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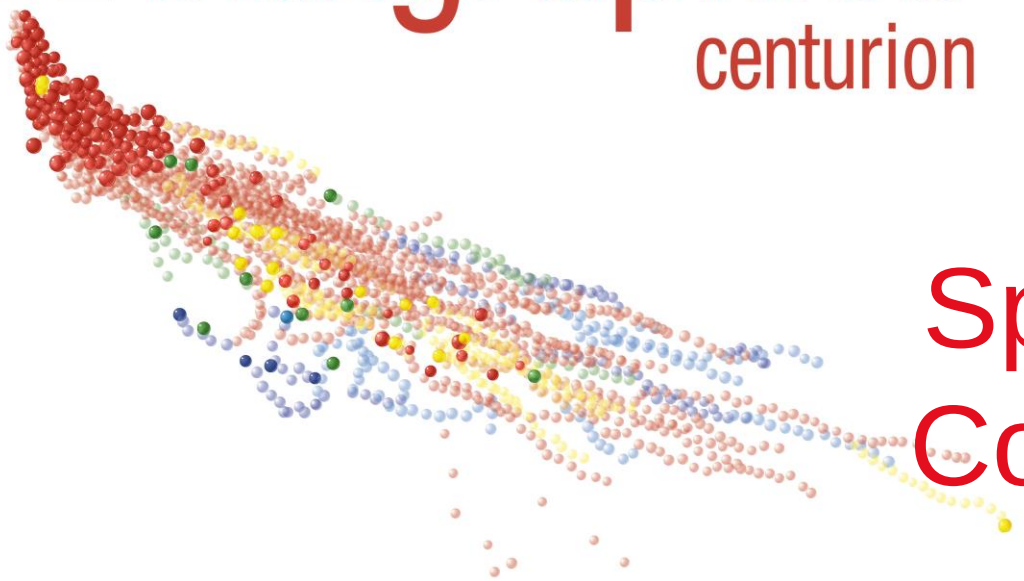


Special Purpose Control Charts in Statgraphics Centurion

Presented by Dr. Neil W. Polhemus

Control Charts

- A control chart:
  - Plots a statistic versus time.
  - Contains control limits to detect unusual values.

- Used for 2 primary purposes:
  - To determine whether a process is in a state of statistical control. (Phase 1 or initial study)
  - To monitor that process to detect deviations from pre-established parameters. (Phase 2 or control to standard)

Common Control Charts

- For variable data:
  - X-bar and R, X-bar and S, X and MR
  - Median and MR
  - Moving average and EWMA
  - Cumulative sum charts
- For attribute data:
  - P or NP for binomial proportions and counts
  - C or U for Poisson rates and counts
  - P-prime and U-prime charts for overdispersed data

Special Purpose Control Charts

1. ARIMA charts
2. Acceptance charts
3. Toolwear charts
4. Charts for rare events
5. Cuscore charts

Usual Model for Variable Data

Let X_t be measurement made at time t. Then

$$X_t = \mu + \varepsilon_t$$

where μ is the process mean and the deviations ε_t are assumed to be NID(0,σ^2).

Assumptions:

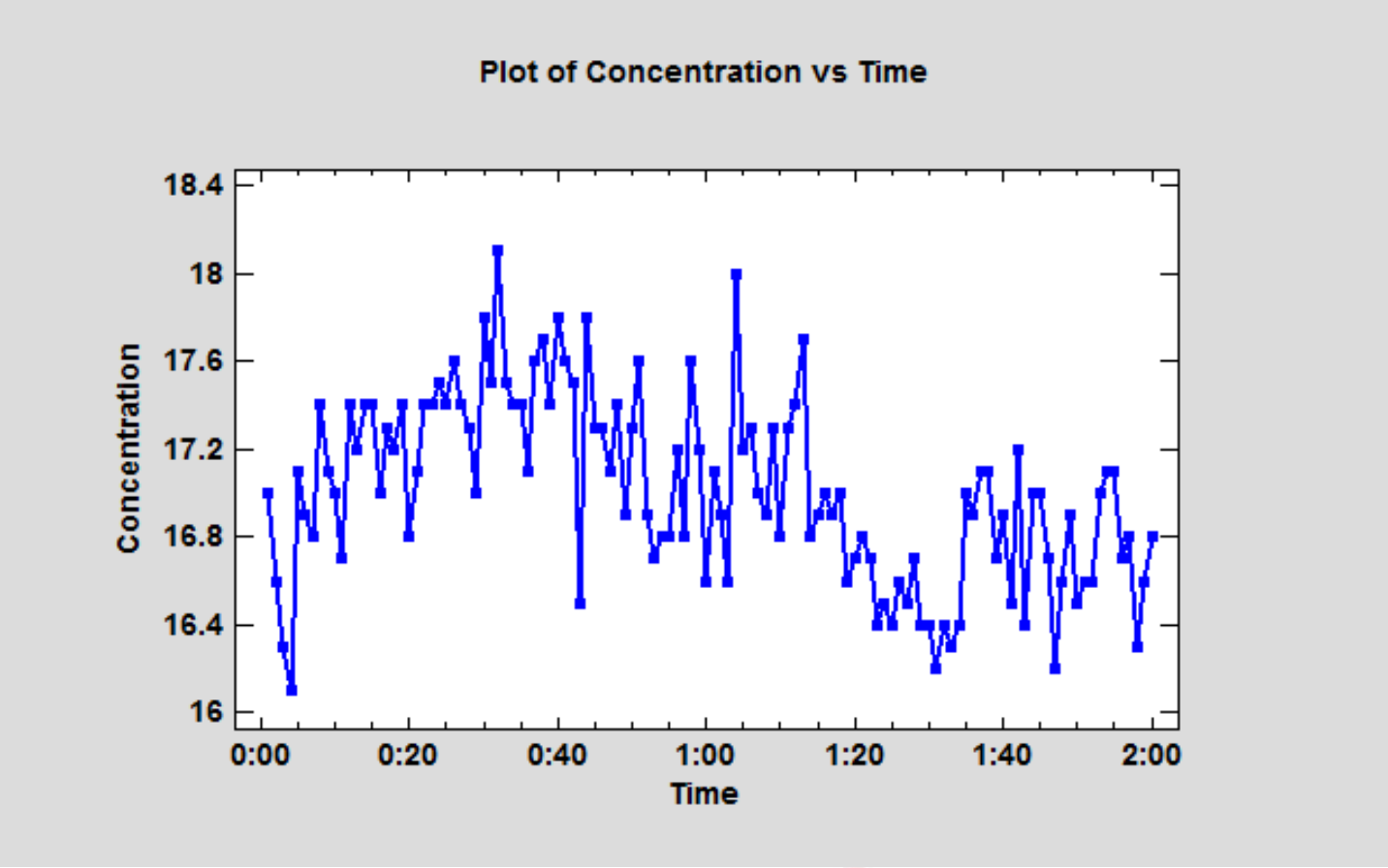
- Stationarity of mean and variance.
- Independence of successive measurements.
- Normal distribution for deviations from mean.

ARIMA Control Charts

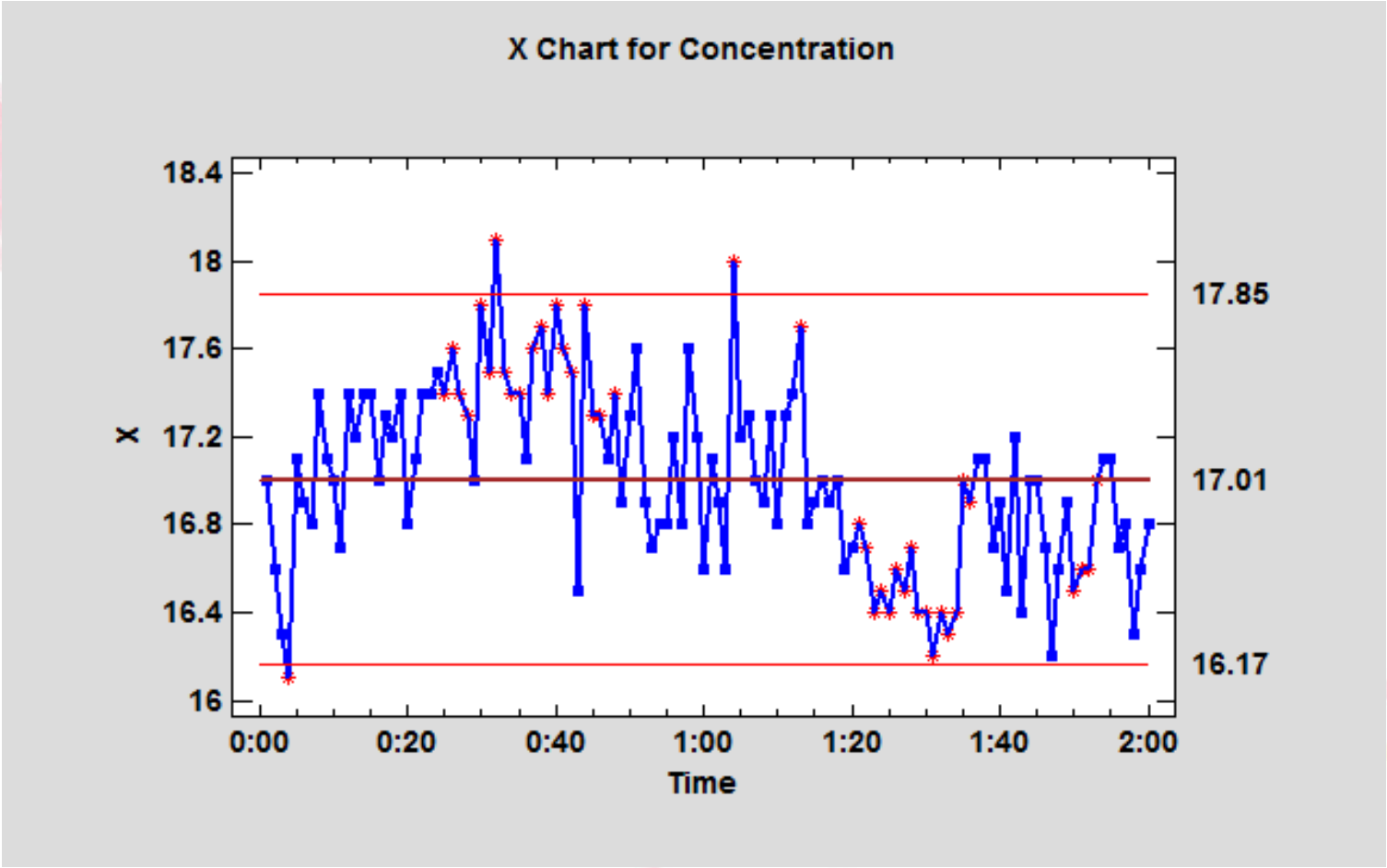
- Used for data that are serially correlated (successive observations are not independent).

- Very common when measurements are taken close together in time from a continuous process.

Example: Chemical concentration



Standard Phase 1 X Chart



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ARIMA Charts Model

- Relax the assumption that successive observations are independent.

- Instead, assume that the process can be described by an autoregressive moving average model.

Autoregressive Models

- AR(1)

$$X_t = \mu + \phi_1(X_{t-1} - \mu) + \varepsilon_t$$

- AR(2)

$$X_t = \mu + \phi_1(X_{t-1} - \mu) + \phi_2(X_{t-2} - \mu) + \varepsilon_t$$

Moving Average Models

- MA(1)

$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

- MA(2)

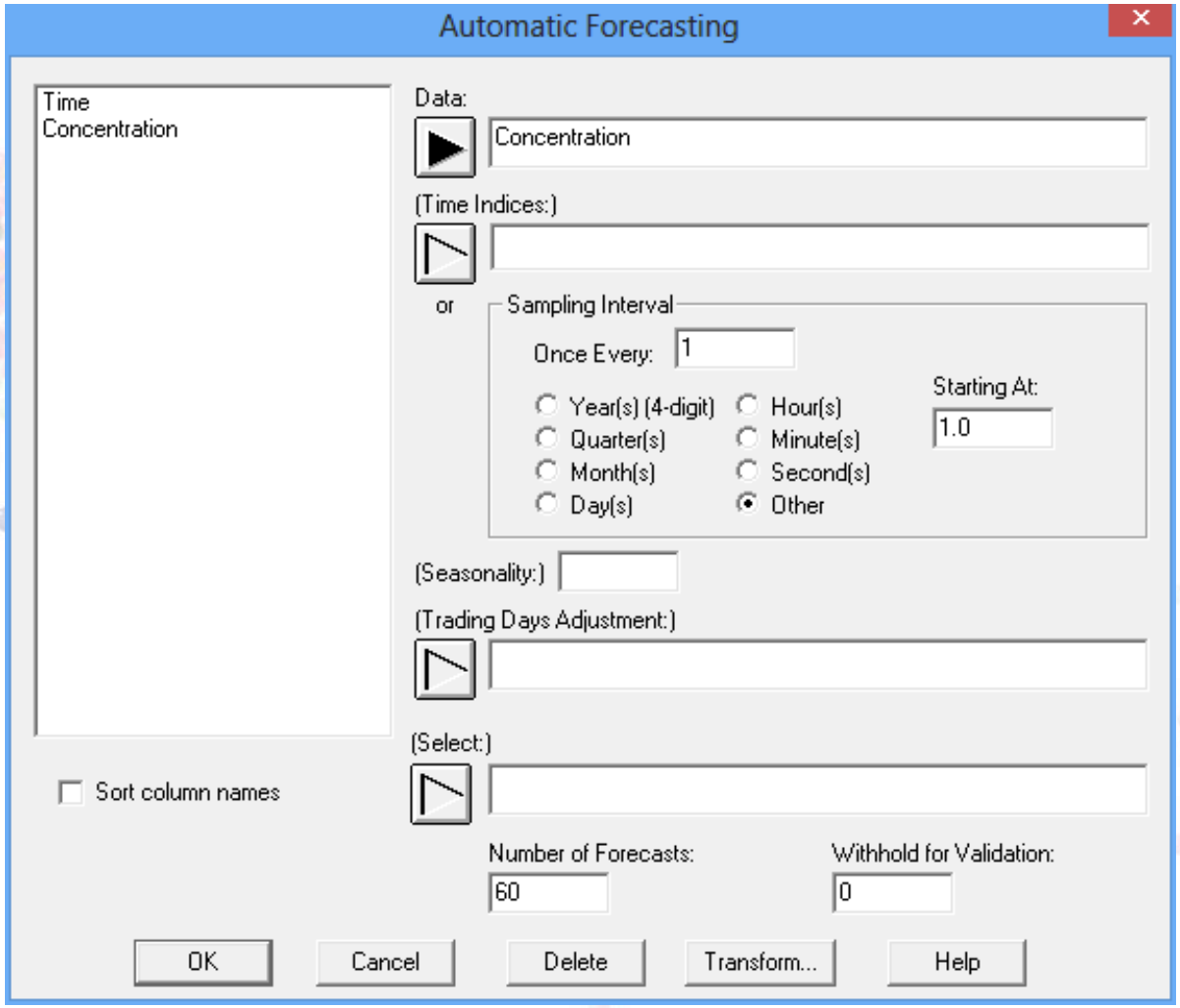
$$X_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

ARMA Models

- ARMA(1,1)

$$X_t = \mu + \phi_1 (X_{t-1} - \mu) + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

Selecting the Best ARIMA Model



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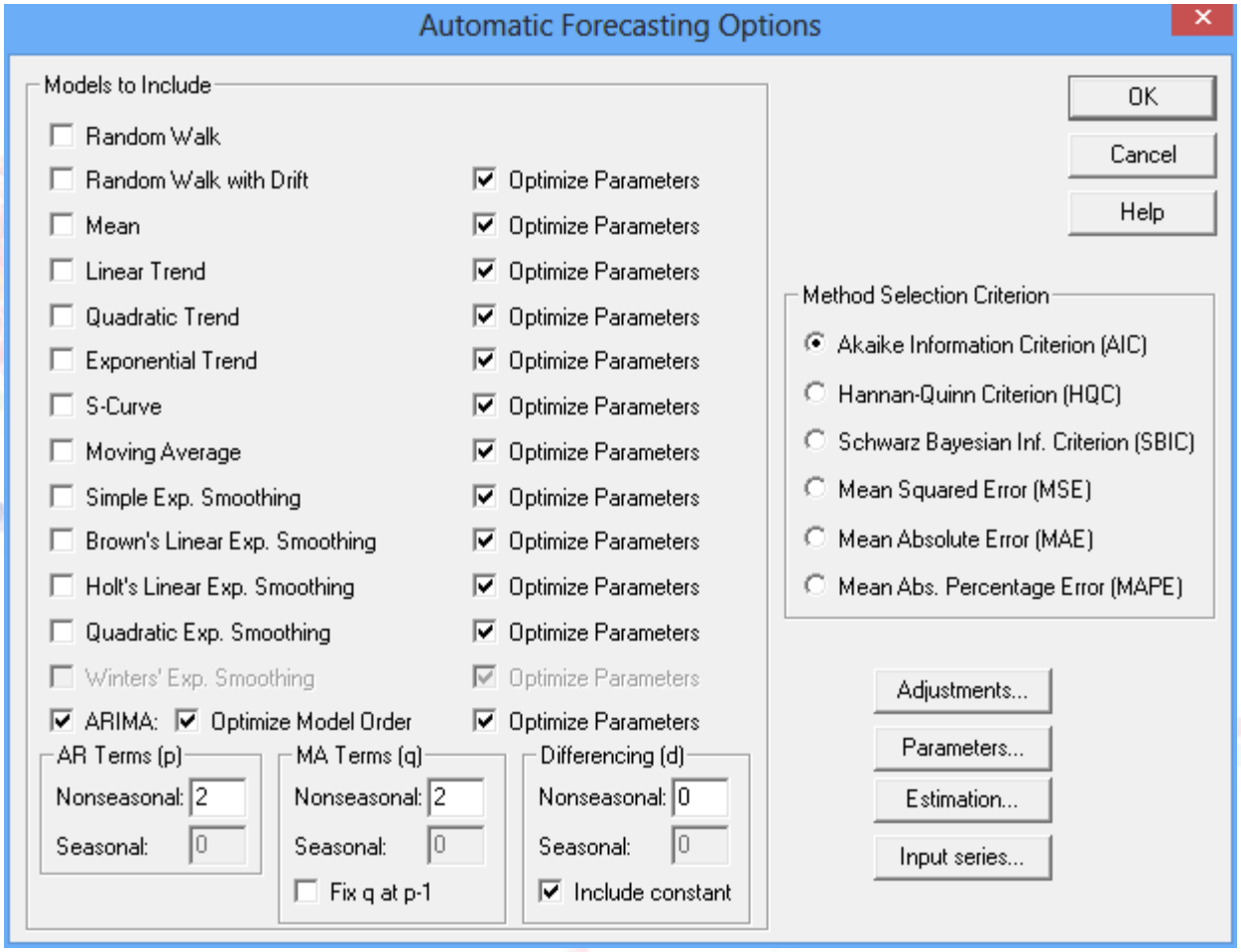
Selecting the Best ARIMA Model



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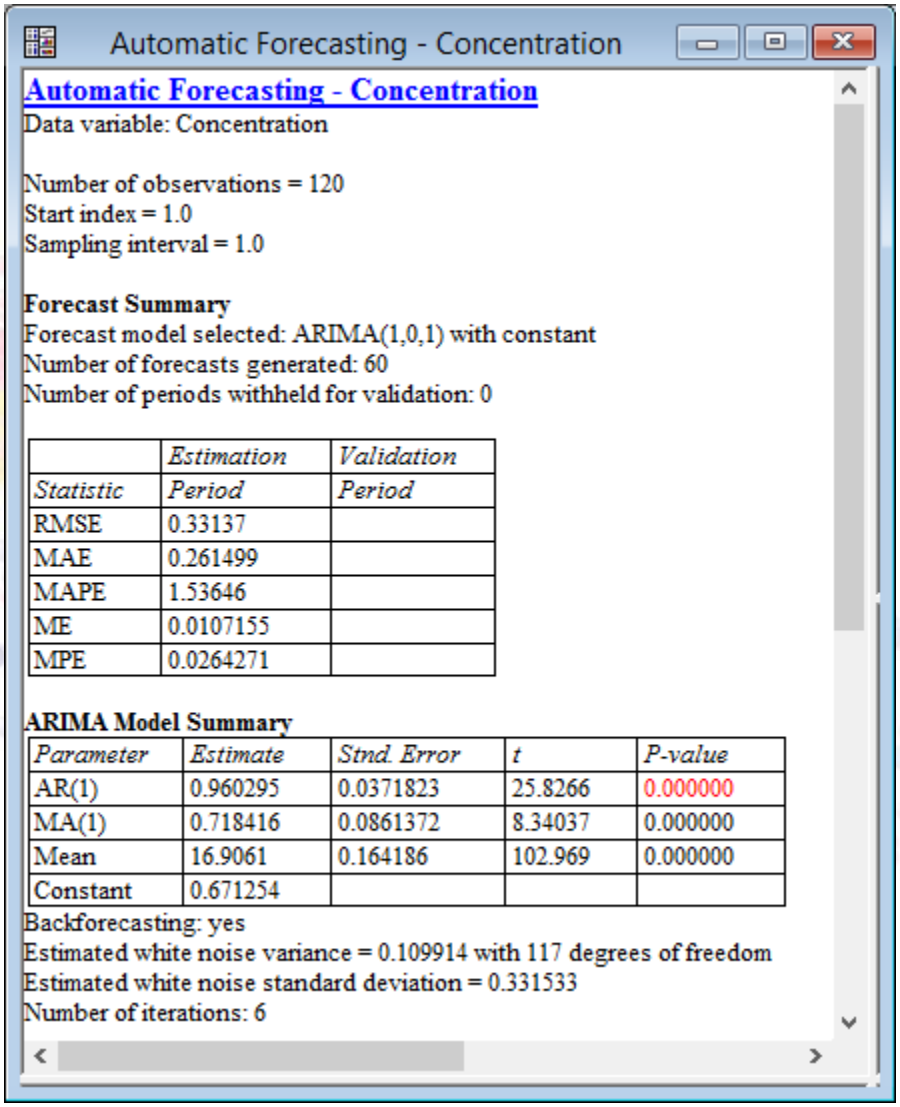
Selecting the Best ARIMA Model



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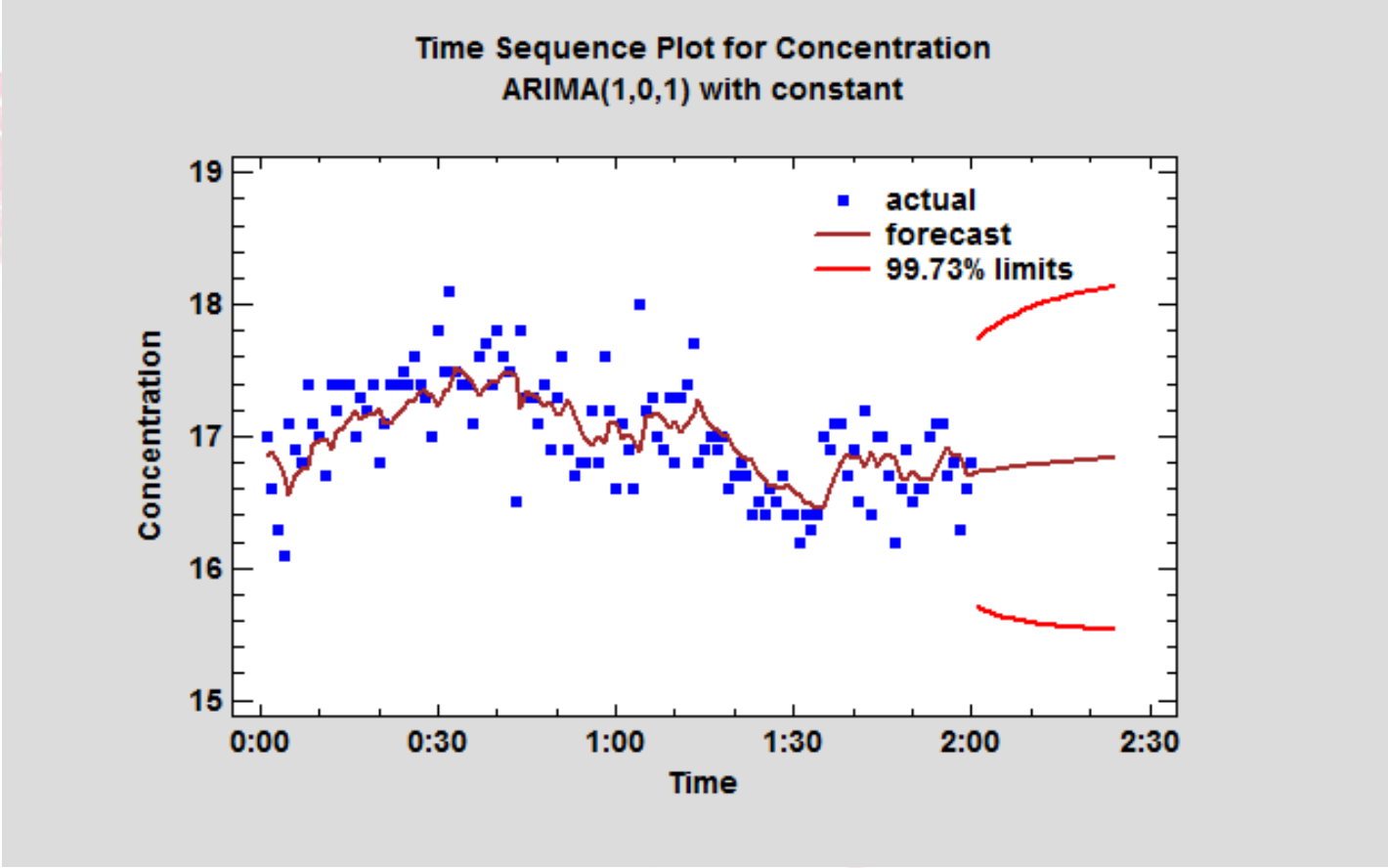
Selected Model



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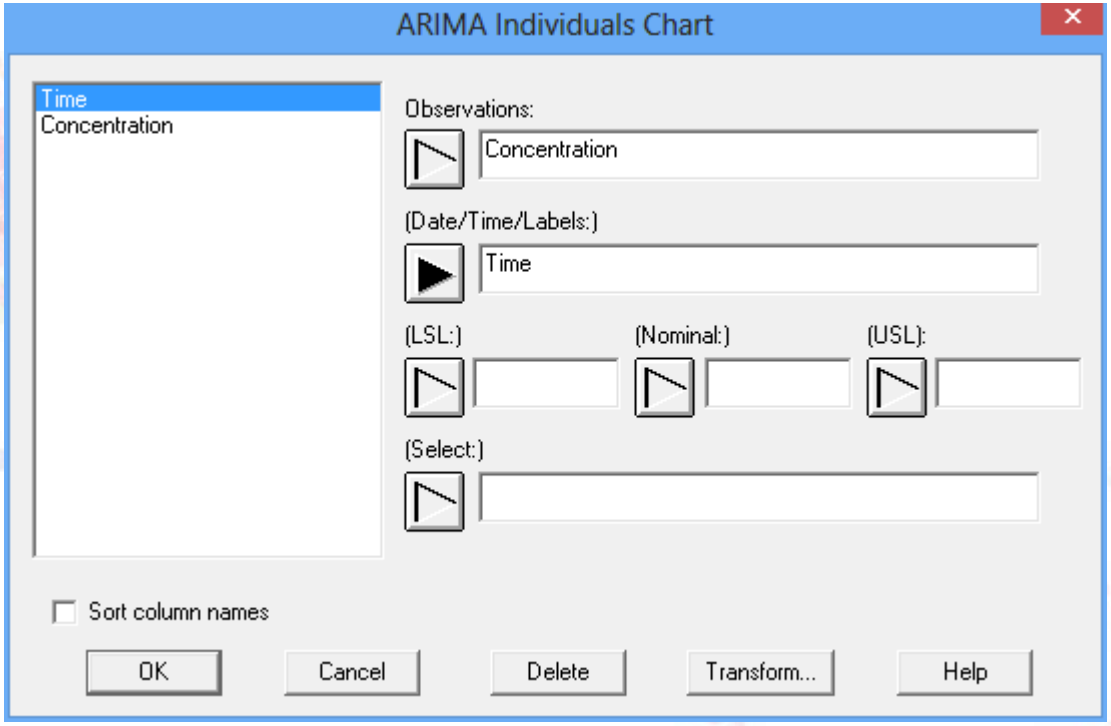
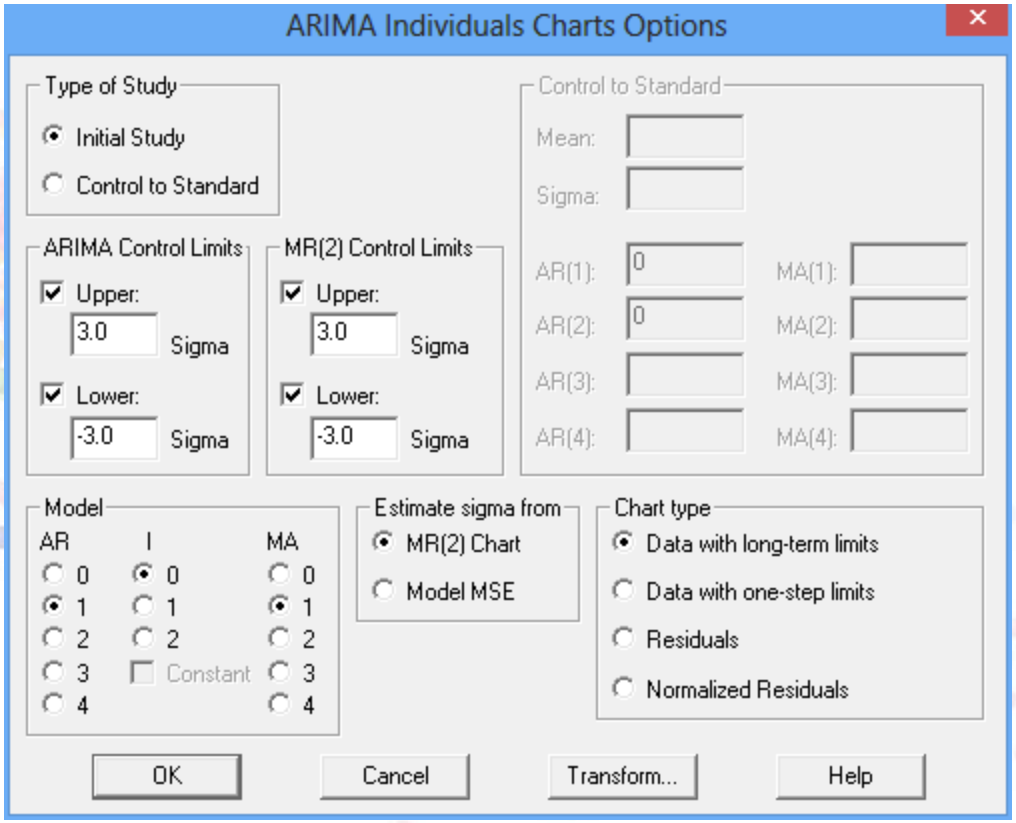
ARIMA Chart – Data Input



ARIMA Chart – Analysis Options



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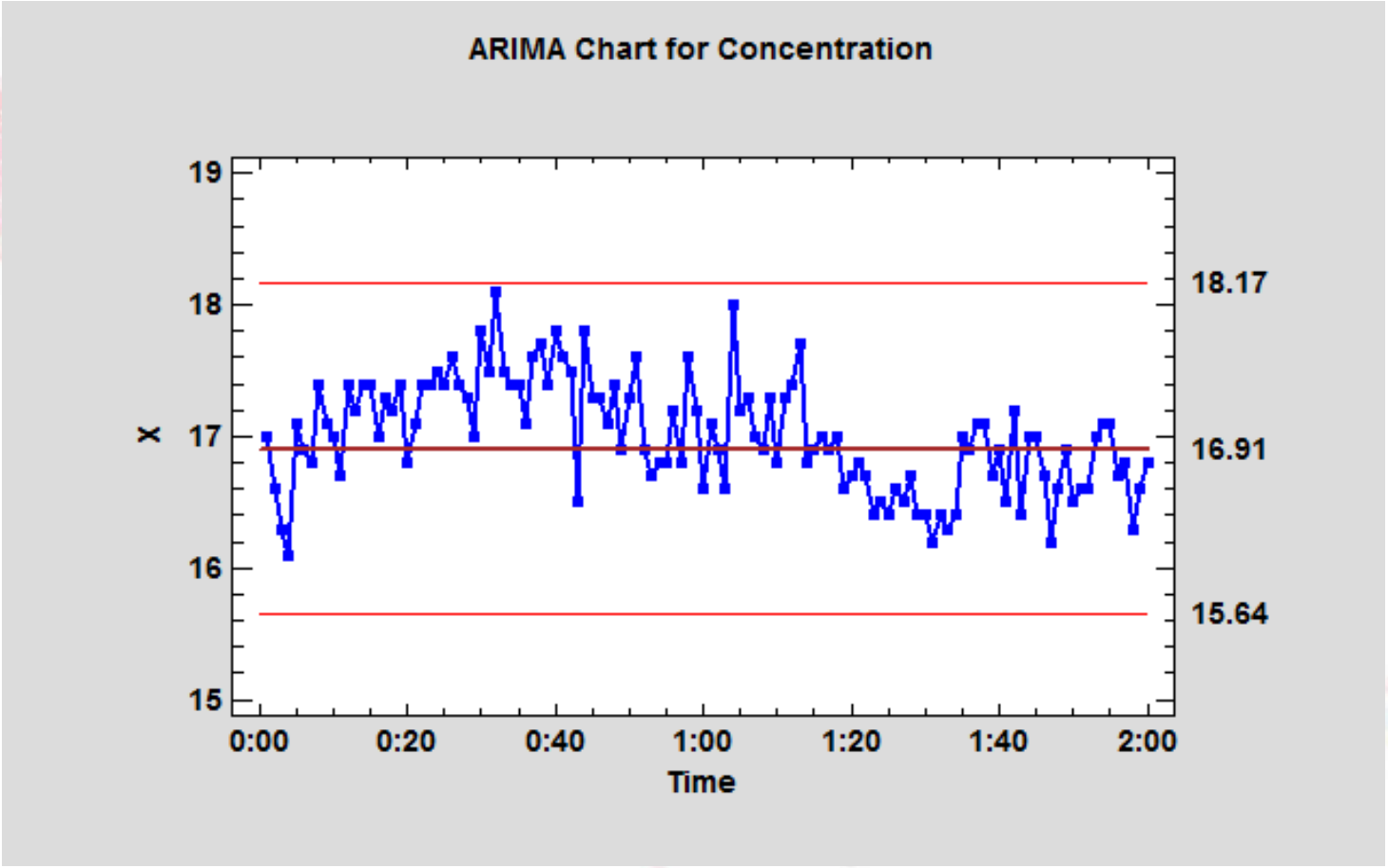
Data with Long-Term Limits



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Data with One-Step Limits

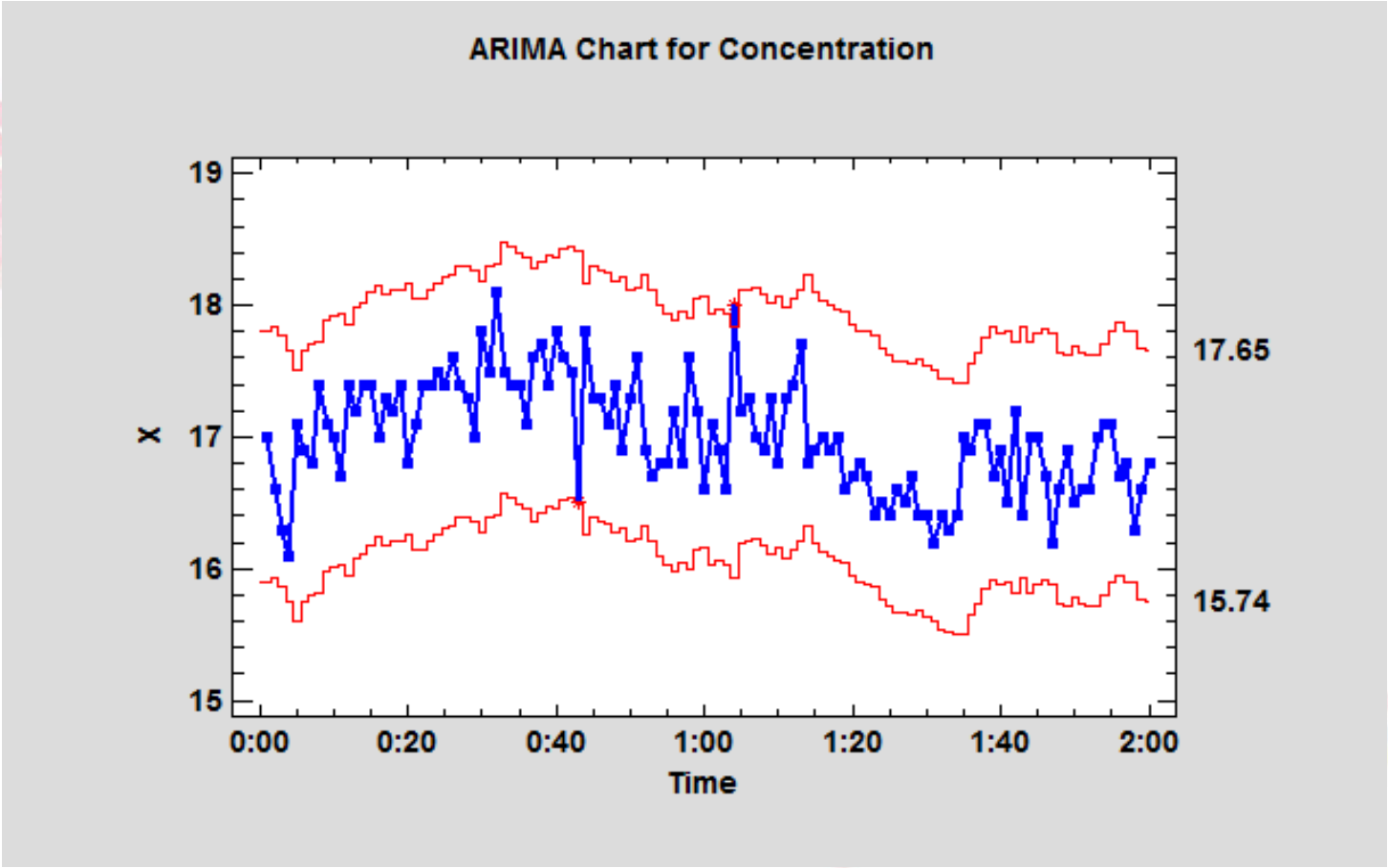

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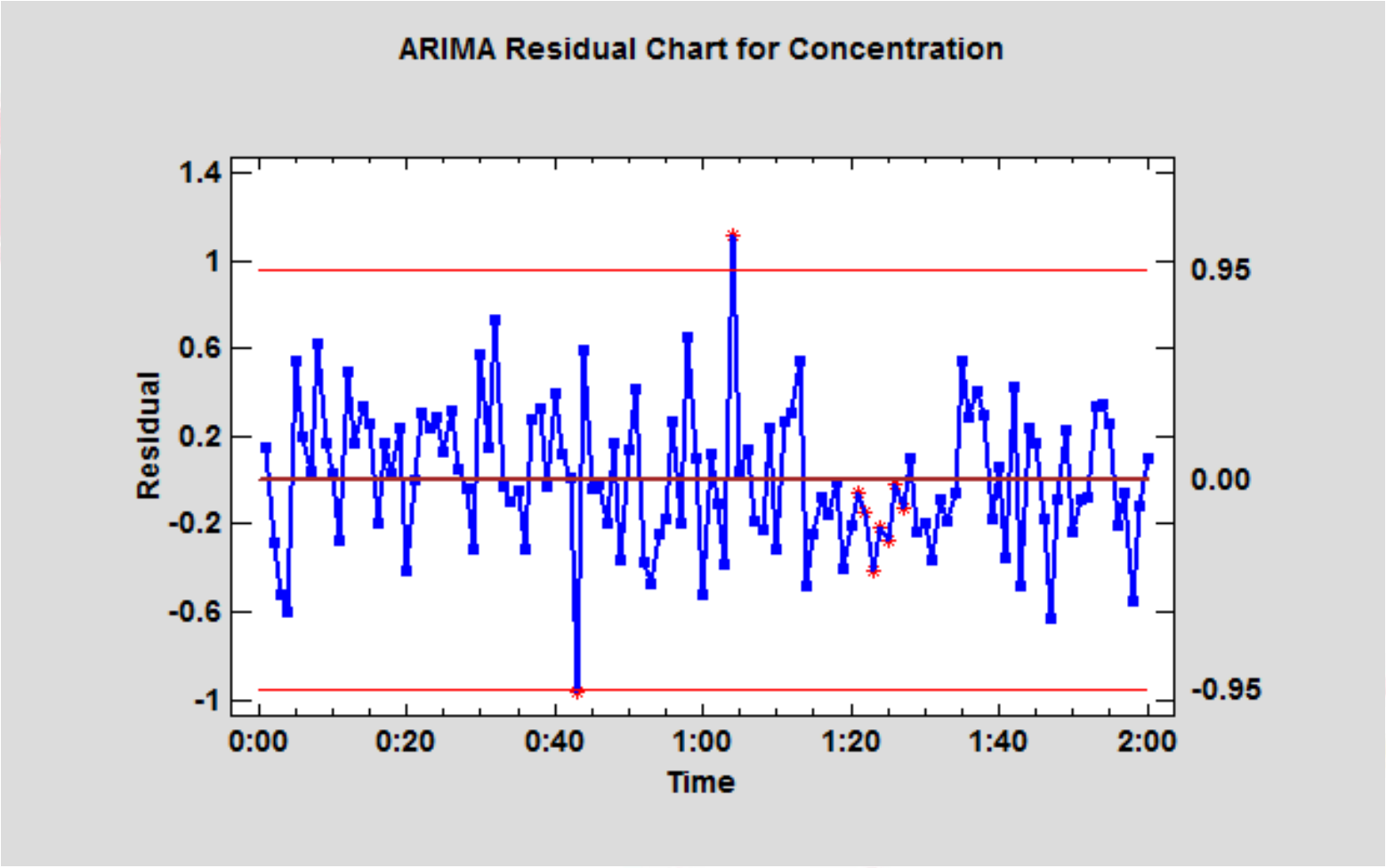
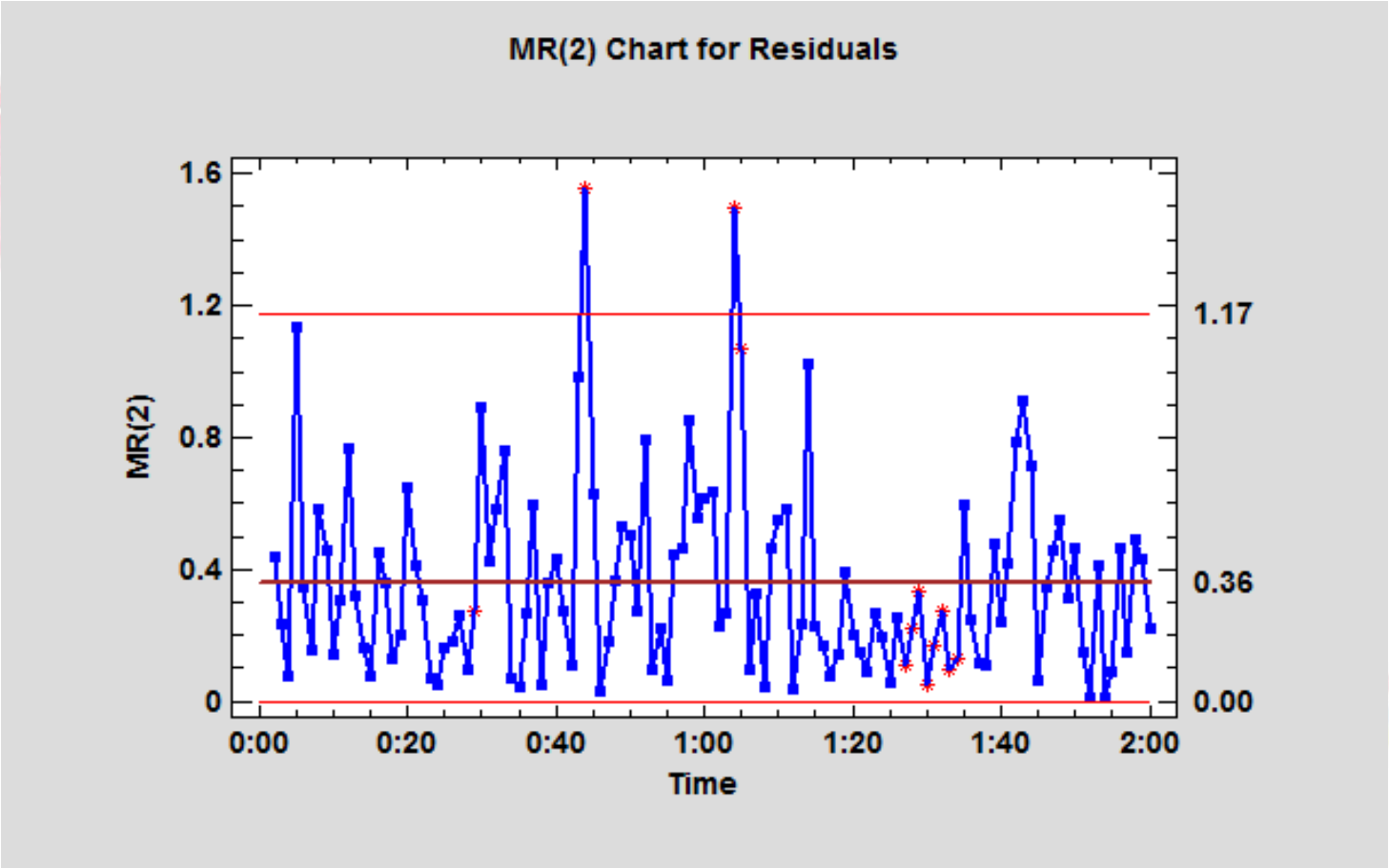
Chart of Residuals



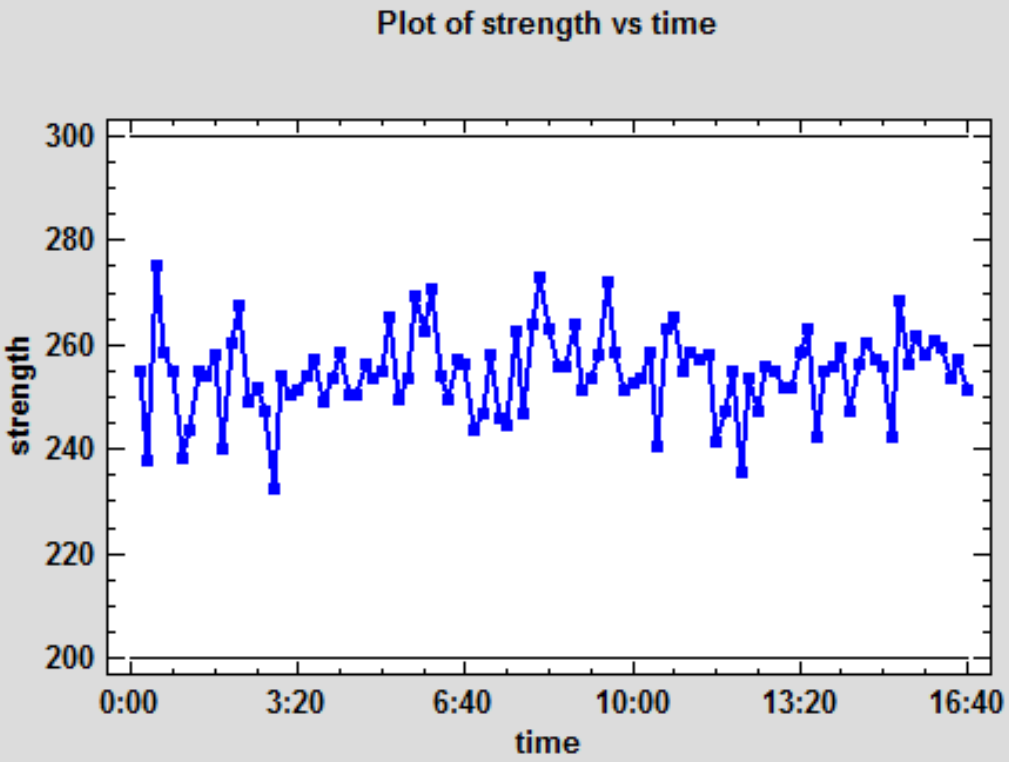
MR(2) Chart for Residuals



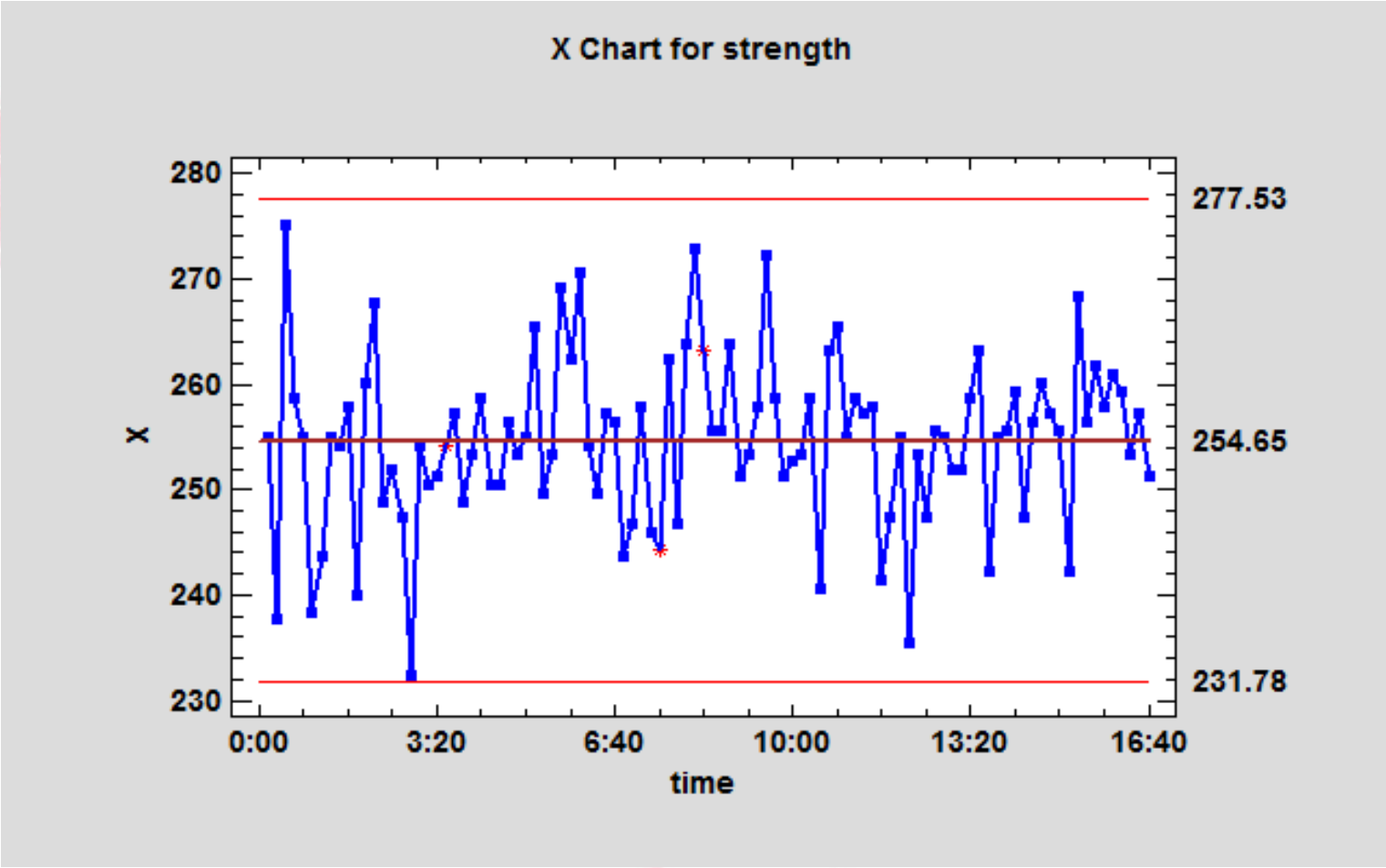
Acceptance Control Charts

- Used to insure that product remains within specification limits.
- Position control limits with respect to the specifications rather than the process mean.
- Useful for high C_{pk} processes which can tolerate some wandering of the process mean.

Sample Data



X Chart



Acceptance Charts

- Begin by specifying *f*, the maximum allowable fraction of nonconforming items.

- Find most extreme values of process mean that would yield no more than *f* nonconformities:

  Largest allowable mean: $\mu_U = USL - Z_\delta\sigma$

  Smallest allowable mean: $\mu_L = LSL + Z_\delta\sigma$

- Position control limits with respect to these extremes.

Positioning Limits

- *Sigma multiple method*: add “3-sigma” to the extreme values for the mean.

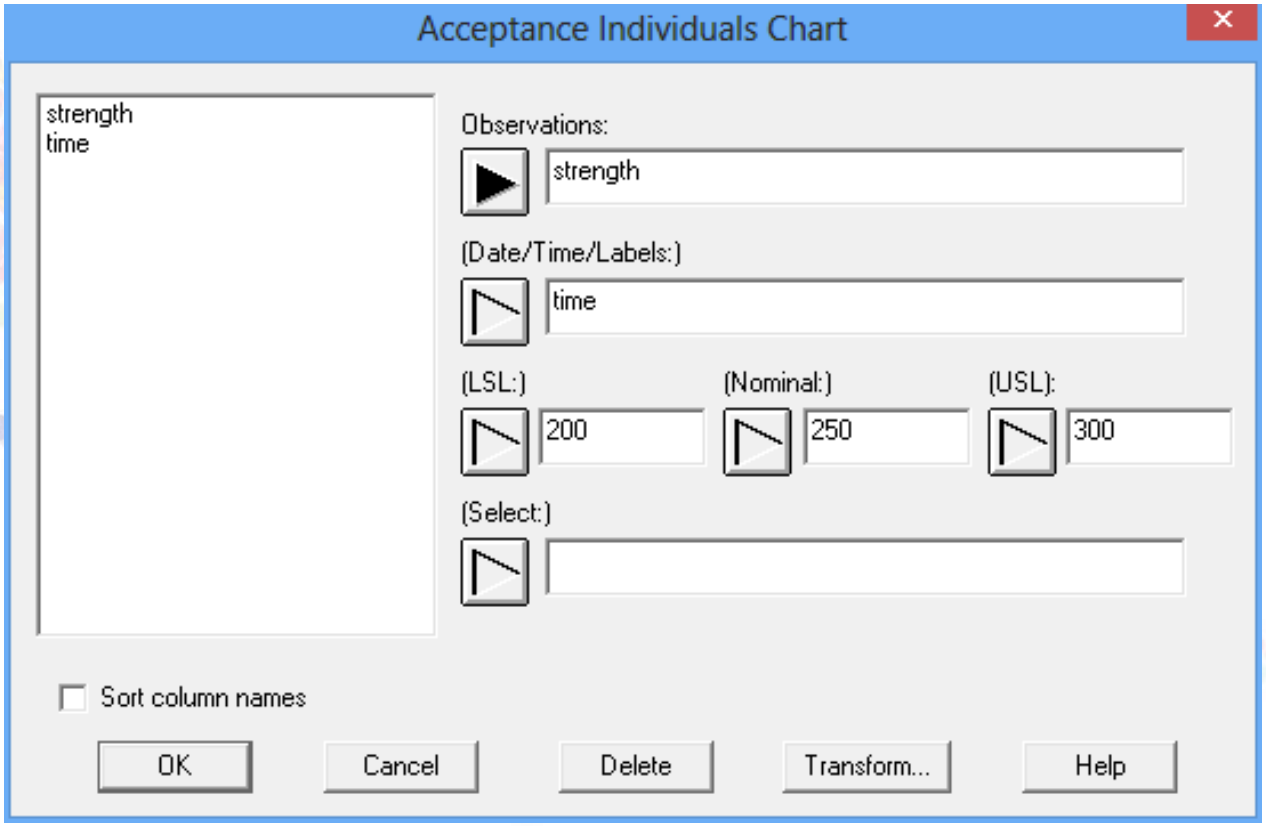
$$UCL = \mu_U + 3\sigma$$
$$LCL = \mu_L - 3\sigma$$

Note: Montgomery calls these “modified control limits”.

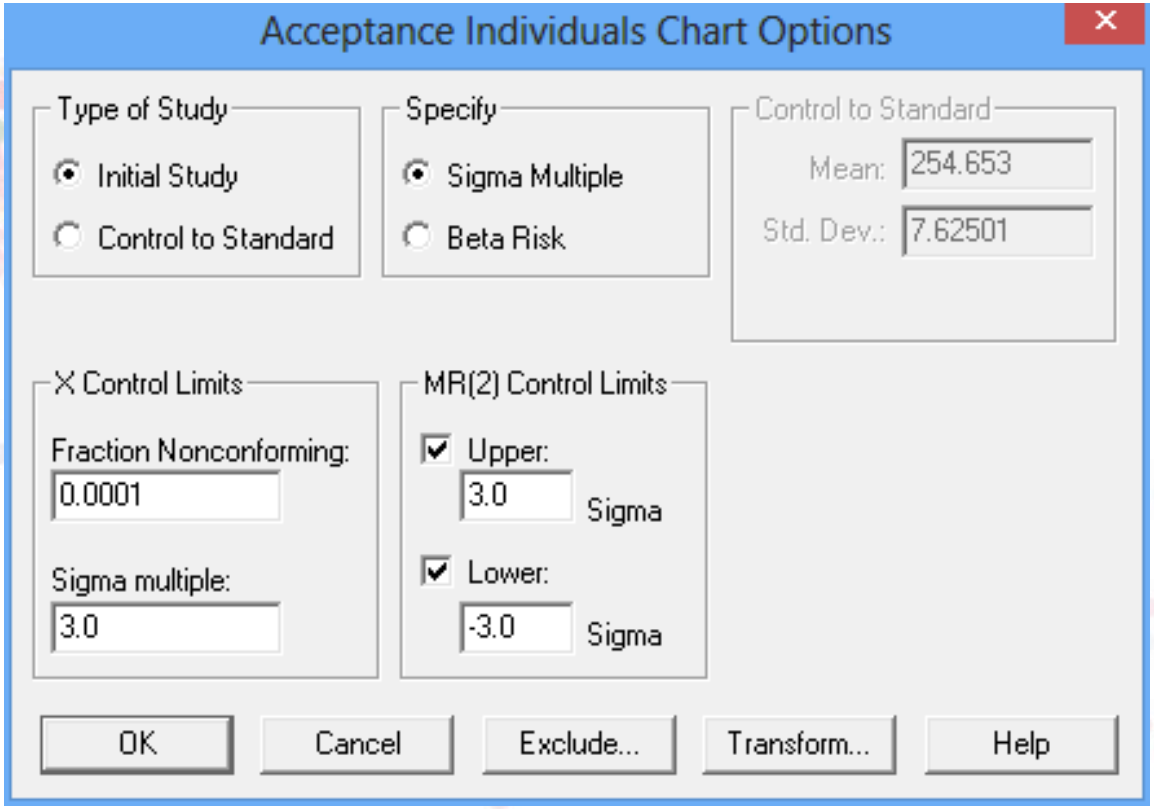
- *Beta risk method*: specify β, probability of not generating an out of control signal. Position control limits at:

$$UCL = \mu_U - Z_\beta\sigma$$
$$LCL = \mu_L + Z_\beta\sigma$$

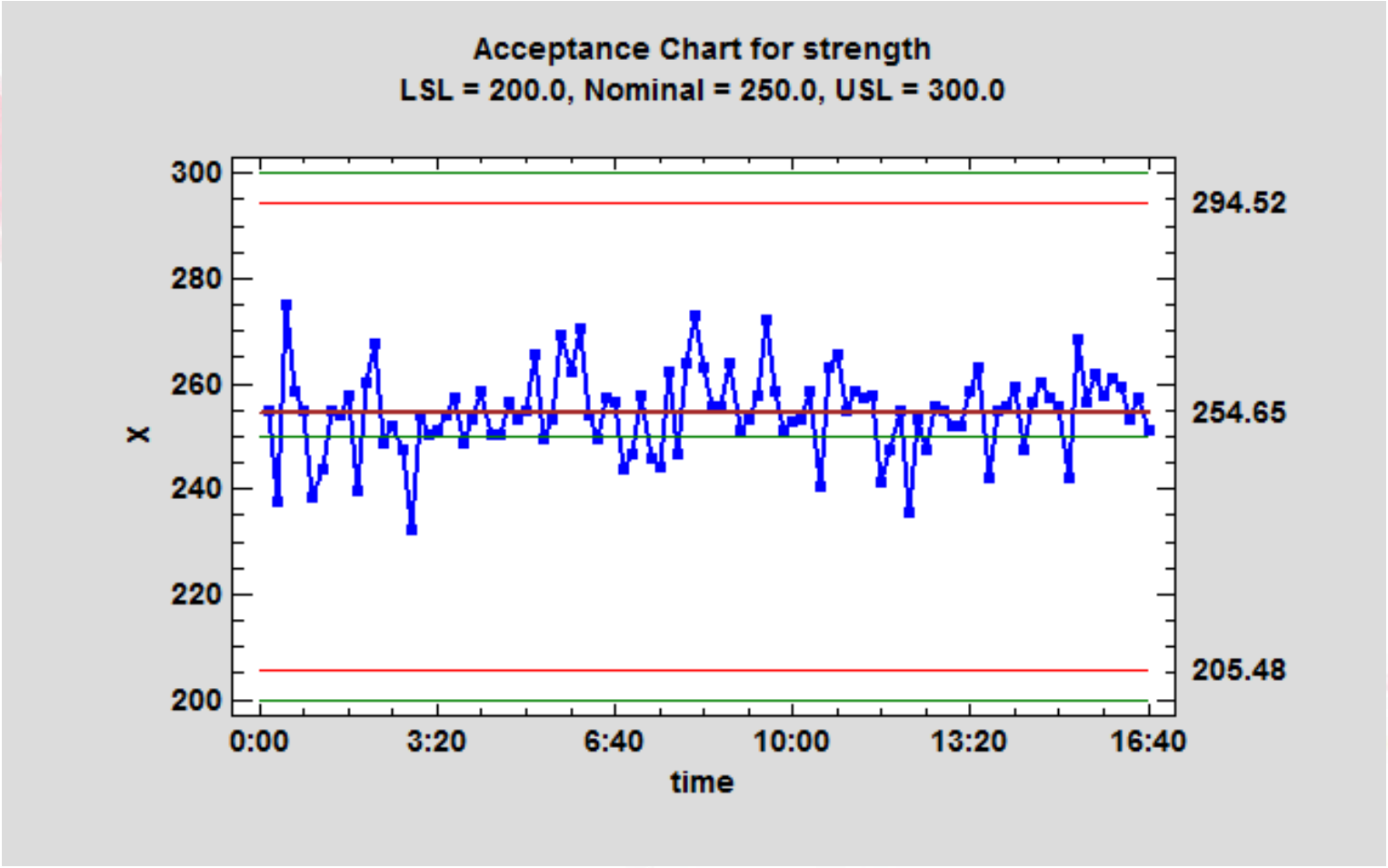
Acceptance Chart – Data Input



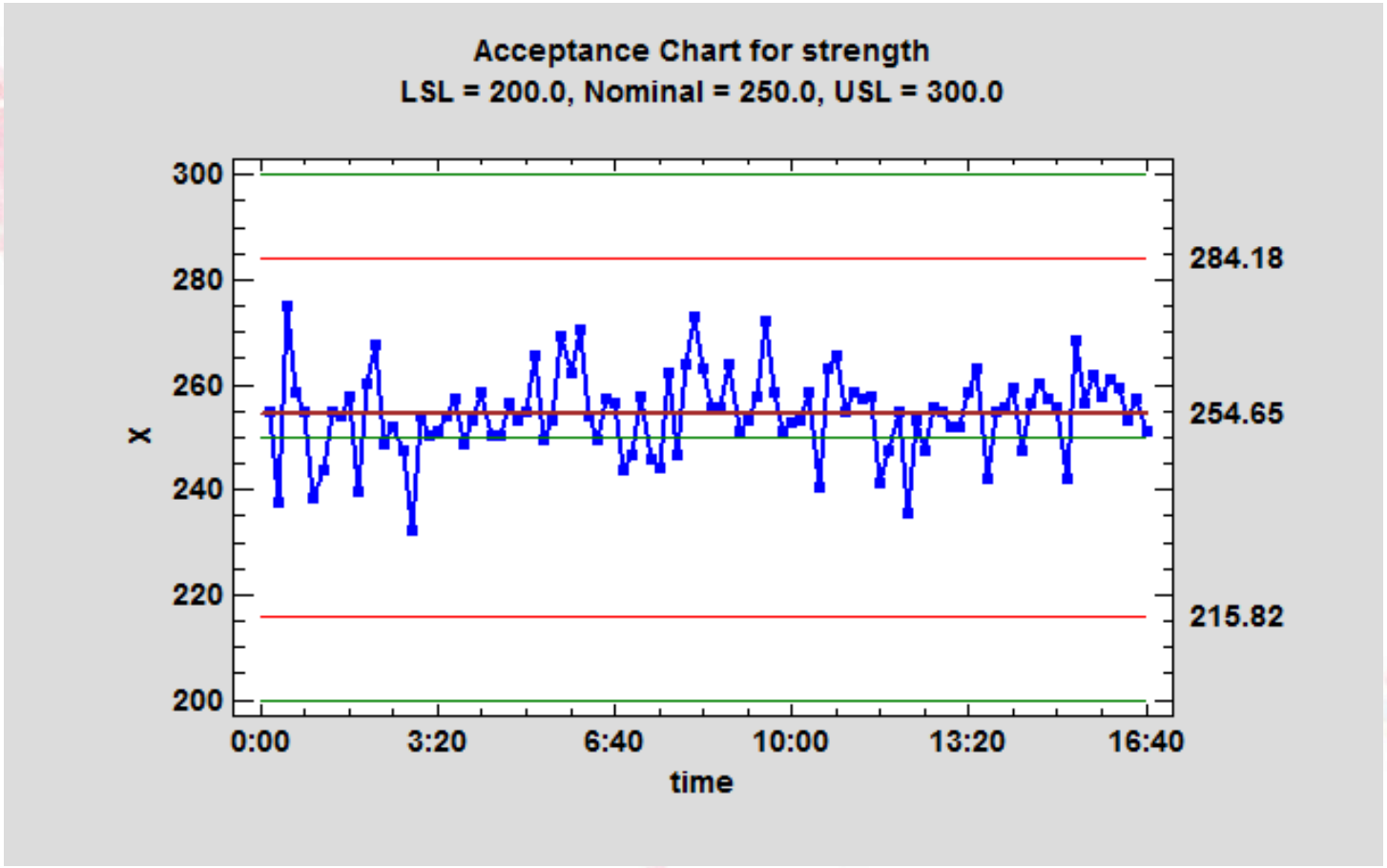
Acceptance Chart – Analysis Options



Based on Sigma Multiple



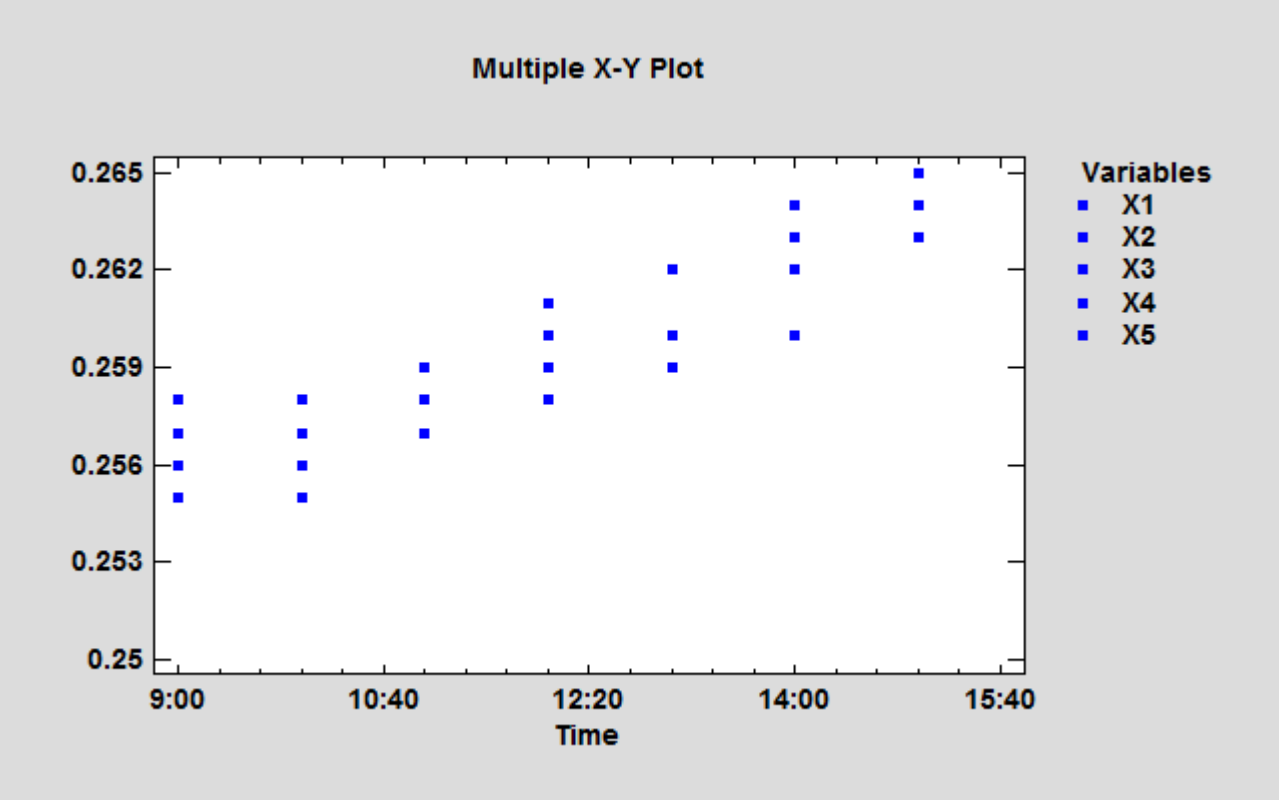
Based on Beta Risk



Toolwear Control Charts

- Used for data that do not have a constant mean, such as measurements affected by wear on a tool.
- Used to determine whether mean is changing at a constant or expected rate.
- Can also add specification limits to help determine when tool should be changed.

Sample Data (from Duncan)



- Specification: 0.255 to 0.265 inches

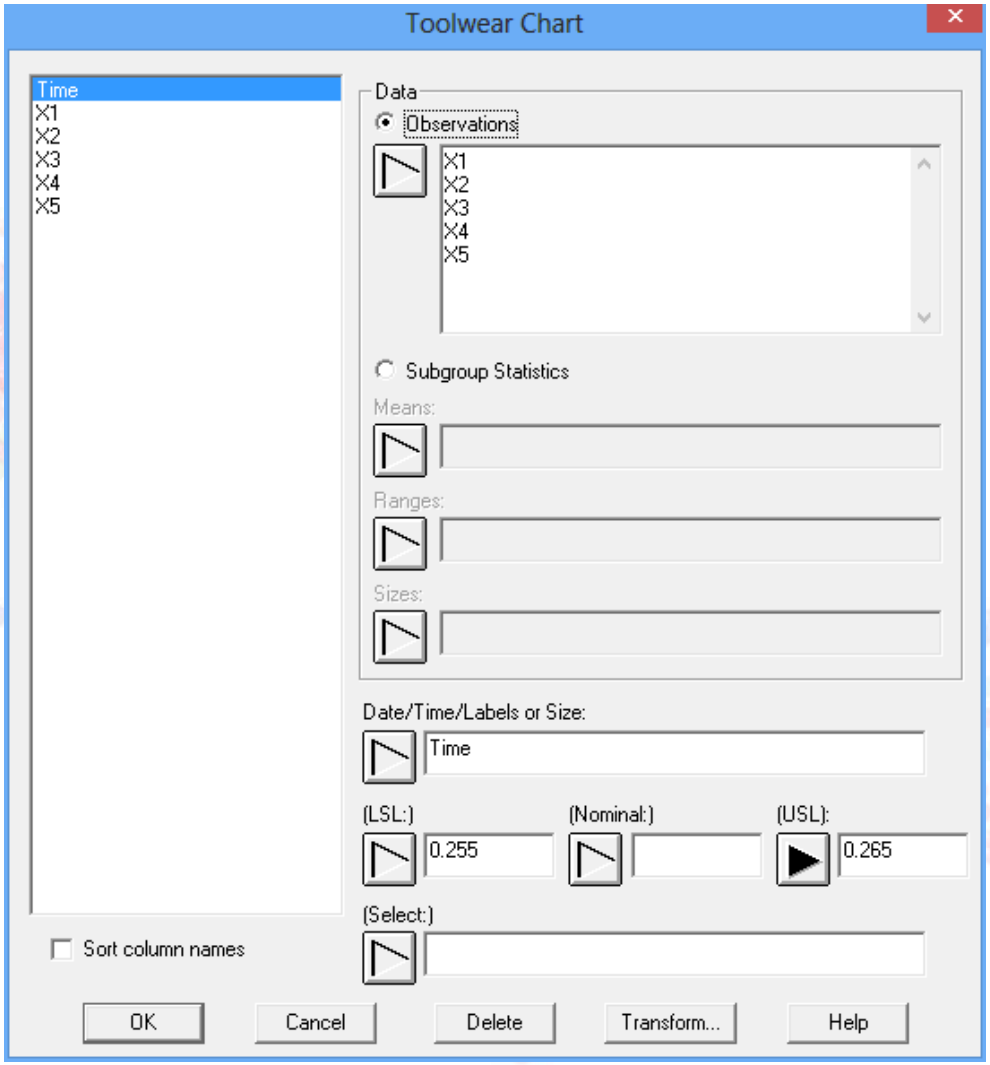
Toolwear Chart Model

Let $X_{t,j}$ be measurement on j^{th} sample made at time t. Then

$$X_{t,j} = \alpha + \beta t + \varepsilon_{t,j}$$

where $\mu = \alpha + \beta t$ is the process mean and the deviations ε_t are assumed to be NID(0,σ^2).

Toolwear Chart – Data Input



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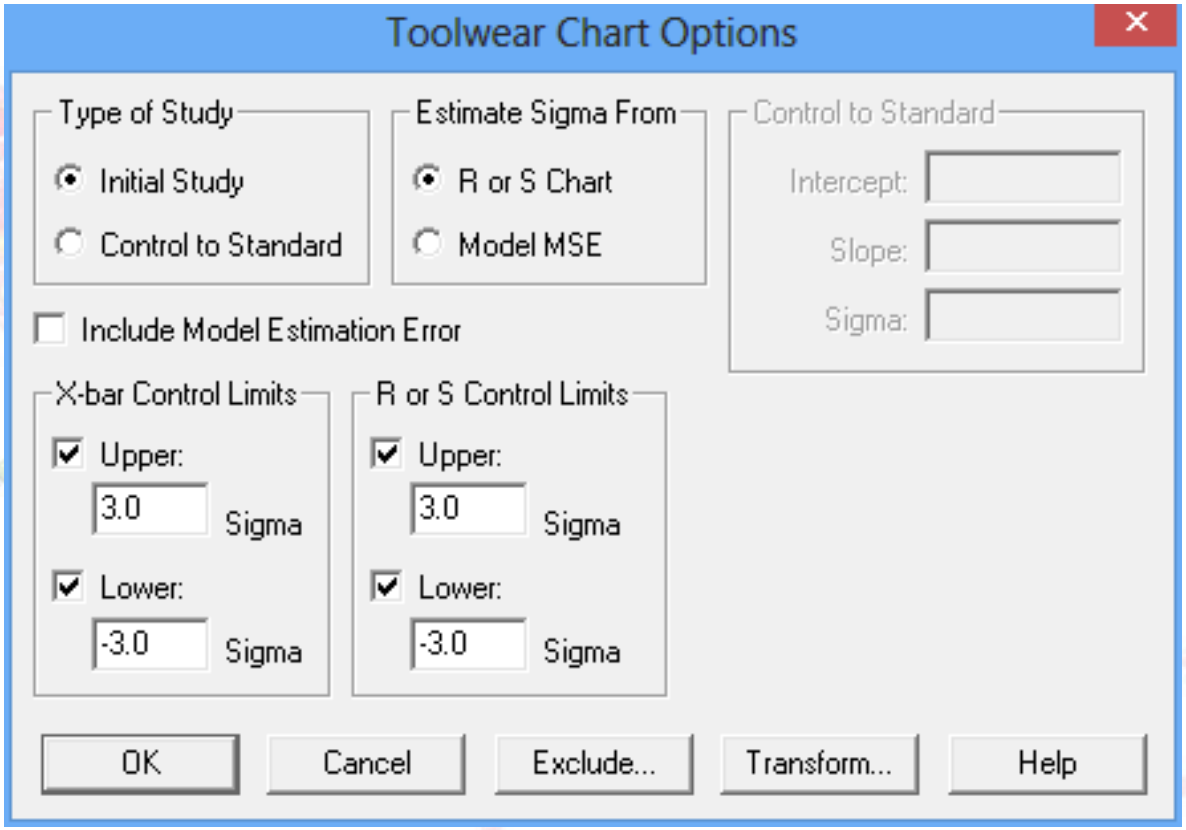
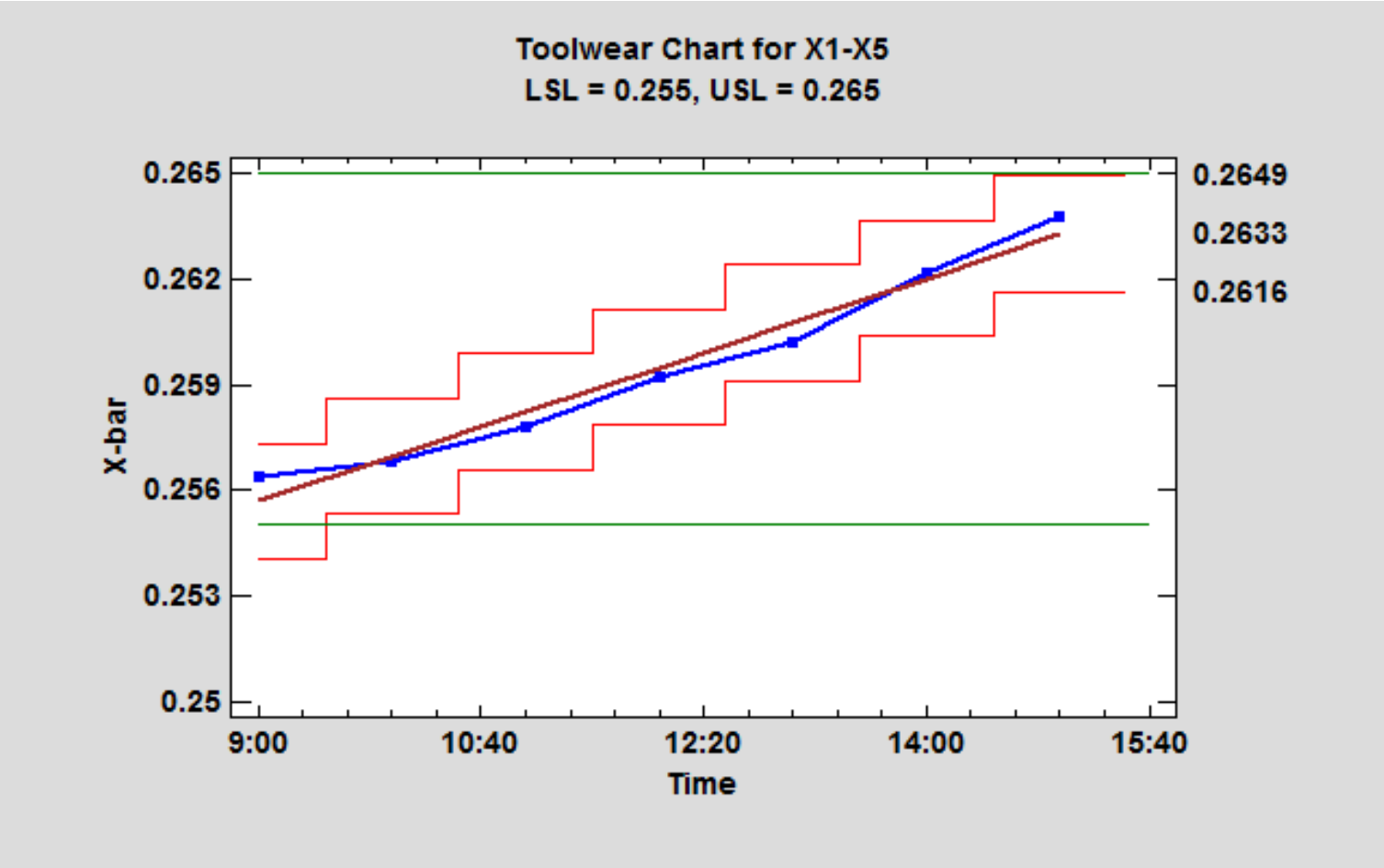
Analysis Options



Toolwear Chart



Charts for Rare Events

- Used for monitoring occurrence of rare events
  - Hospital infections
  - Factory or mining accidents
  - Unexpected shutdowns

- Two primary charts
  - t chart: for events recorded in continuous time
  - g chart: for events recorded in discrete time

Examples

C:\Data\webinar\infections.sgd

	Infection time	Date of infection	Col_3
1	1/1/12 8:11	1/4/11	
2	1/2/12 5:42	1/5/11	
3	1/3/12 21:55	1/7/11	
4	1/5/12 5:24	1/8/11	
5	1/7/12 10:14	1/10/11	
6	1/9/12 11:11	1/13/11	
7	1/12/12 16:56	1/14/11	
8	1/15/12 13:39	1/14/11	
9	1/17/12 14:26	1/20/11	
10	1/18/12 13:26	1/31/11	
11	1/21/12 10:53	2/2/11	
12	1/22/12 23:58	2/2/11	
13	1/25/12 6:34	2/15/11	
14	1/27/12 6:31	2/19/11	
15	1/28/12 14:18	2/19/11	
16	1/30/12 16:20	2/21/11	
17	2/4/12 9:15	2/23/11	
18	2/8/12 23:19	3/5/11	
19	2/9/12 5:15	3/8/11	
20	2/12/12 13:34	3/9/11	
21	2/15/12 8:46	3/9/11	
22	2/18/12 4:37	3/12/11	

infections B C

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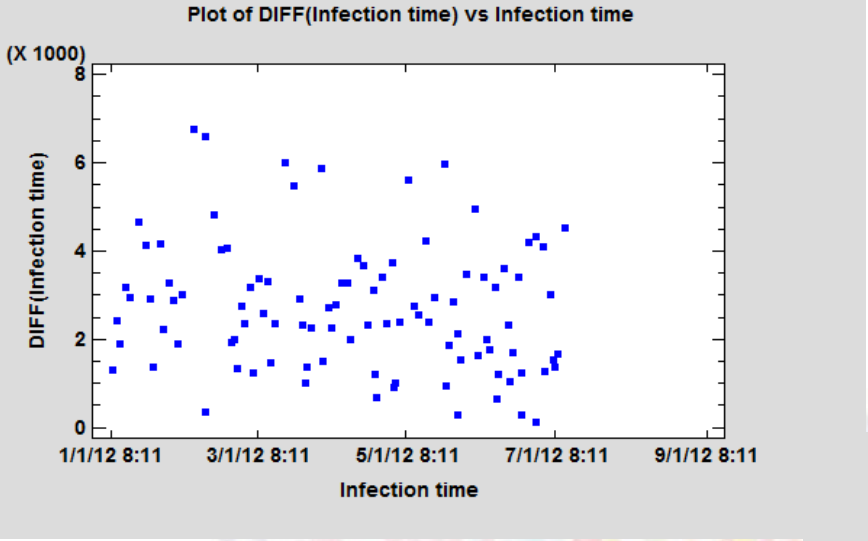
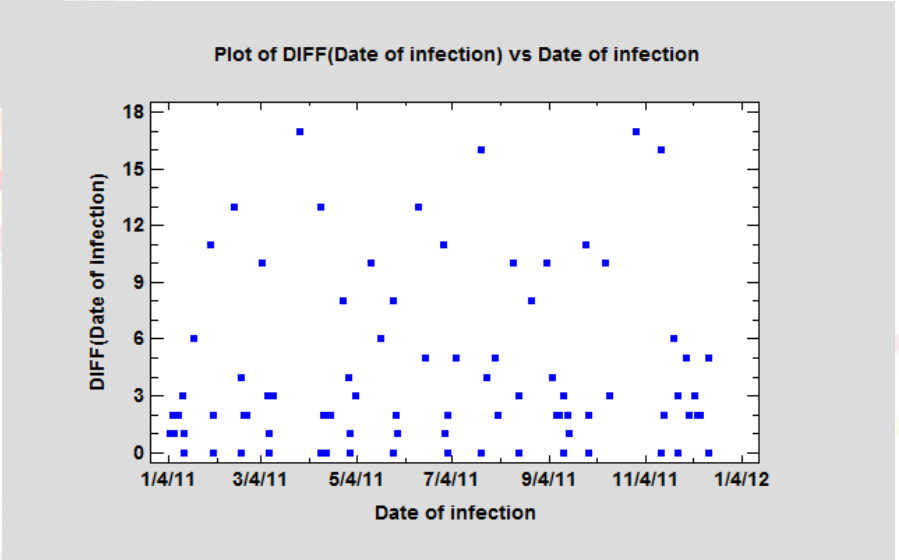
Time Between Events

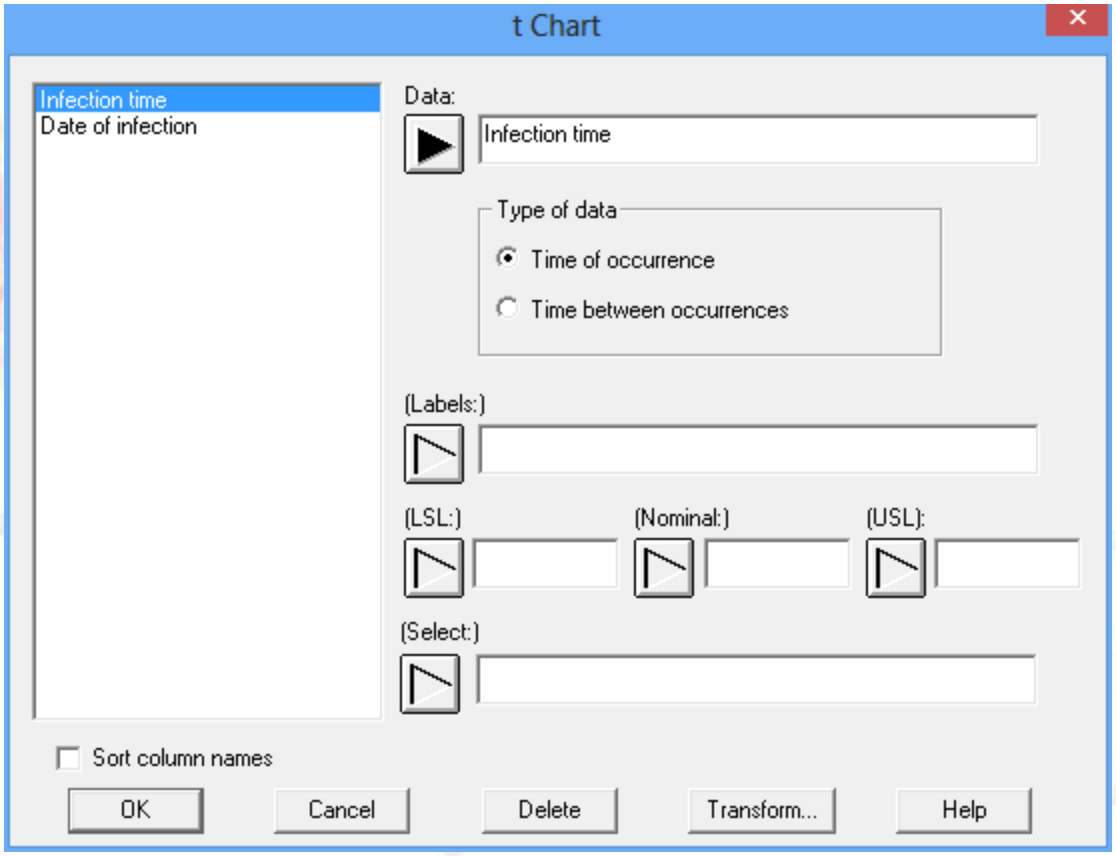
- When events are rare, standard charts such as a C chart will have many 0's.
- Better approach than plotting counts is to plot the times between consecutive events.
- For a continuous time process, the interevent times are modeled by a Weibull distribution.
- For a discrete time process, the interevent times are modeled by a geometric distribution.

Scatterplots

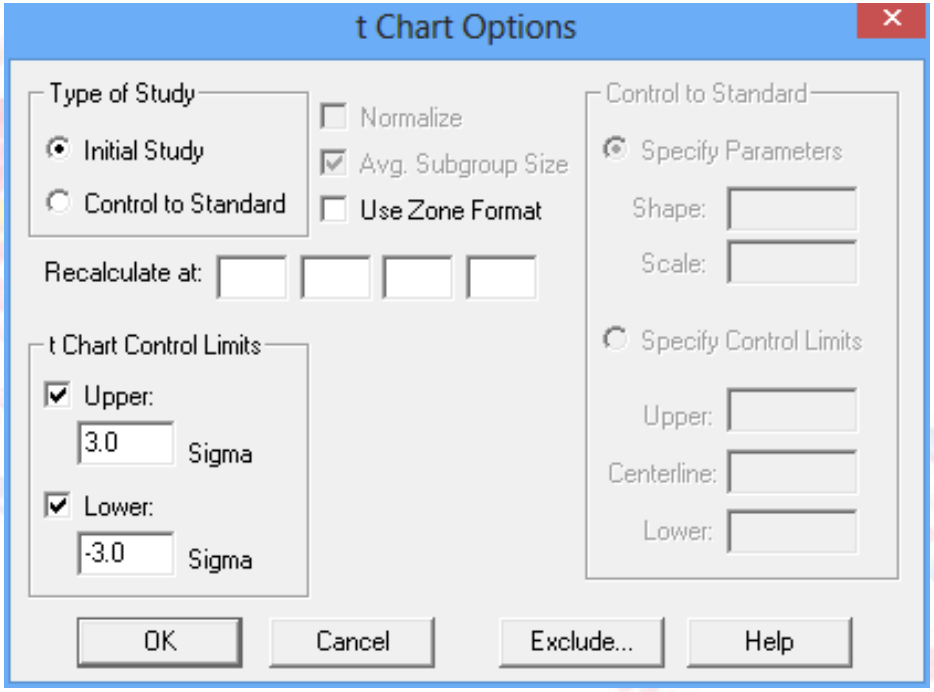




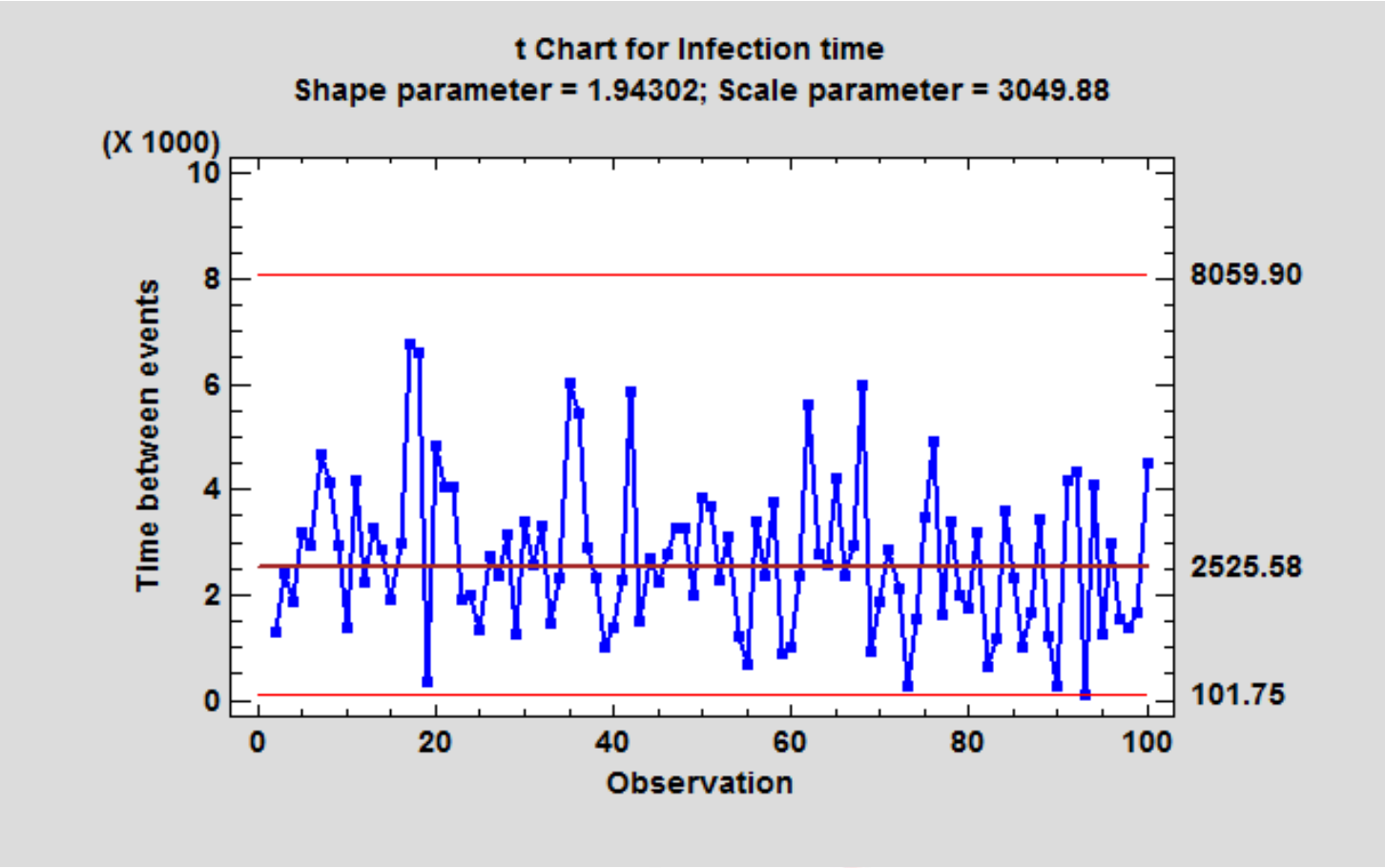
t Chart Data Input



t Chart Analysis Options



t Chart



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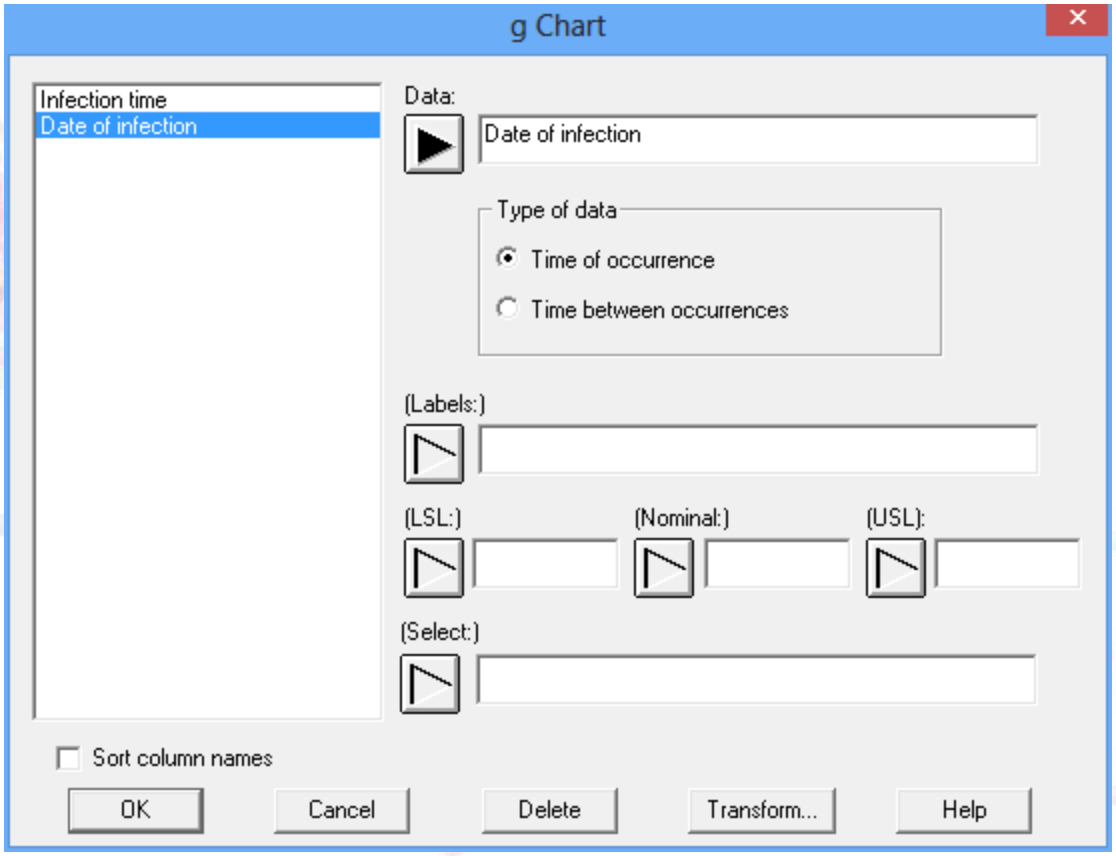
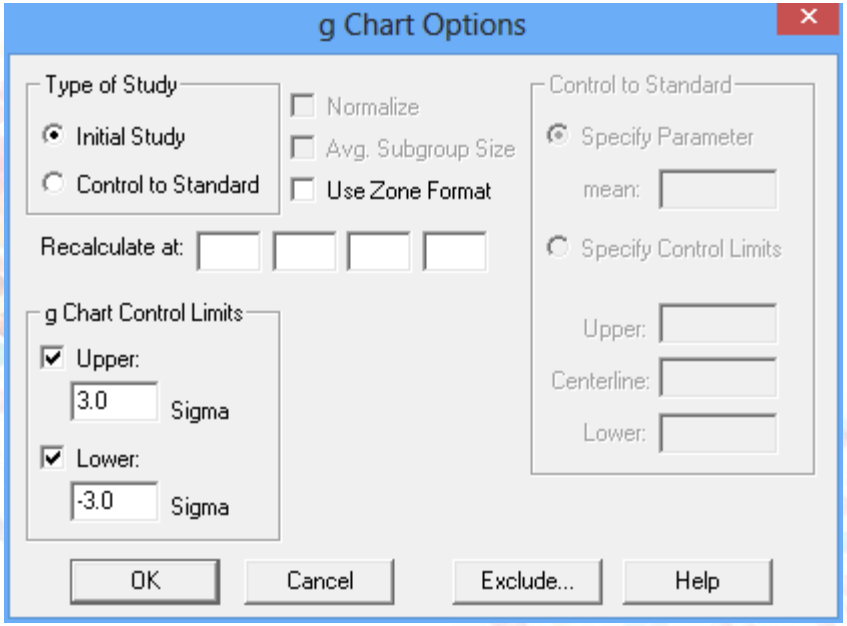
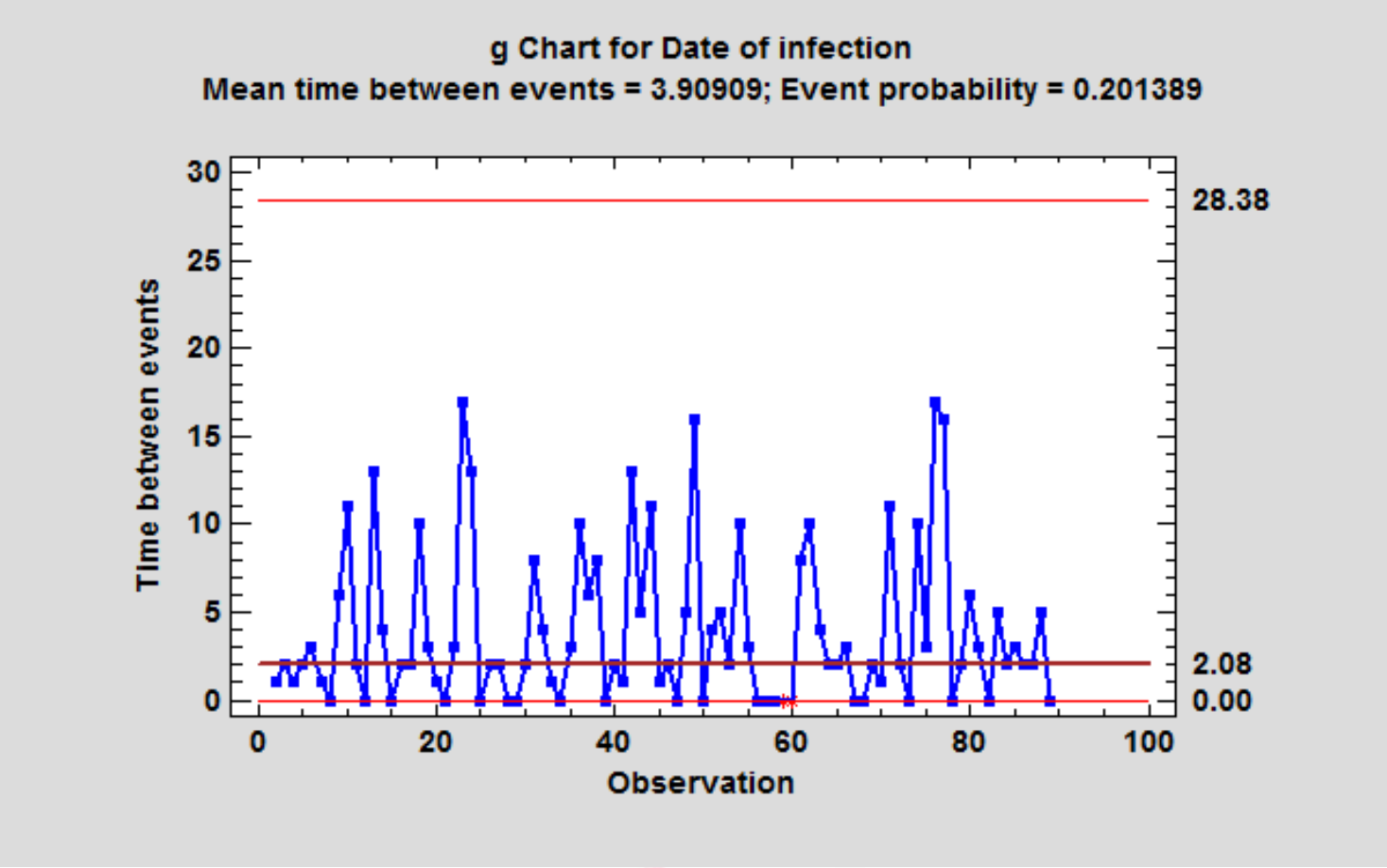
g Chart Data Input



g Chart Analysis Options



g Chart



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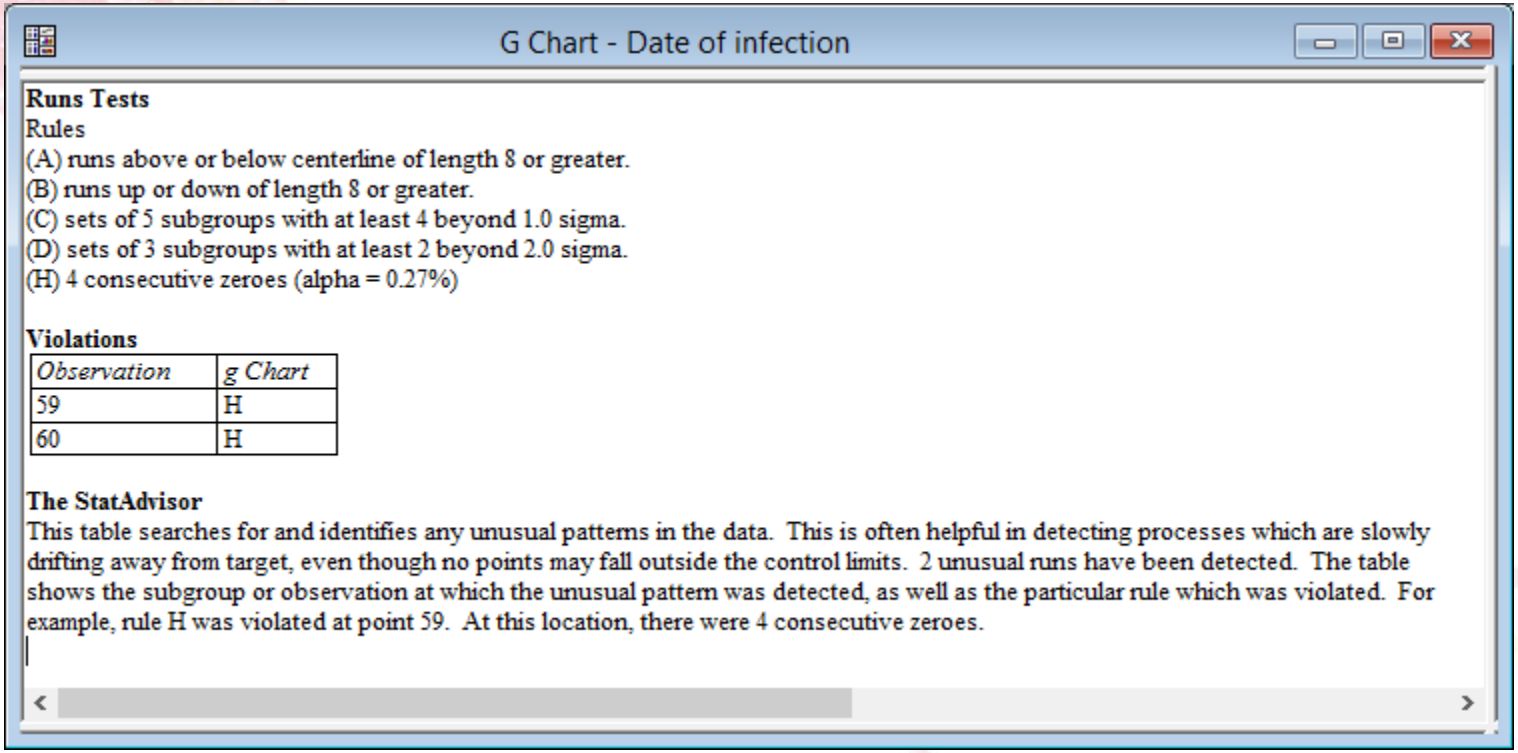
G Chart – Special Run Test

- Extra run test for consecutive zeroes:



Cuscore Charts

- Designed to detect specific types of disturbances:
  - Spike
  - Ramp
  - Bump
  - Step change
  - Exponential increase
  - Sine wave

- Automatically selects the best type of control chart to detect that pattern

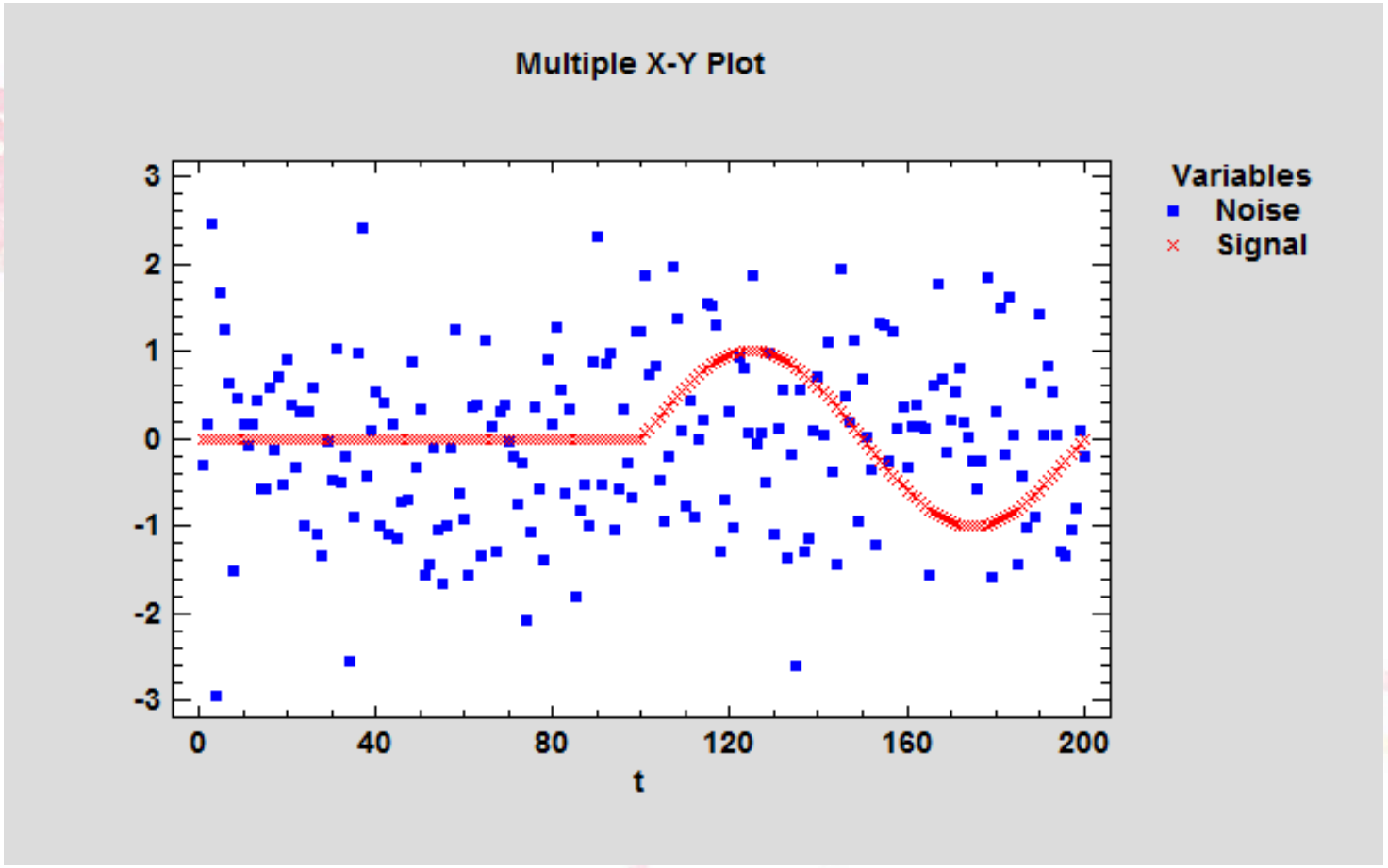
Example: sine wave plus noise

signal plus noise.sgd

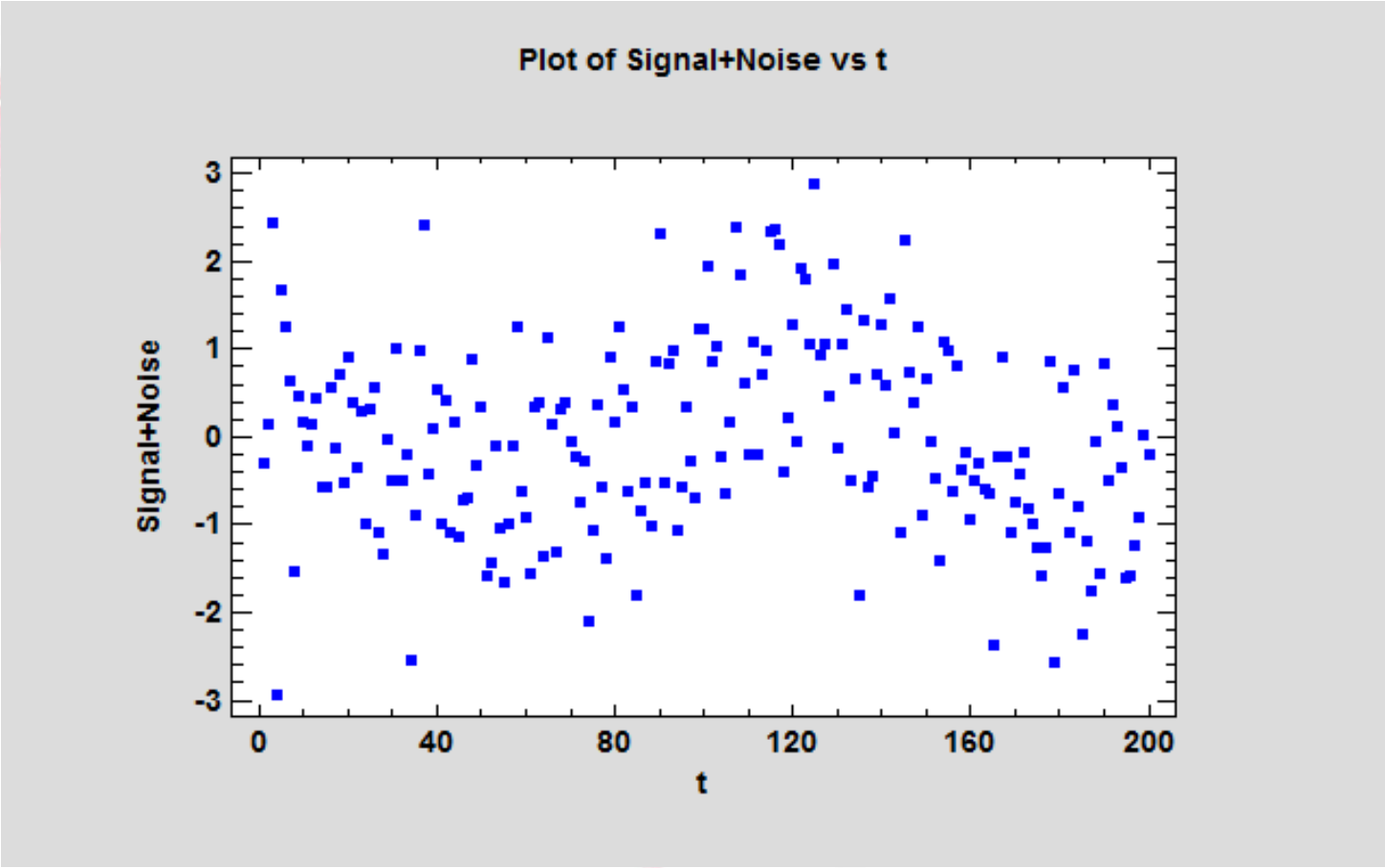
	t	Noise	Signal	Col_4
1	1	-0.298475889641	0	
2	2	0.158540926563	0	
3	3	2.44921673896	0	
4	4	-2.92901952736	0	
5	5	1.66838656126	0	
6	6	1.25353294413	0	
7	7	0.645732200693	0	
8	8	-1.51840745872	0	
9	9	0.470703738286	0	
10	10	0.162318392636	0	
11	11	-0.0911150761078	0	
12	12	0.156118205884	0	
13	13	0.434819344846	0	
14	14	-0.573646766803	0	
15	15	-0.56808795381	0	
16	16	0.572478593534	0	
17	17	-0.13202972842	0	
18	18	0.719853740624	0	

signal plus noise | B | C

Signal and Noise



Signal Plus Noise



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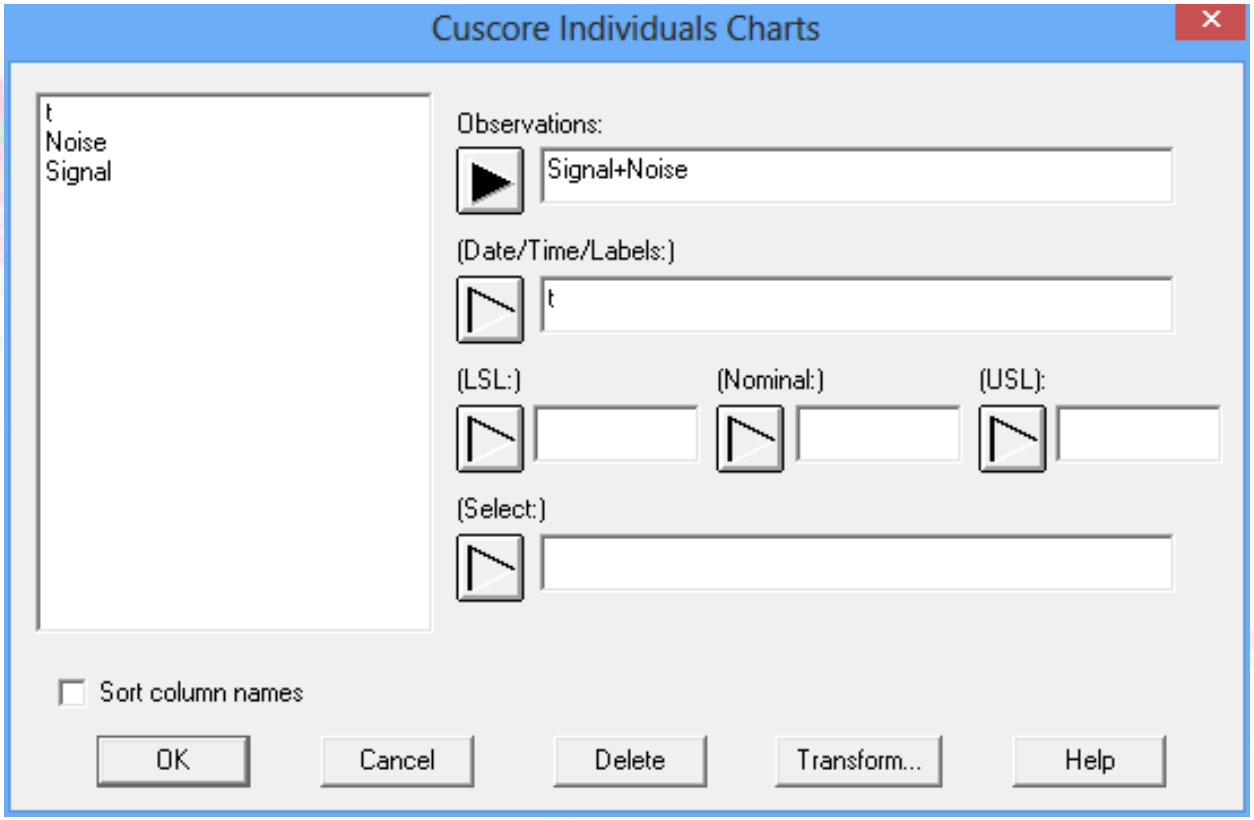
Q Score Statistic

- Cuscore chart plots a cumulative "Q score" statistic defined by:

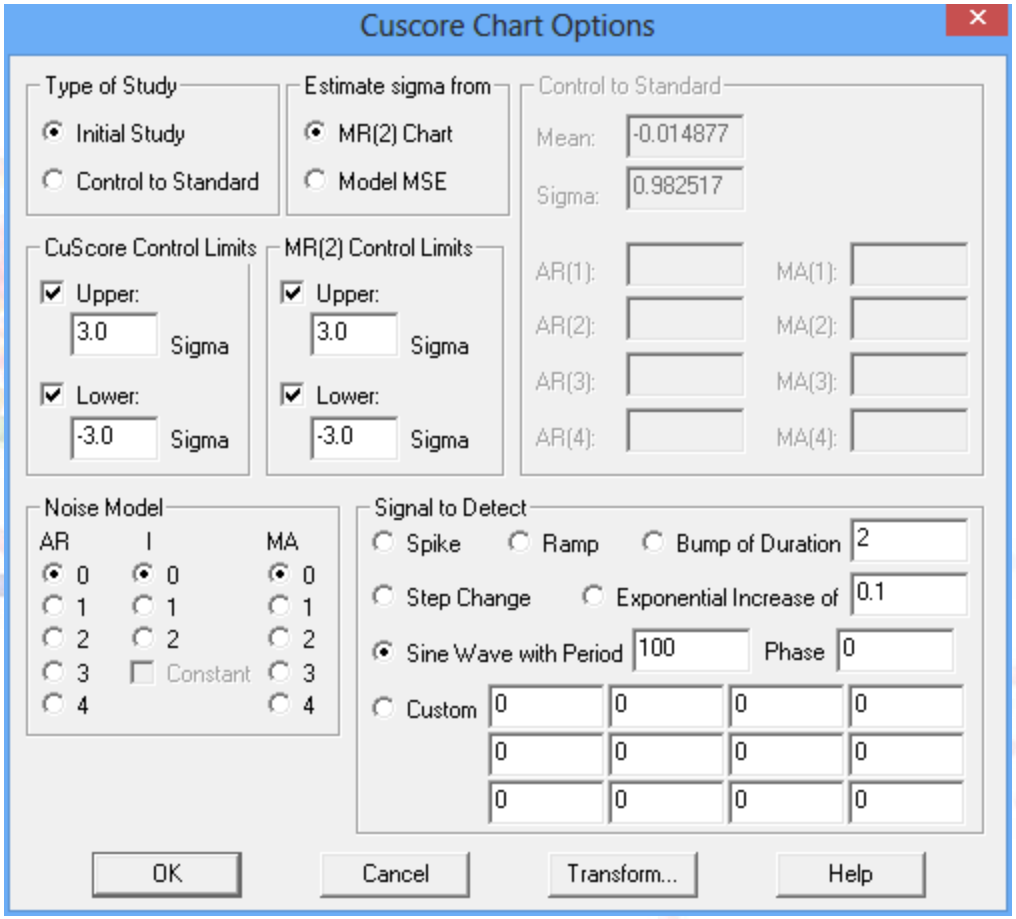
$$Q_t = \sum_{i=1}^{t} \hat{a}_i r_i$$

where the r_i values come from a "detector" series specifically designed to detect the presence of the expected disturbance and the a_i values are the residuals from an ARIMA model.

Cuscore Chart – Data Input



Cuscore Chart – Analysis Options



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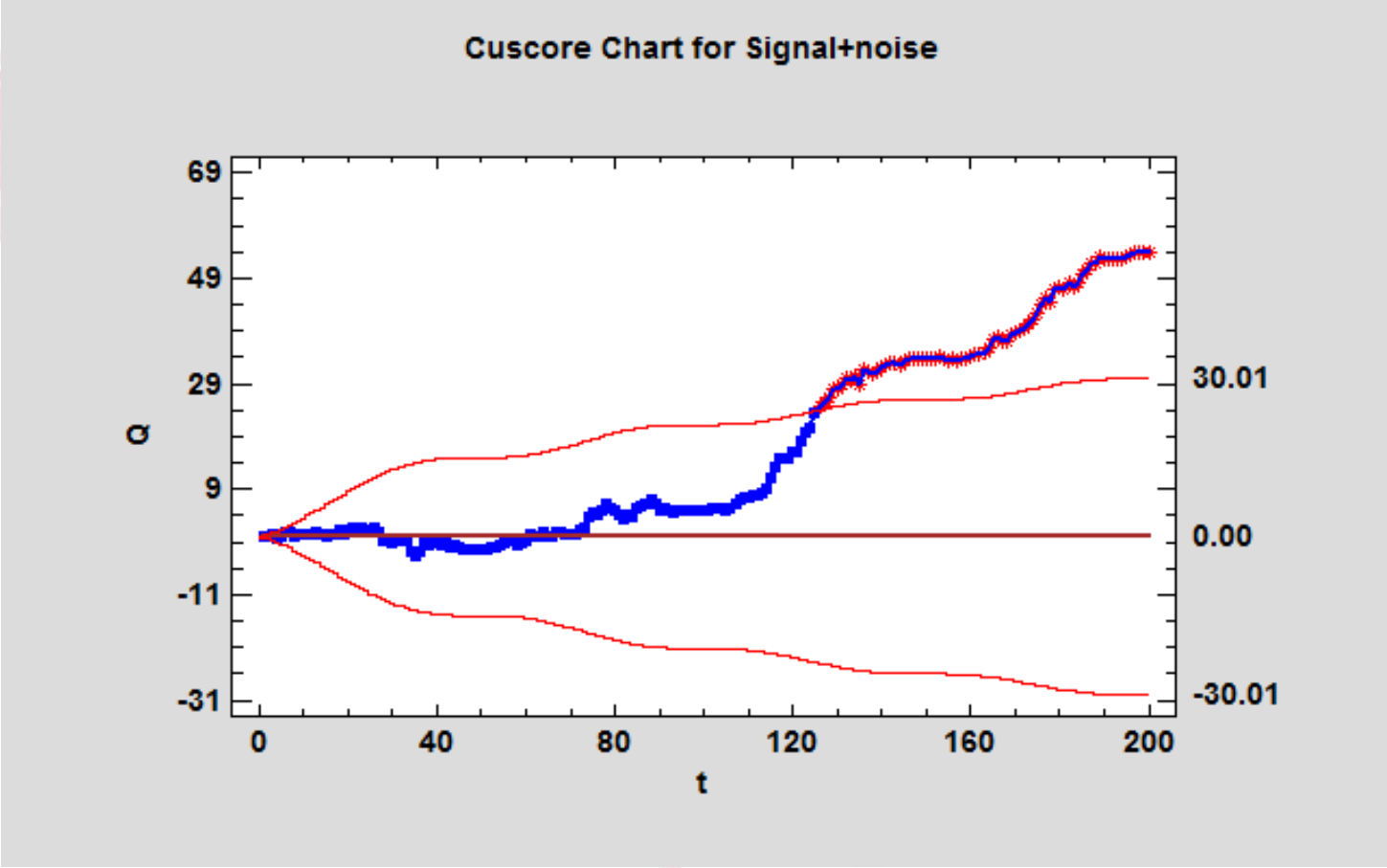
Cuscore Chart



References

- Introduction to Statistical Quality Control (7th edition) by Douglas Montgomery (2012)

- Quality Control and Industrial Statistics (5th edition) by Acheson Duncan (1986)

- Statistical Control by Monitoring and Adjustment (2nd edition) by George Box, Alberto Luceño, and Maria del Carmen Paniagua-Quinones (2009)

More Information

Go to www.statgraphics.com

- Click on “Learn” and then “Instructional Videos”.

Go to www.youtube.com

- Search for “Statgraphics control charts”.