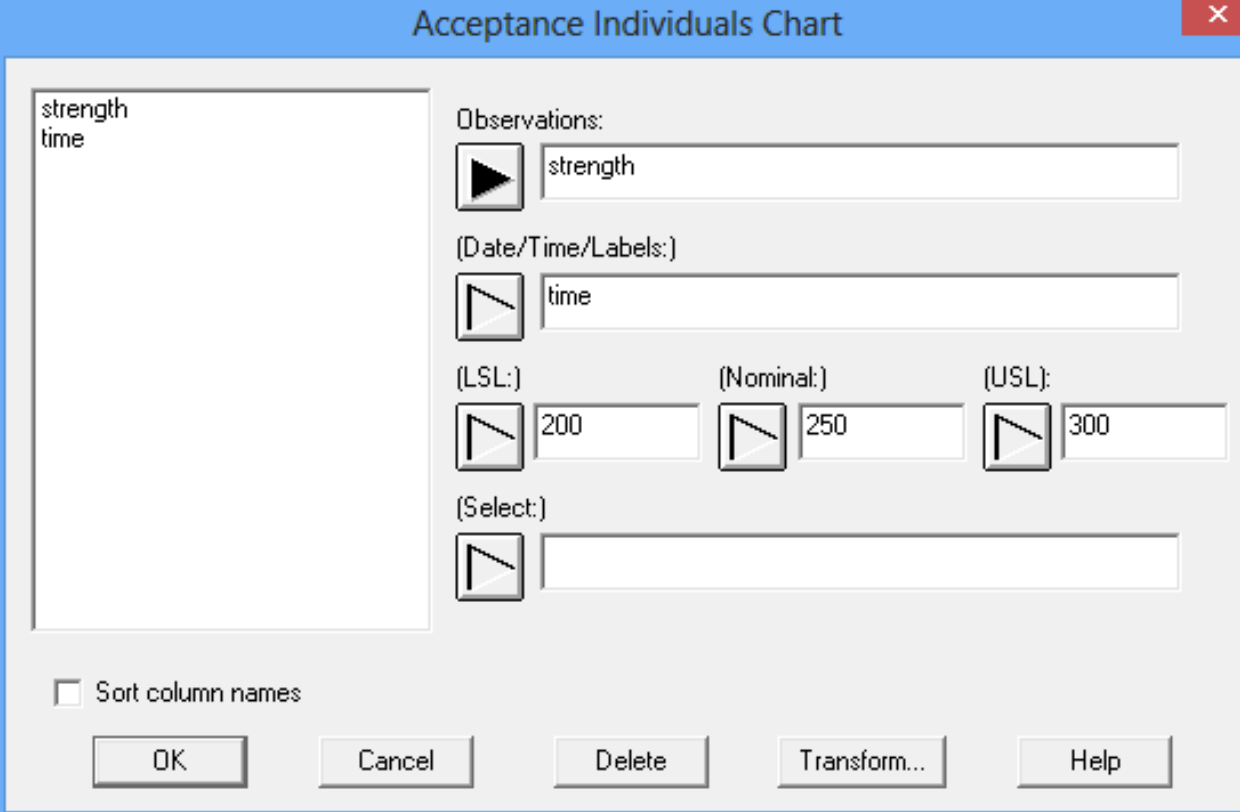
Note: Montgomery calls these “modified control limits”.

- *Beta risk method*: specify β , probability of not generating an out of control signal. Position control limits at:

$$UCL = \mu_U - Z_\beta\sigma$$

$$LCL = \mu_L + Z_\beta\sigma$$

Acceptance Chart – Data Input

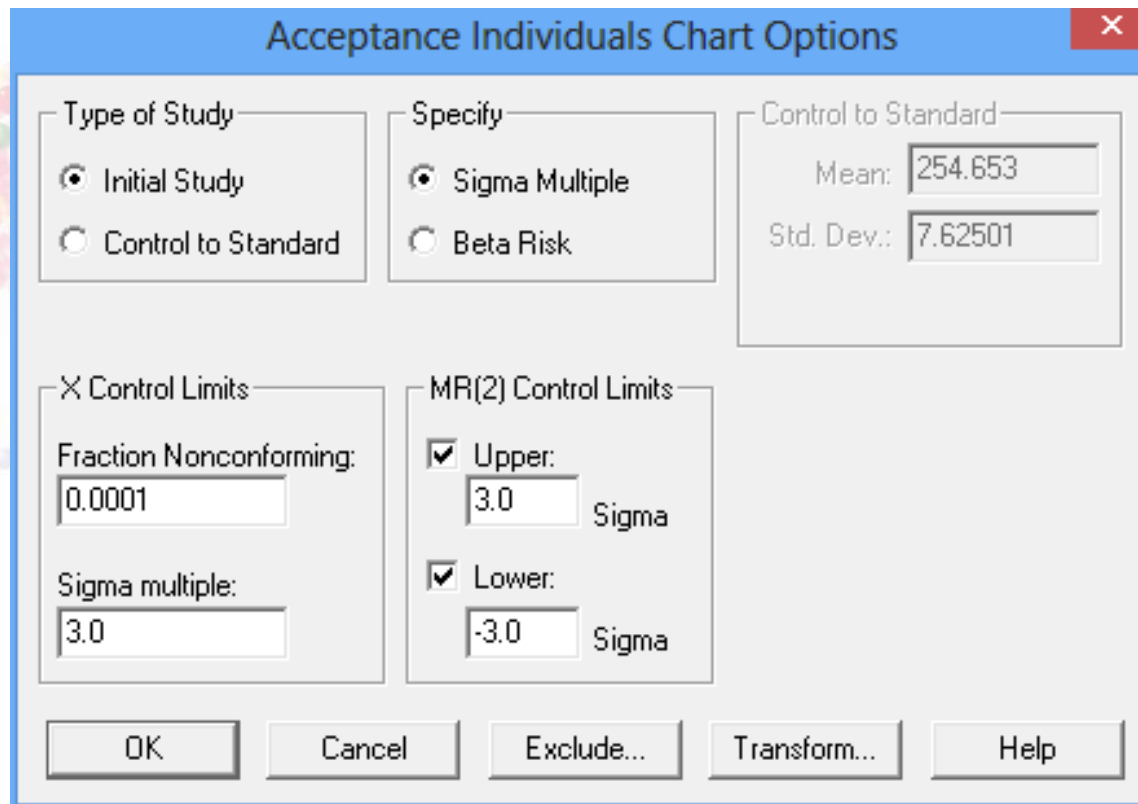


The image shows a software dialog box titled "Acceptance Individuals Chart". On the left is a list box containing "strength" and "time". On the right, there are several input fields with dropdown arrows:

- Observations:** A text box containing "strength".
- (Date/Time/Labels:)** A text box containing "time".
- (LSL:)** A text box containing "200".
- (Nominal:)** A text box containing "250".
- (USL:)** A text box containing "300".
- (Select:)** An empty text box.

At the bottom left, there is a checkbox labeled "Sort column names" which is currently unchecked. At the bottom, there are five buttons: "OK", "Cancel", "Delete", "Transform...", and "Help".

Acceptance Chart – Analysis Options

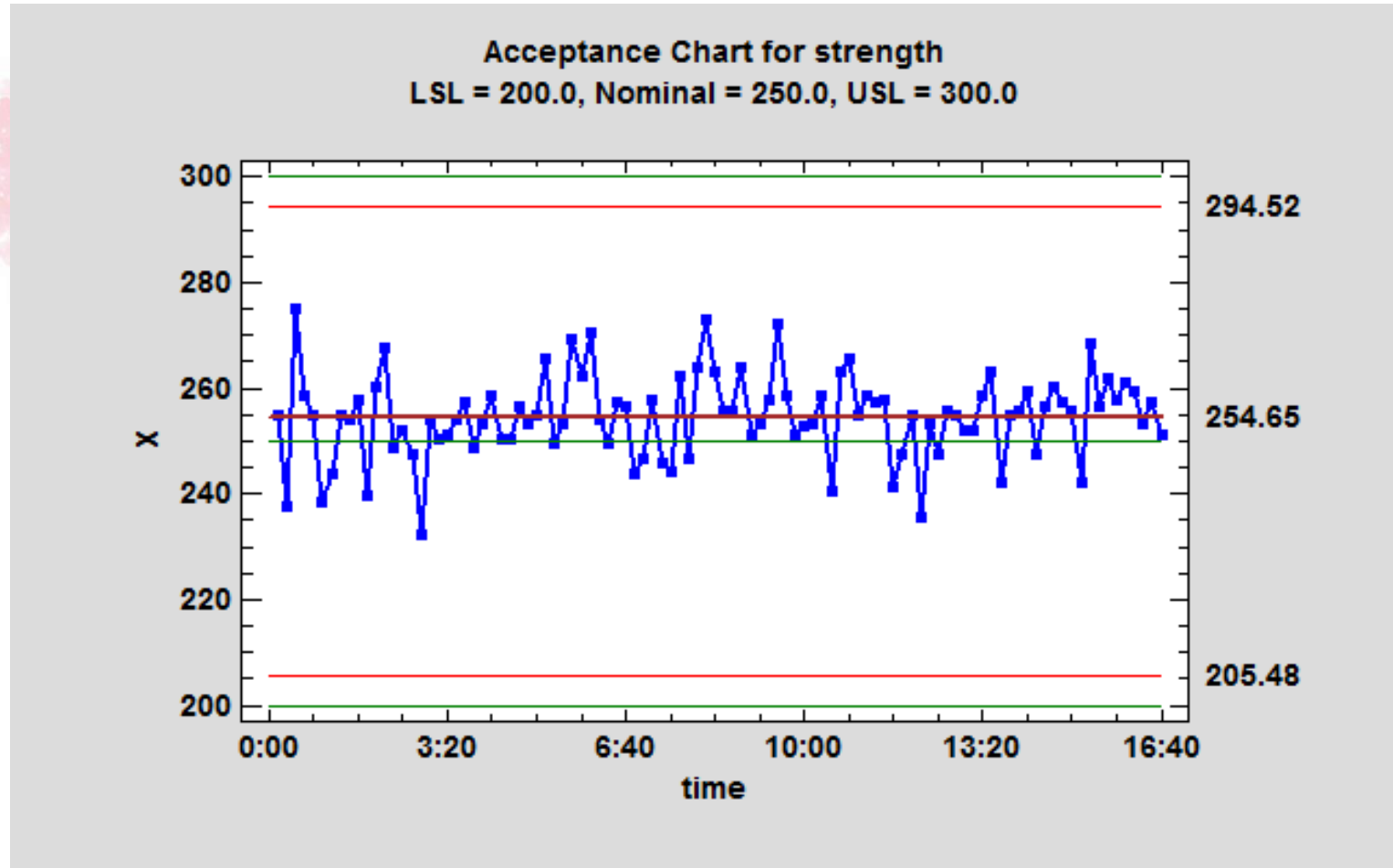


The dialog box is titled "Acceptance Individuals Chart Options" and contains the following sections:

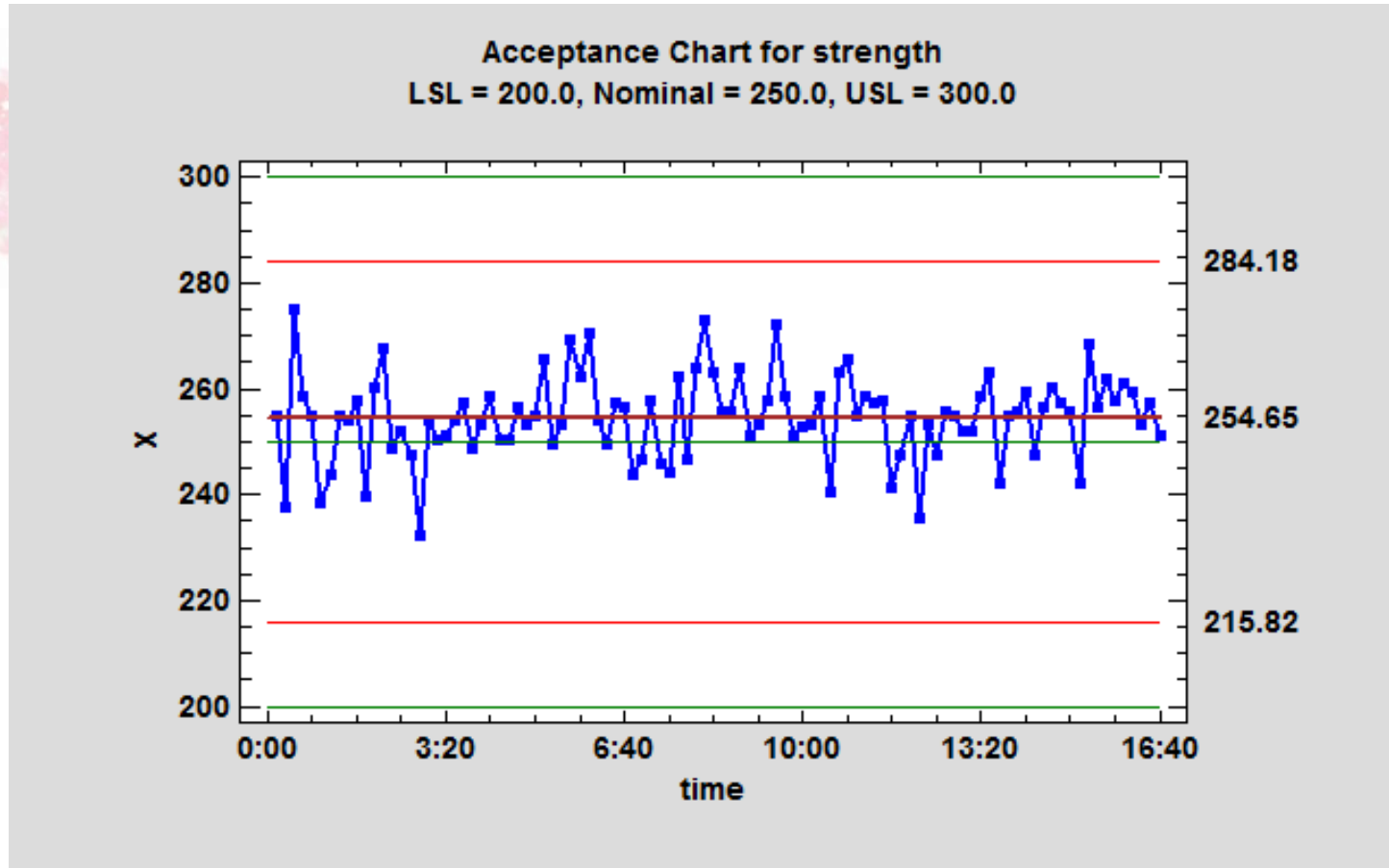
- Type of Study:** Radio buttons for "Initial Study" (selected) and "Control to Standard".
- Specify:** Radio buttons for "Sigma Multiple" (selected) and "Beta Risk".
- Control to Standard:** Text boxes for "Mean:" (254.653) and "Std. Dev.:" (7.62501).
- X Control Limits:** Text boxes for "Fraction Nonconforming:" (0.0001) and "Sigma multiple:" (3.0).
- MR(2) Control Limits:** Checkboxes for "Upper:" (checked) and "Lower:" (checked), each with a text box for "Sigma" (3.0 and -3.0 respectively).

Buttons at the bottom: OK, Cancel, Exclude..., Transform..., Help.

Based on Sigma Multiple



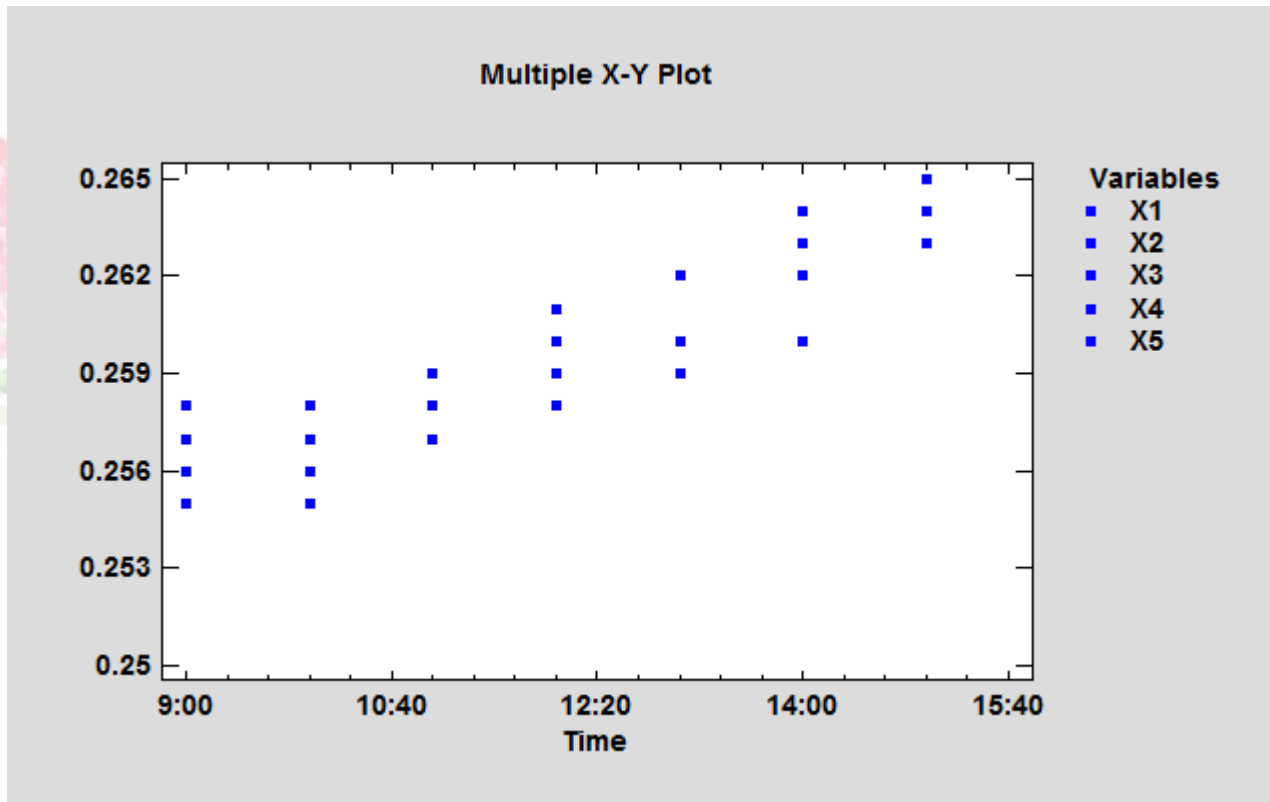
Based on Beta Risk



Toolwear Control Charts

- Used for data that do not have a constant mean, such as measurements affected by wear on a tool.
- Used to determine whether mean is changing at a constant or expected rate.
- Can also add specification limits to help determine when tool should be changed.

Sample Data (from Duncan)



- Specification: 0.255 to 0.265 inches

Toolwear Chart Model

Let $X_{t,j}$ be measurement on j^{th} sample made at time t . Then

$$X_{t,j} = \alpha + \beta t + \varepsilon_{t,j}$$

where $\mu = \alpha + \beta t$ is the process mean and the deviations ε_t are assumed to be $\text{NID}(0, \sigma^2)$.

Toolwear Chart – Data Input

The screenshot shows the 'Toolwear Chart' dialog box. On the left, a list of variables includes 'Time' (highlighted) and 'X1' through 'X5'. The 'Data' section on the right has 'Observations' selected, with a list of 'X1' through 'X5' in a scrollable box. Below this are 'Subgroup Statistics' options for Means, Ranges, and Sizes, each with a selection icon and an empty input field. The 'Date/Time/Labels or Size:' section has a selection icon and a text field containing 'Time'. At the bottom, there are input fields for (LSL:) with value 0.255, (Nominal:) with an empty field, and (USL:) with value 0.265. A (Select:) field is also present. A 'Sort column names' checkbox is unchecked. At the bottom are buttons for 'OK', 'Cancel', 'Delete', 'Transform...', and 'Help'.

Analysis Options

Toolwear Chart Options

Type of Study

- Initial Study
- Control to Standard

Estimate Sigma From

- R or S Chart
- Model MSE

Control to Standard

Intercept:

Slope:

Sigma:

Include Model Estimation Error

X-bar Control Limits

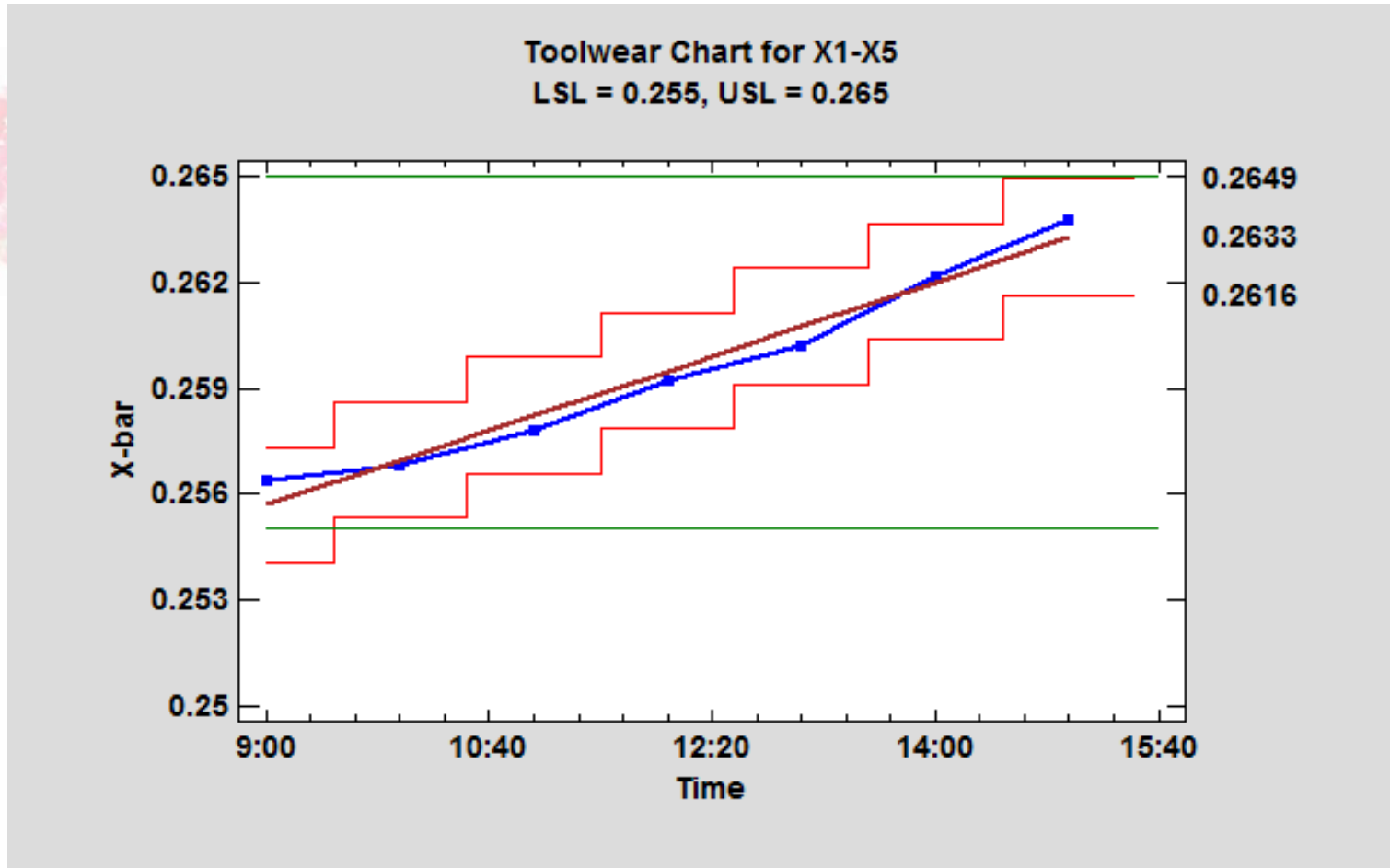
- Upper: Sigma
- Lower: Sigma

R or S Control Limits

- Upper: Sigma
- Lower: Sigma

OK Cancel Exclude... Transform... Help

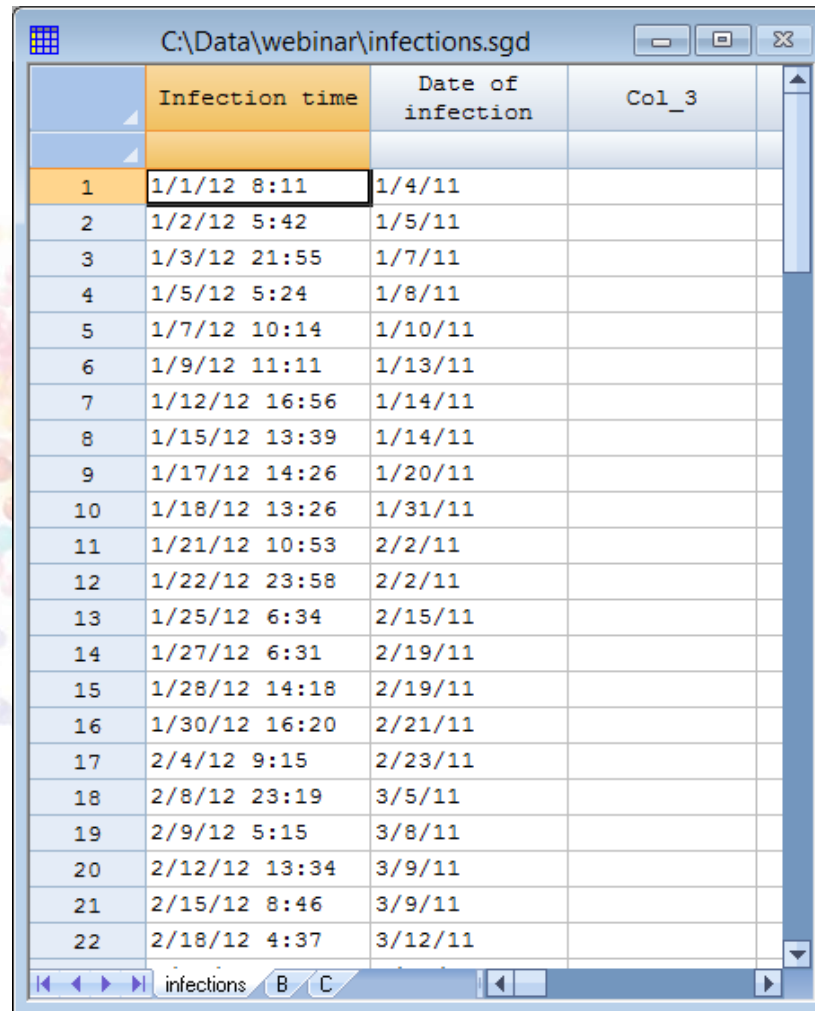
Toolwear Chart



Charts for Rare Events

- Used for monitoring occurrence of rare events
 - Hospital infections
 - Factory or mining accidents
 - Unexpected shutdowns
- Two primary charts
 - t chart: for events recorded in continuous time
 - g chart: for events recorded in discrete time

Examples

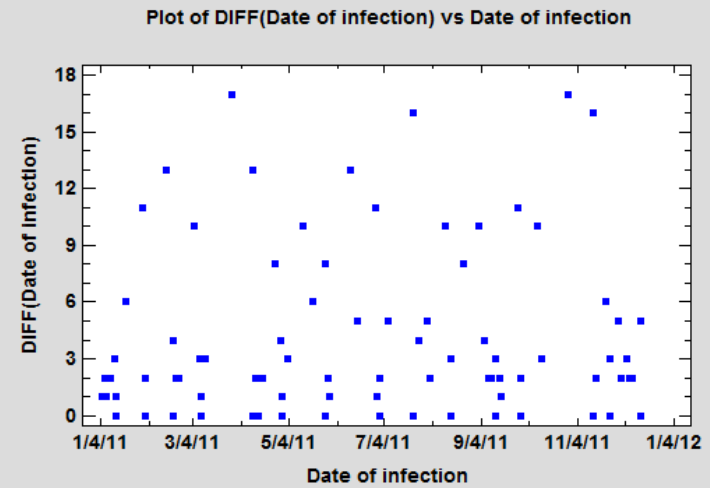
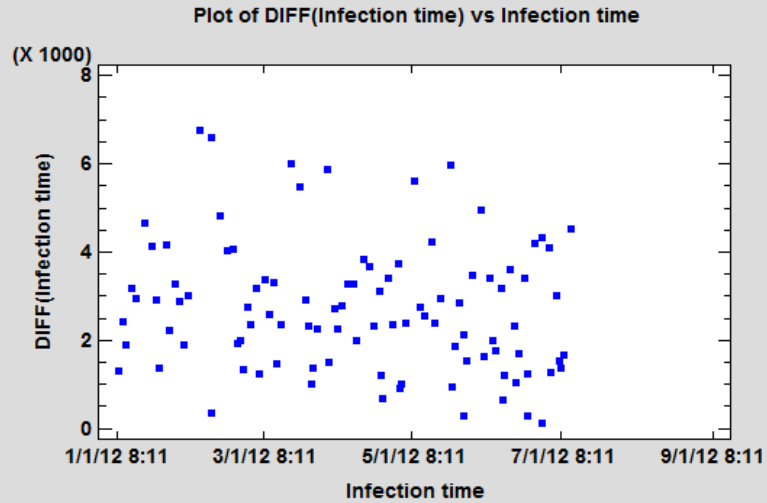


	Infection time	Date of infection	Col_3
1	1/1/12 8:11	1/4/11	
2	1/2/12 5:42	1/5/11	
3	1/3/12 21:55	1/7/11	
4	1/5/12 5:24	1/8/11	
5	1/7/12 10:14	1/10/11	
6	1/9/12 11:11	1/13/11	
7	1/12/12 16:56	1/14/11	
8	1/15/12 13:39	1/14/11	
9	1/17/12 14:26	1/20/11	
10	1/18/12 13:26	1/31/11	
11	1/21/12 10:53	2/2/11	
12	1/22/12 23:58	2/2/11	
13	1/25/12 6:34	2/15/11	
14	1/27/12 6:31	2/19/11	
15	1/28/12 14:18	2/19/11	
16	1/30/12 16:20	2/21/11	
17	2/4/12 9:15	2/23/11	
18	2/8/12 23:19	3/5/11	
19	2/9/12 5:15	3/8/11	
20	2/12/12 13:34	3/9/11	
21	2/15/12 8:46	3/9/11	
22	2/18/12 4:37	3/12/11	

Time Between Events

- When events are rare, standard charts such as a C chart will have many 0's.
- Better approach than plotting counts is to plot the times between consecutive events.
- For a continuous time process, the interevent times are modeled by a Weibull distribution.
- For a discrete time process, the interevent times are modeled by a geometric distribution.

Scatterplots



t Chart Data Input

t Chart

Infection time
Date of infection

Data:
▶ Infection time

Type of data:
 Time of occurrence
 Time between occurrences

(Labels:)
▶

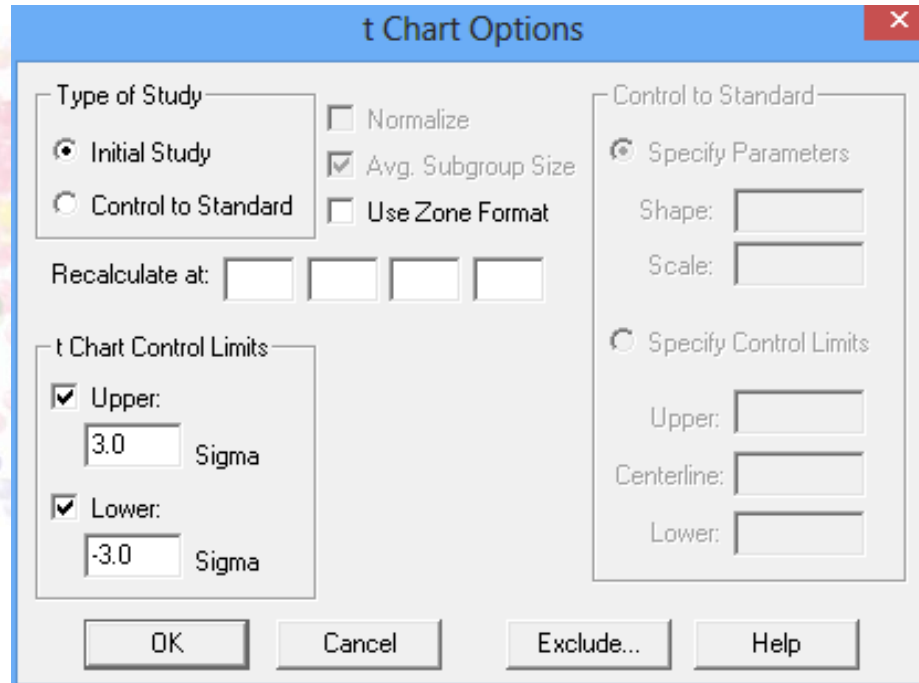
(LSL:) ▶ (Nominal:) ▶ (USL:) ▶

(Select:)
▶

Sort column names

OK Cancel Delete Transform... Help

t Chart Analysis Options

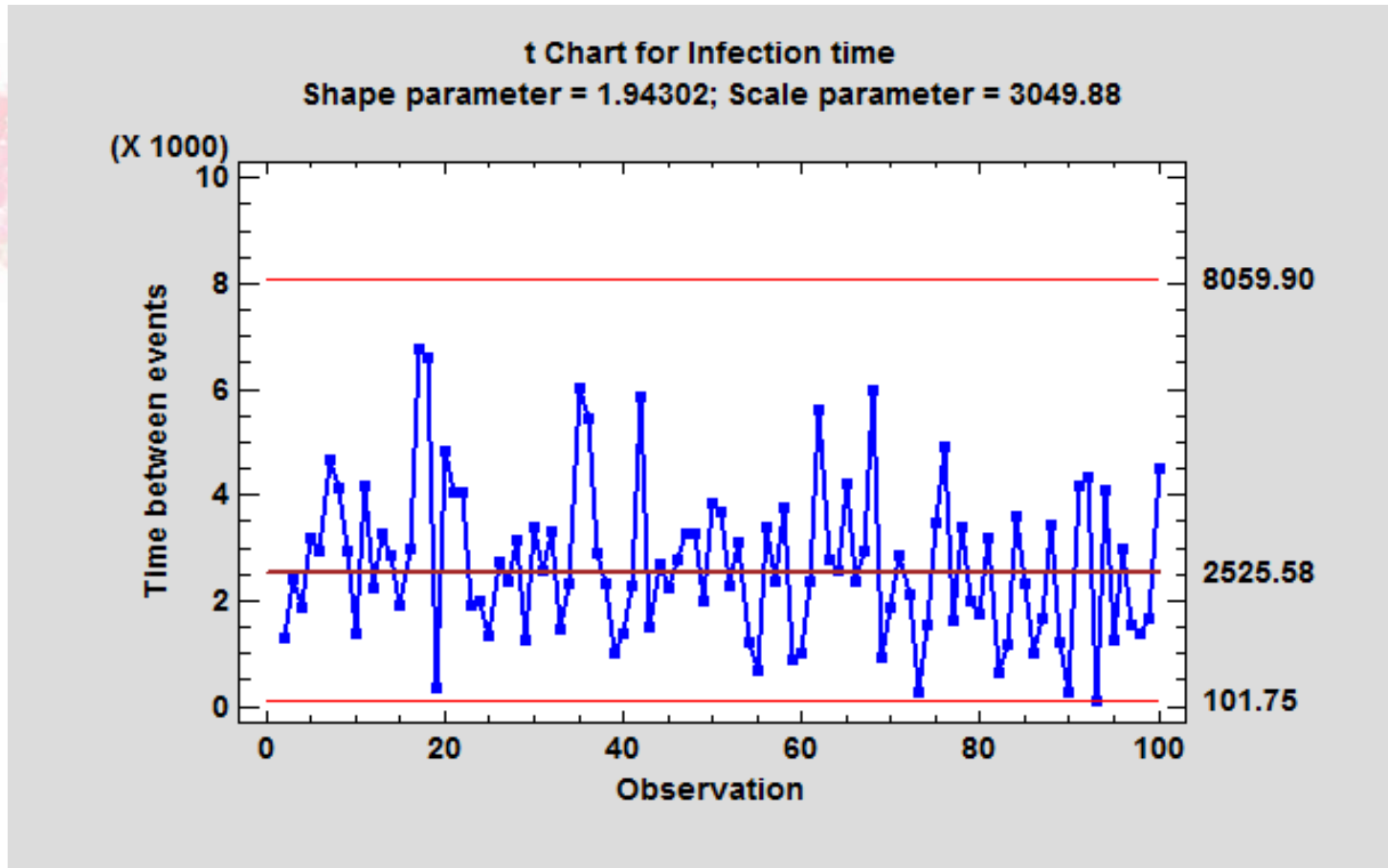


The image shows a software dialog box titled "t Chart Options". The dialog is divided into several sections:

- Type of Study:** Contains two radio buttons: "Initial Study" (selected) and "Control to Standard".
- Control to Standard:** Contains two radio buttons: "Specify Parameters" (selected) and "Specify Control Limits".
- Normalize:** A checkbox that is unchecked.
- Avg. Subgroup Size:** A checkbox that is checked.
- Use Zone Format:** A checkbox that is unchecked.
- Recalculate at:** Four empty input boxes.
- t Chart Control Limits:** Contains two checked checkboxes: "Upper:" and "Lower:". The "Upper:" checkbox has a text box containing "3.0" and the word "Sigma" next to it. The "Lower:" checkbox has a text box containing "-3.0" and the word "Sigma" next to it.
- Specify Parameters:** Contains two text boxes labeled "Shape:" and "Scale:".
- Specify Control Limits:** Contains three text boxes labeled "Upper:", "Centerline:", and "Lower:".

At the bottom of the dialog are four buttons: "OK", "Cancel", "Exclude...", and "Help".

t Chart

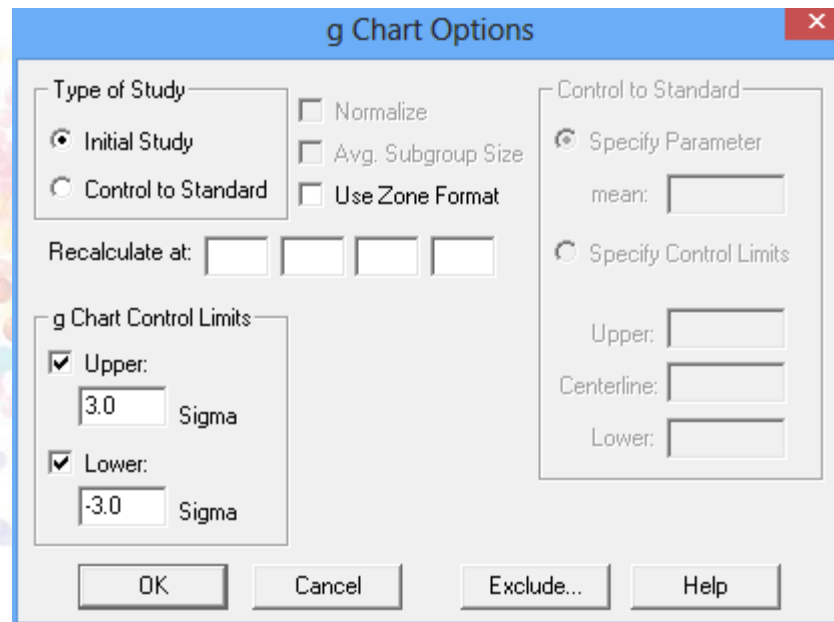


g Chart Data Input

The screenshot shows the 'g Chart' dialog box with the following configuration:

- Data:** Date of infection
- Type of data:** Time of occurrence, Time between occurrences
- (Labels:)** [Empty text box]
- (LSL:)** [Empty text box]
- (Nominal:)** [Empty text box]
- (USL:)** [Empty text box]
- (Select:)** [Empty text box]
- Sort column names
- Buttons:** OK, Cancel, Delete, Transform..., Help

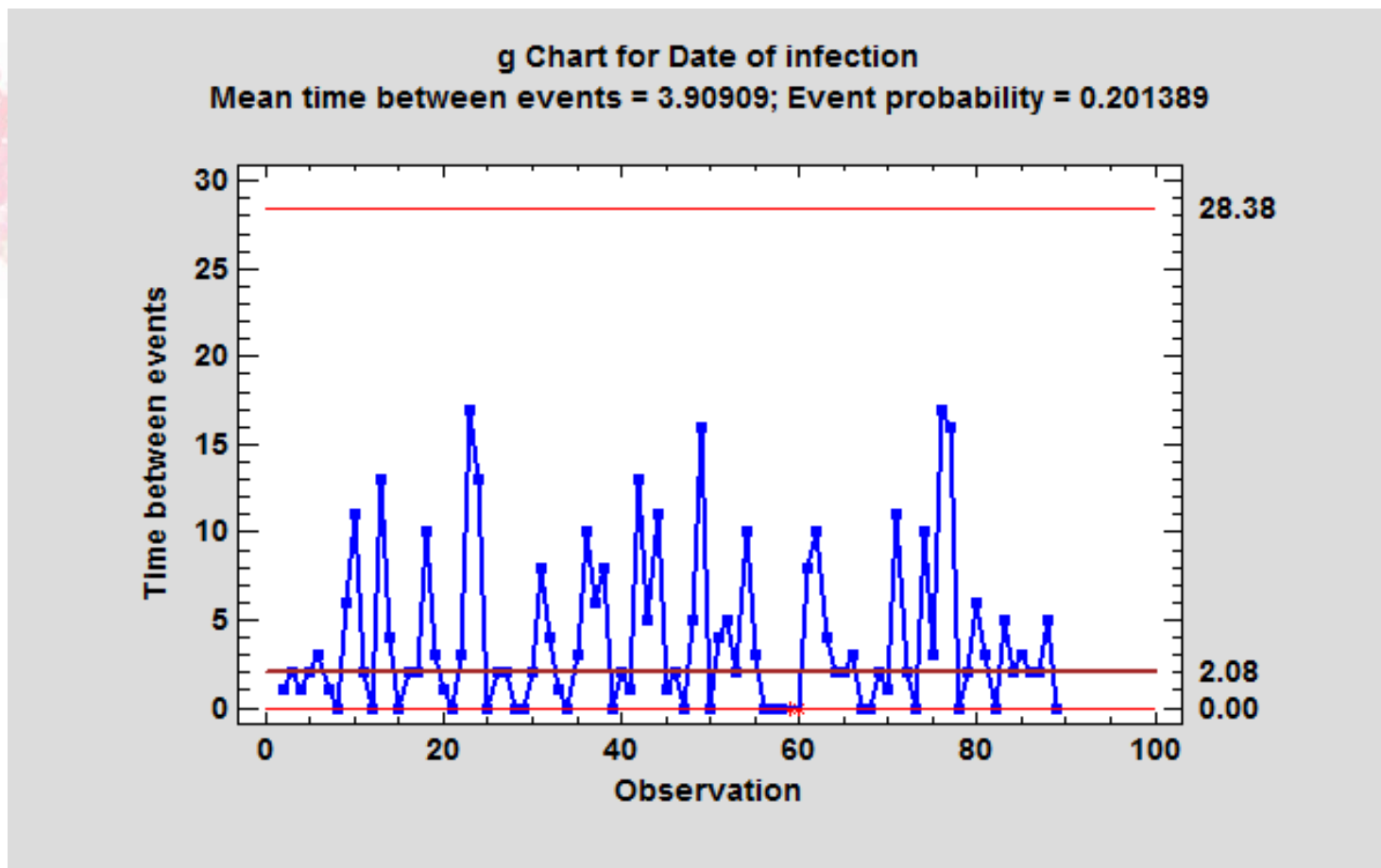
g Chart Analysis Options



The image shows a software dialog box titled "g Chart Options". The dialog is divided into several sections:

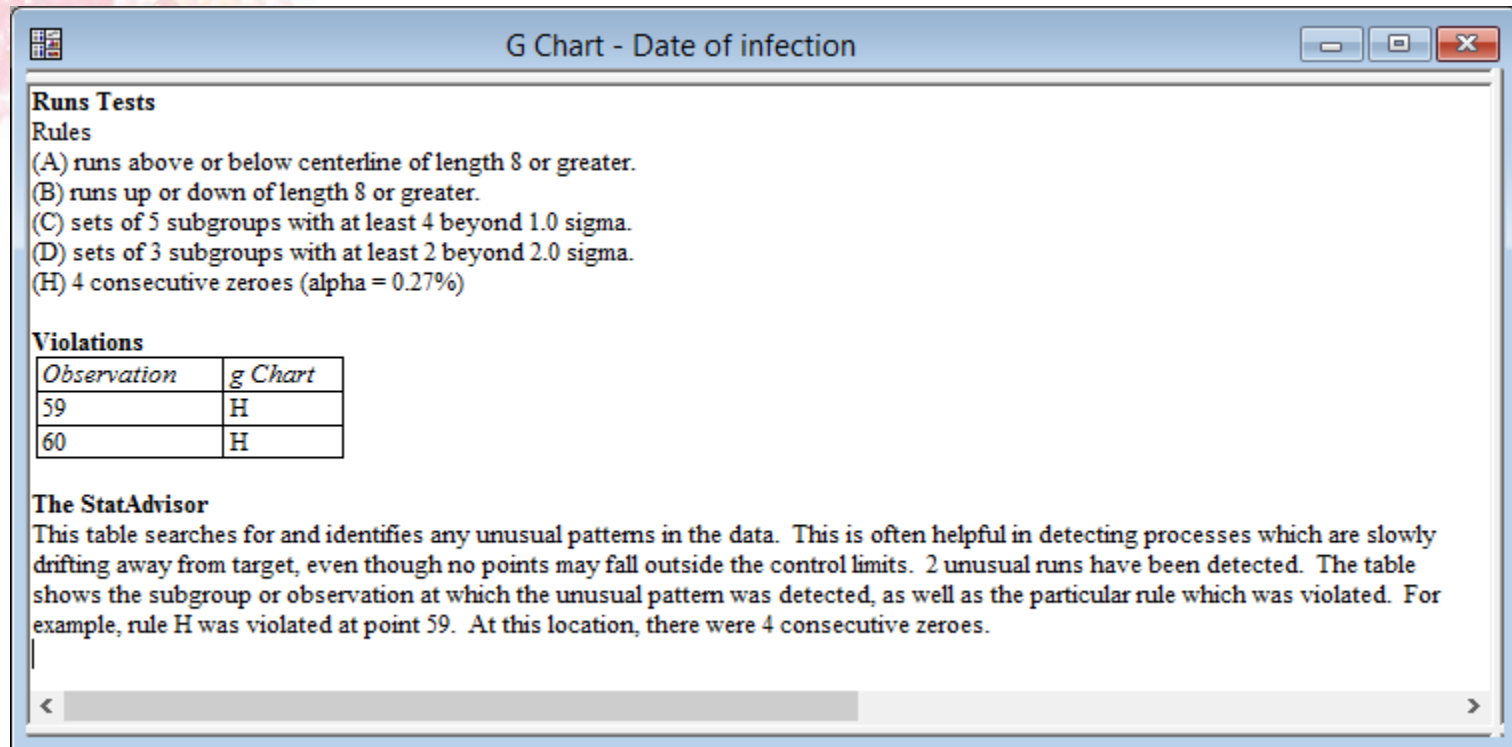
- Type of Study:** Contains two radio buttons: "Initial Study" (selected) and "Control to Standard".
- Control to Standard:** Contains three checkboxes: "Normalize", "Avg. Subgroup Size", and "Use Zone Format", all of which are currently unchecked.
- Recalculate at:** A row of four empty input boxes.
- g Chart Control Limits:** Contains two checked checkboxes: "Upper:" and "Lower:". The "Upper:" checkbox is followed by a text box containing "3.0" and the word "Sigma". The "Lower:" checkbox is followed by a text box containing "-3.0" and the word "Sigma".
- Control to Standard (right side):** Contains two radio buttons: "Specify Parameter" (selected) and "Specify Control Limits". Below "Specify Parameter" are three text boxes labeled "mean:", "Upper:", and "Centerline:". Below "Specify Control Limits" is one text box labeled "Lower:".
- Buttons:** At the bottom are four buttons: "OK", "Cancel", "Exclude...", and "Help".

g Chart



G Chart – Special Run Test

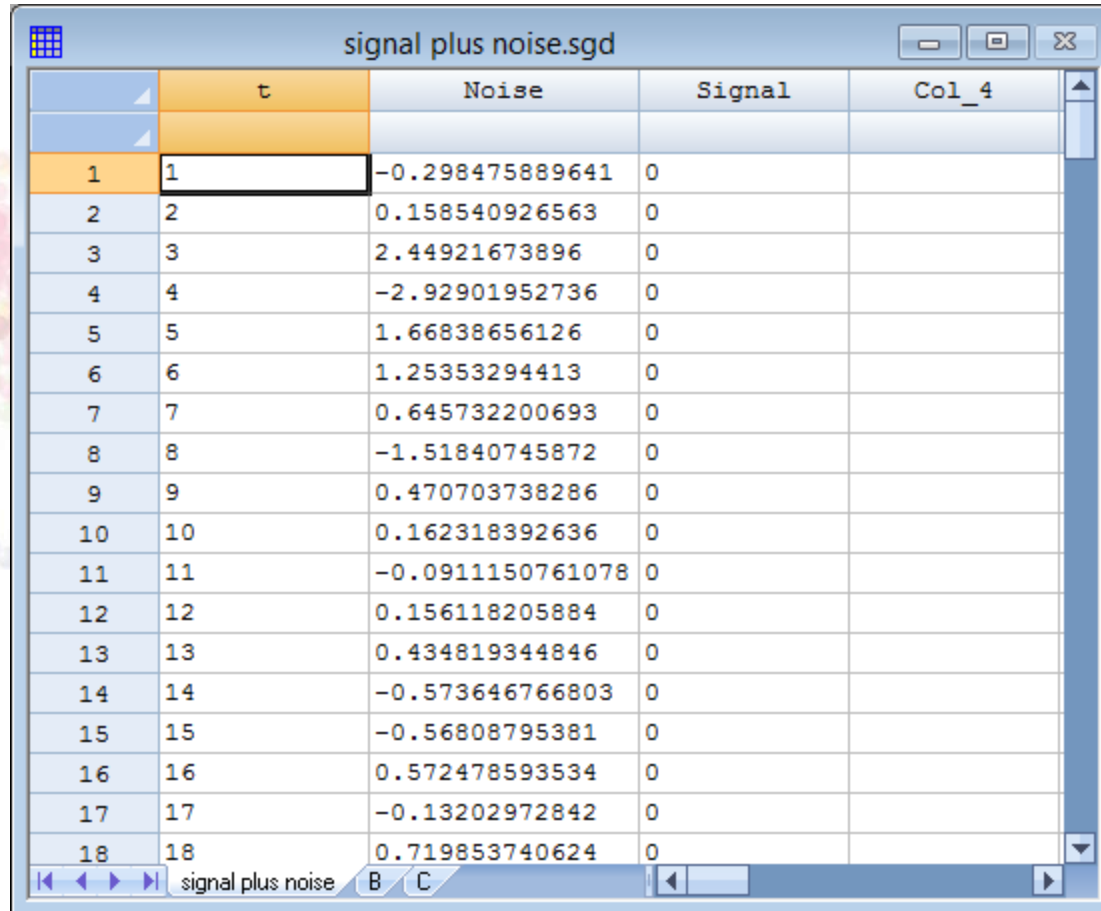
- Extra run test for consecutive zeroes:



Cuscore Charts

- Designed to detect specific types of disturbances:
 - Spike
 - Ramp
 - Bump
 - Step change
 - Exponential increase
 - Sine wave
- Automatically selects the best type of control chart to detect that pattern

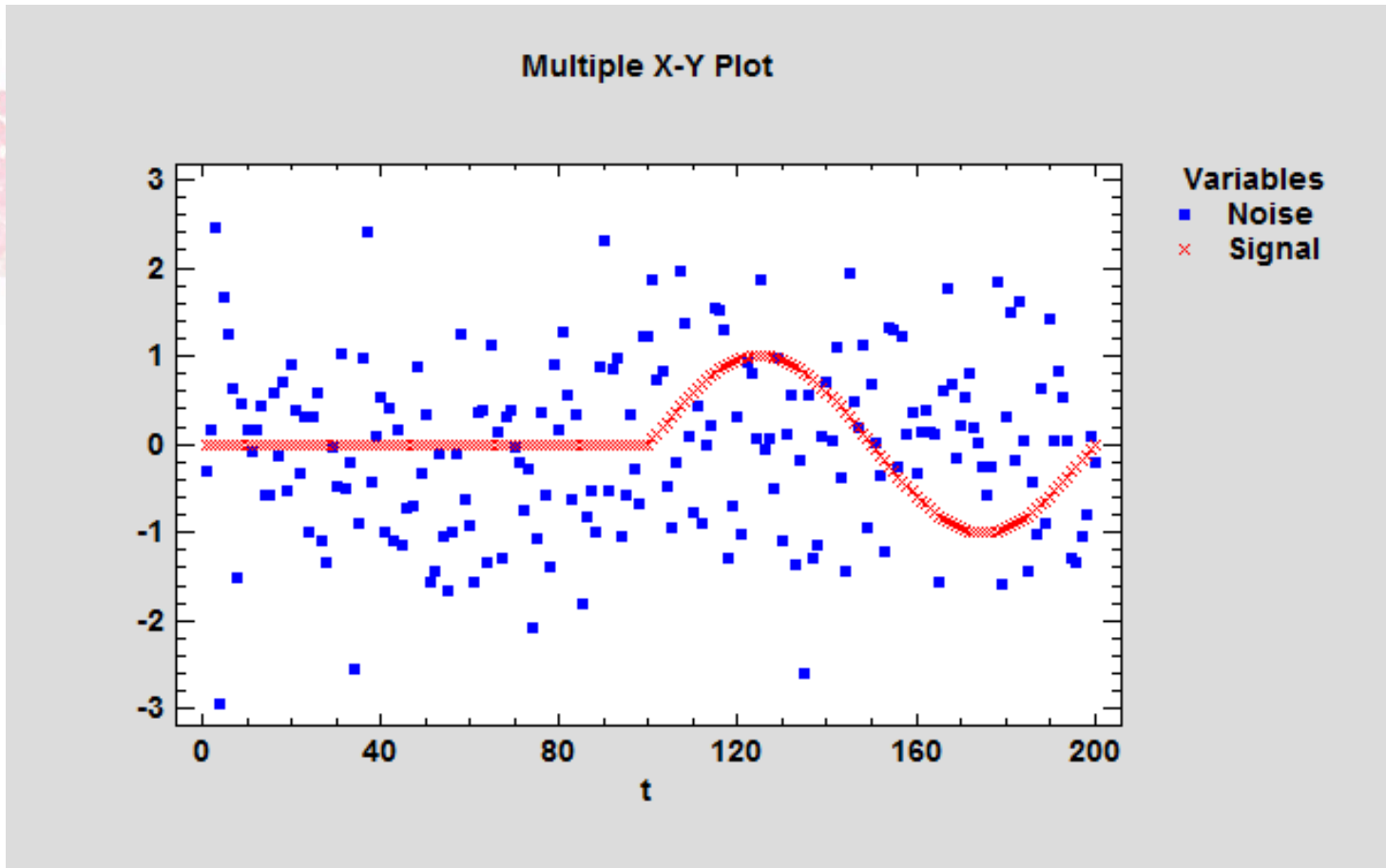
Example: sine wave plus noise



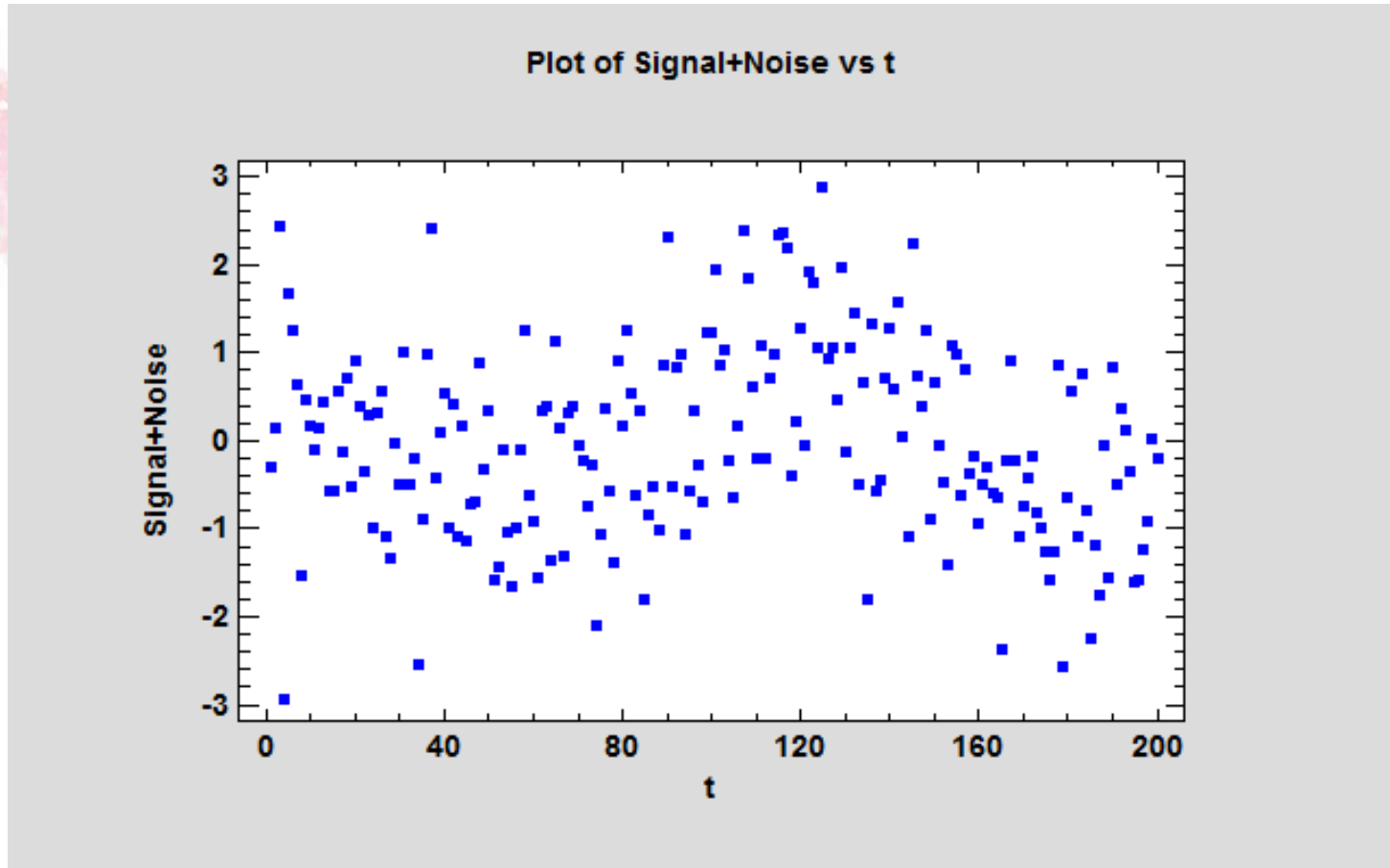
The screenshot shows a data table window titled "signal plus noise.sgd". The table contains 18 rows of data. The columns are labeled "t", "Noise", "Signal", and "Col_4". The "Signal" column is consistently zero for all rows. The "Noise" column contains random values ranging from approximately -2.9 to 0.7. The "t" column contains integers from 1 to 18. The "Col_4" column is empty.

	t	Noise	Signal	Col_4
1	1	-0.298475889641	0	
2	2	0.158540926563	0	
3	3	2.44921673896	0	
4	4	-2.92901952736	0	
5	5	1.66838656126	0	
6	6	1.25353294413	0	
7	7	0.645732200693	0	
8	8	-1.51840745872	0	
9	9	0.470703738286	0	
10	10	0.162318392636	0	
11	11	-0.0911150761078	0	
12	12	0.156118205884	0	
13	13	0.434819344846	0	
14	14	-0.573646766803	0	
15	15	-0.56808795381	0	
16	16	0.572478593534	0	
17	17	-0.13202972842	0	
18	18	0.719853740624	0	

Signal and Noise



Signal Plus Noise



Q Score Statistic

- Cuscore chart plots a cumulative “Q score” statistic defined by:

$$Q_t = \sum_{i=1}^t \hat{a}_i r_i$$

where the r_i values come from a “detector” series specifically designed to detect the presence of the expected disturbance and the a_i values are the residuals from an ARIMA model.

Cuscore Chart – Data Input

Cuscore Individuals Charts

t
Noise
Signal

Observations:
▶ Signal+Noise

(Date/Time/Labels:)
▶ t

(LSL:) (Nominal:) (USL:)
▶ ▶ ▶

(Select:)
▶

Sort column names

OK Cancel Delete Transform... Help

Cuscore Chart – Analysis Options

Cuscore Chart Options

Type of Study
 Initial Study
 Control to Standard

Estimate sigma from
 MR(2) Chart
 Model MSE

Control to Standard
Mean:
Sigma:

CuScore Control Limits
 Upper: Sigma
 Lower: Sigma

MR(2) Control Limits
 Upper: Sigma
 Lower: Sigma

AR(1):
AR(2):
AR(3):
AR(4):

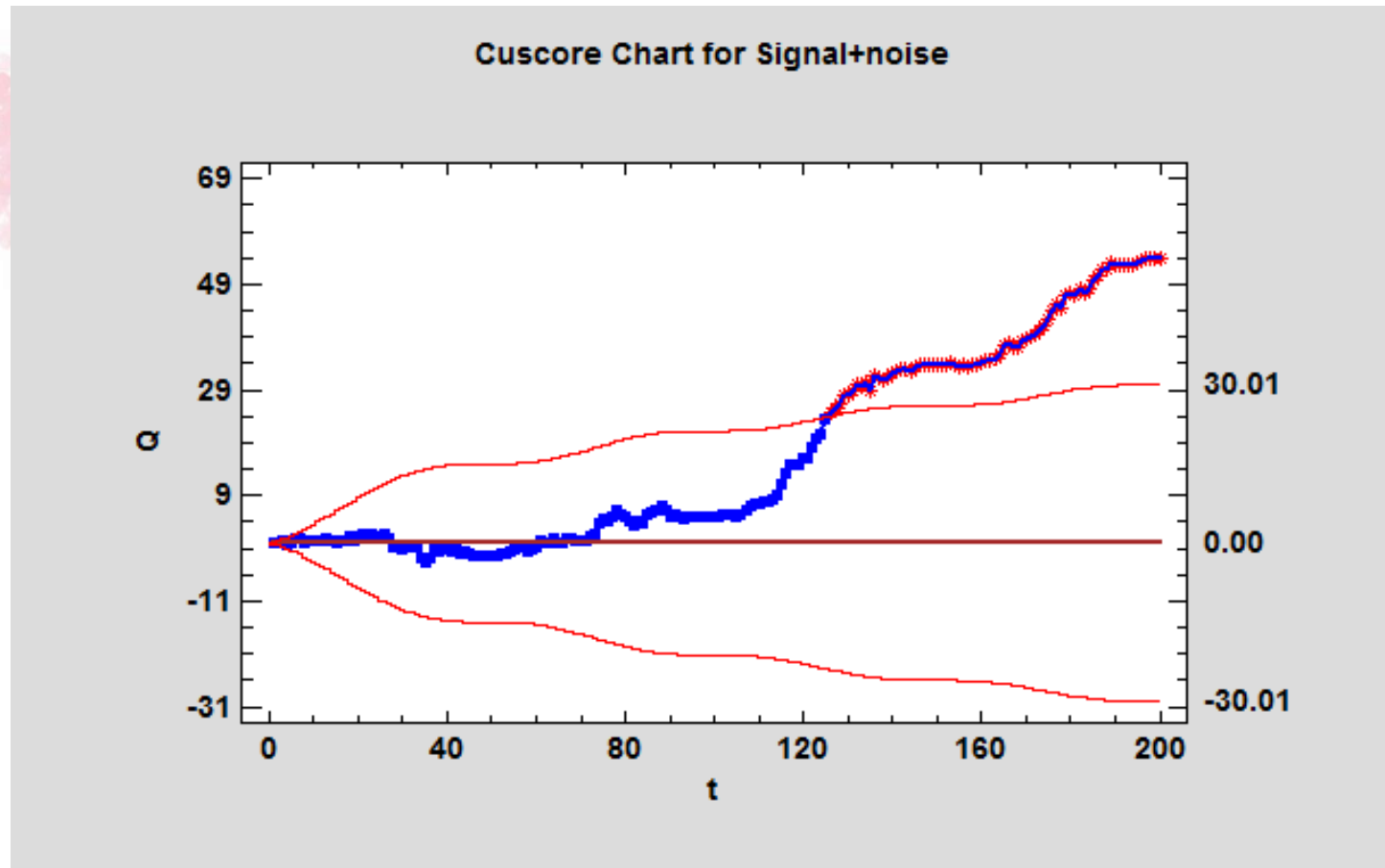
MA(1):
MA(2):
MA(3):
MA(4):

Noise Model
AR: 0 1 2 3 4
I: 0 1 2 3 Constant
MA: 0 1 2 3 4

Signal to Detect
 Spike Ramp Bump of Duration
 Step Change Exponential Increase of
 Sine Wave with Period Phase
 Custom

OK Cancel Transform... Help

Cuscore Chart



References

- Introduction to Statistical Quality Control (7th edition) by Douglas Montgomery (2012)
- Quality Control and Industrial Statistics (5th edition) by Acheson Duncan (1986)
- Statistical Control by Monitoring and Adjustment (2nd edition) by George Box, Alberto Luceño, and Maria del Carmen Paniagua-Quinones (2009)

More Information

Go to www.statgraphics.com

- Click on “Learn” and then “Instructional Videos”.

Go to www.youtube.com

- Search for “Statgraphics control charts”.