

# Robust Parameter Design Using Statgraphics Centurion

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# Robust Parameter Designs

Experimental designs containing 2 types of factors:

- *controllable factors* that the experimenter can manipulate both during the experiment and during production.
- *noise factors* that can be manipulated during the experiment but are normally uncontrollable.

Goal: To find *robust operating conditions*, i.e., levels of the controllable factors where the values of the response variables are both desirable and relatively insensitive to variation in the noise factors.

# Two Approaches

## 1. Crossed arrays (Genichi Taguchi)

- 2 designs are created, 1 for the controllable factors and 1 for the noise factors.
- Taguchi called these designs inner and outer arrays.
- An inner array is created for the controllable factors.
- At each location of the inner array, the outer array is performed.

## 2. Combined arrays (Doug Montgomery's response surface method)

- A single design is created for both the controllable and noise factors.
- Interactions between controllable factors and noise factors reveal location of robust operating conditions.

# Example #1 – Crossed Arrays

- Myers, Montgomery, and Anderson-Cook (2009) provide an example aimed at minimizing the rate of soldering defects.
- Response:  $Y$  = defects per million joints
- Controllable factors:
  - $X1$  = temperature
  - $X2$  = speed
  - $X3$  = flux density
  - $X4$  = preheat temperature
  - $X5$  = wave height
- Noise factors:
  - $Z1$  = variation in temperature
  - $Z2$  = variation in conveyor speed
  - $Z3$  = type of assembly

# Step 1: Specify response

Design of Experiments Wizard - Define Responses

Design file: <untitled>

Comment: Robust parameter soldering study

Number of responses: 1

Response	Name	Units	Analyze	Goal	Target	Impact (1-5)	Sensitivity	Minimum	Maximum
1	defects	per million joints	SNR: smaller	Maximize	0.5	3.0	Medium		
2	Var_2		Mean	Maximize	0.5	3.0	Medium		
3	Var_3		Mean	Maximize	0.5	3.0	Medium		
4	Var_4		Mean	Maximize	0.5	3.0	Medium		
5	Var_5		Mean	Maximize	0.5	3.0	Medium		
6	Var_6		Mean	Maximize	0.5	3.0	Medium		
7	Var_7		Mean	Maximize	0.5	3.0	Medium		
8	Var_8		Mean	Maximize	0.5	3.0	Medium		
9	Var_9		Mean	Maximize	0.5	3.0	Medium		
10	Var_10		Mean	Maximize	0.5	3.0	Medium		
11	Var_11		Mean	Maximize	0.5	3.0	Medium		
12	Var_12		Mean	Maximize	0.5	3.0	Medium		
13	Var_13		Mean	Maximize	0.5	3.0	Medium		
14	Var_14		Mean	Maximize	0.5	3.0	Medium		
15	Var_15		Mean	Maximize	0.5	3.0	Medium		
16	Var_16		Mean	Maximize	0.5	3.0	Medium		

OK Cancel Help

# Step 2: Specify factors

Design of Experiments Wizard - Define Factors

Design file: <untitled>

Comment: Robust parameter soldering study

Number of controllable process factors: 5    Number of controllable mixture components: 0    Number of noise factors: 3

Factor	Name	Units	Type	Role	Low	High	Levels
A	temperature	degrees F	Continuous	Controllable	480	510	1,2,3,4
B	speed	ft/min	Continuous	Controllable	7.2	10	1,2,3,4
C	flux density		Continuous	Controllable	0.9	1.0	1,2,3,4
D	preheat temperature	degrees F	Continuous	Controllable	150	200	1,2,3,4
E	wave height	inches	Continuous	Controllable	0.5	0.6	1,2,3,4
F	temperature deviation	degrees F	Continuous	Noise	-5	5	1,2,3,4
G	speed deviation	ft/min	Continuous	Noise	-0.2	0.2	1,2,3,4
H	assembly type		Continuous	Noise	1	2	1,2,3,4
I	Factor_I		Continuous		-1.0	1.0	1,2,3,4
J	Factor_J		Continuous		-1.0	1.0	1,2,3,4
K	Factor_K		Continuous		-1.0	1.0	1,2,3,4
L	Factor_L		Continuous		-1.0	1.0	1,2,3,4
M	Factor_M		Continuous		-1.0	1.0	1,2,3,4

Total for controllable mixture components: 1.0

Factors A-M    Factors N-Z

OK    Back    Cancel    Help

# Step 3: Select design

Design of Experiments Wizard - Select Design

Design file: solder.sgx  
Comment: Robust parameter soldering study

Robust Parameter Design  
Process factors:  
 Combined array  
 Crossed array

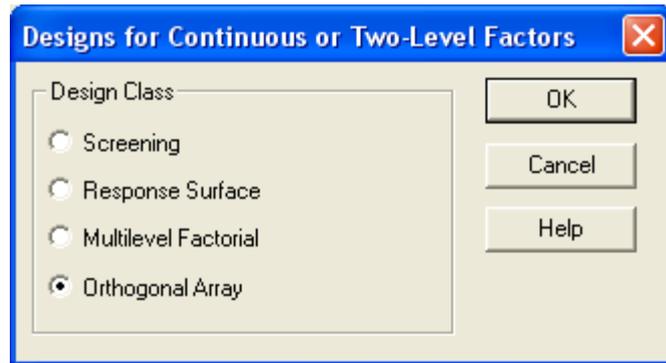
Options...	Segment	Factors	Runs	Blocks	Design
Options...	Controllable process	5	0	0	Press the Options button.
Options...	Mixture components	0	0	0	
Options...	Noise factors	3	0	0	Press the Options button.
	COMBINED	8	0	0	Samples per run: 1

BLOCK	temperature degrees F	speed ft/min	flux density	preheat temperature degrees F	wave height inches	te
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OK Cancel Rerandomize Help

# Step 3: Select design (cont.)

First select the design for the controllable factors.



# Step 3: Select design (cont.)

**Orthogonal Array Options**

Design

L4 ( $2^3$ )       L32 ( $2^{31}$ )  
 L8 ( $2^7$ )       L32 ( $2^{1 \times 4^9}$ )  
 L9 ( $3^4$ )       L36 ( $2^{11} \times 3^{12}$ )  
 L12 ( $2^{11}$ )       L36 ( $2^3 \times 3^{13}$ )  
 L16 ( $2^{15}$ )       L50 ( $2^1 \times 5^{11}$ )  
 L16 ( $4^5$ )       L54 ( $2^1 \times 3^{25}$ )  
 L18 ( $2^1 \times 3^7$ )       L64 ( $2^{63}$ )  
 L25 ( $5^6$ )       L64 ( $4 \times 21$ )  
 L27 ( $3^{13}$ )       L81 ( $3^{40}$ )

Replicate Design  
 Number:

Randomize

OK  
 Cancel  
 Back  
 Help

**Column Assignments**

Factor	Column
A	<input type="text" value="1"/>
B	<input type="text" value="2"/>
C	<input type="text" value="4"/>
D	<input type="text" value="5"/>
E	<input type="text" value="6"/>

OK  
 Cancel  
 Back  
 Help

Column Run	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

# Step 3: Select design (cont.)

**Design of Experiments Wizard - Select Design**

Design file: C:\DocData16\solder.sgx  
Comment: Robust parameter soldering study

Robust Parameter Design  
Process factors:  
 Combined array  
 Crossed array

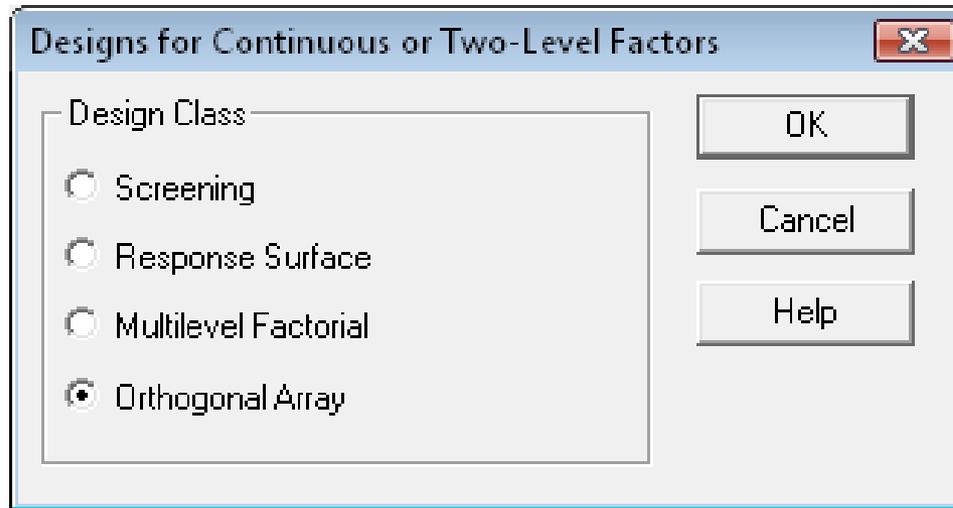
	Segment	Factors	Runs	Blocks	Design
Options...	Controllable process	5	8	1	L8 (2 <sup>7</sup> )
Options...	Mixture components	0	0	0	
Options...	Noise factors	3	0	0	Press the Options button.
	COMBINED	8	8	1	Samples per run: 1

	BLOCK	temperature degrees F	speed ft/min	flux density	preheat temperature degrees F	wave height inches	te ▲
1	1	480.0	7.2	0.9	150.0	0.5	
2	1	480.0	7.2	1.0	200.0	0.6	
3	1	480.0	10.0	0.9	150.0	0.6	
4	1	480.0	10.0	1.0	200.0	0.5	
5	1	510.0	7.2	0.9	200.0	0.5	
6	1	510.0	7.2	1.0	150.0	0.6	
7	1	510.0	10.0	0.9	200.0	0.6	
8	1	510.0	10.0	1.0	150.0	0.5	

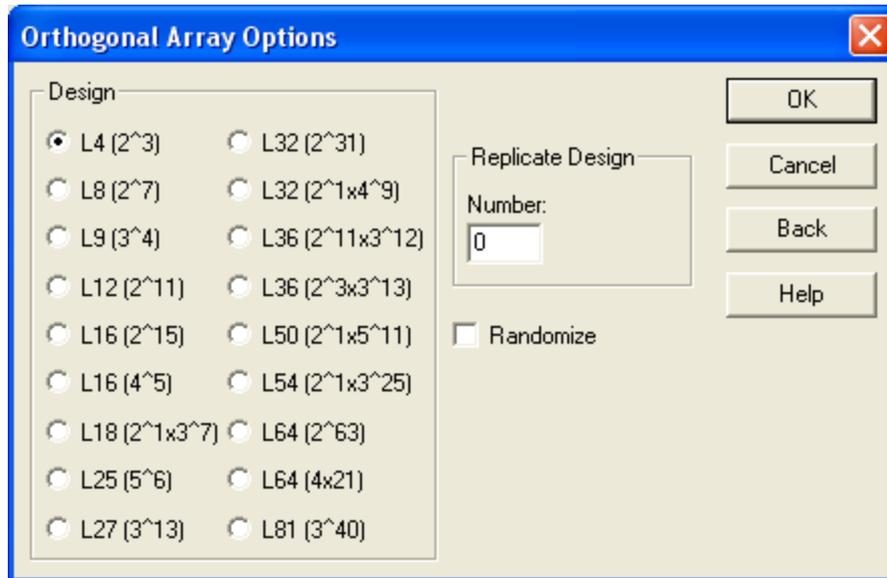
OK Cancel Rerandomize Help

# Step 3: Select design (cont.)

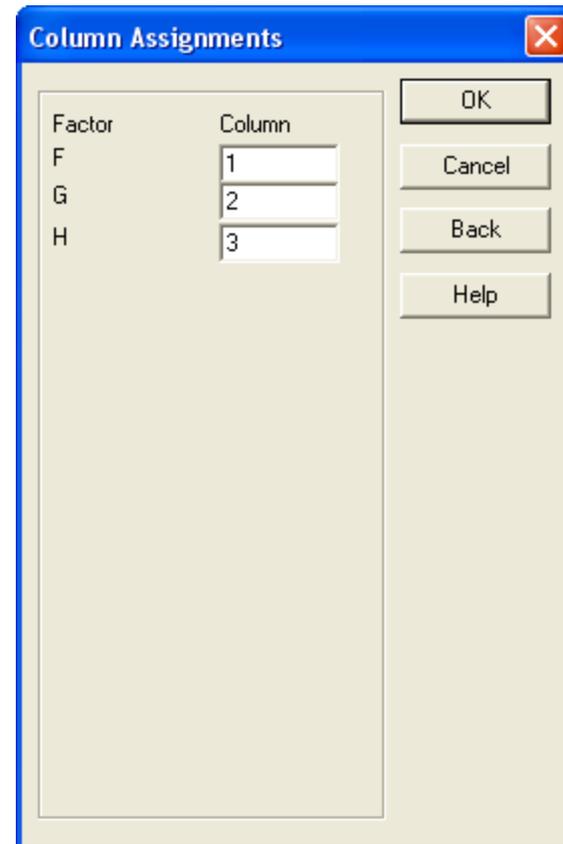
Next select the design for the noise factors.



# Step 3: Select design (cont.)



The 'Orthogonal Array Options' dialog box shows the 'Design' section with several radio button options. The 'L4 (2^3)' option is selected. Other options include L8 (2^7), L9 (3^4), L12 (2^11), L16 (2^15), L16 (4^5), L18 (2^1x3^7), L25 (5^6), L27 (3^13), L32 (2^31), L32 (2^1x4^9), L36 (2^11x3^12), L36 (2^3x3^13), L50 (2^1x5^11), L54 (2^1x3^25), L64 (2^63), L64 (4x21), and L81 (3^40). There is a 'Replicate Design' section with a 'Number:' field set to 0 and a 'Randomize' checkbox which is unchecked. Buttons for 'OK', 'Cancel', 'Back', and 'Help' are on the right.



The 'Column Assignments' dialog box shows a table with 'Factor' and 'Column' columns. Factor F is assigned to Column 1, Factor G to Column 2, and Factor H to Column 3. Buttons for 'OK', 'Cancel', 'Back', and 'Help' are on the right.

Factor	Column
F	1
G	2
H	3

Column Run	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

# Step 3: Select design (cont.)

**Design of Experiments Wizard - Select Design**

Design file: C:\DocData16\solder.sgx  
 Comment: Robust parameter soldering study

Robust Parameter Design  
 Process factors:  
 Combined array  
 Crossed array

	Segment	Factors	Runs	Blocks	Design
Options...	Controllable process	5	8	1	L8 (2 <sup>7</sup> )
Options...	Mixture components	0	0	0	
Options...	Noise factors	3	4	1	L4 (2 <sup>3</sup> )
	COMBINED	8	32	1	Samples per run: 1

	BLOCK	temperature degrees F	speed ft/min	flux density	preheat temperature degrees F	wave height inches	te
1	1	480.0	7.2	0.9	150.0	0.5	-5.0
2	1	480.0	7.2	0.9	150.0	0.5	-5.0
3	1	480.0	7.2	0.9	150.0	0.5	5.0
4	1	480.0	7.2	0.9	150.0	0.5	5.0
5	2	480.0	7.2	1.0	200.0	0.6	-5.0
6	2	480.0	7.2	1.0	200.0	0.6	-5.0
7	2	480.0	7.2	1.0	200.0	0.6	5.0
8	2	480.0	7.2	1.0	200.0	0.6	5.0
9	3	480.0	10.0	0.9	150.0	0.6	-5.0
10	3	480.0	10.0	0.9	150.0	0.6	-5.0
11	3	480.0	10.0	0.9	150.0	0.6	5.0
12	3	480.0	10.0	0.9	150.0	0.6	5.0
13	4	480.0	10.0	1.0	200.0	0.5	-5.0
14	4	480.0	10.0	1.0	200.0	0.5	-5.0
15	4	480.0	10.0	1.0	200.0	0.5	5.0

OK Cancel Rerandomize Help

# Step 8: Analyze data

**Signal-to-Noise Ratio (Smaller the Better)** - For situations in which the response is to be minimized, Taguchi proposed analyzing

$$SNR_S = -10 \log \sum_{i=1}^n \left( \frac{y_i^2}{n} \right)$$

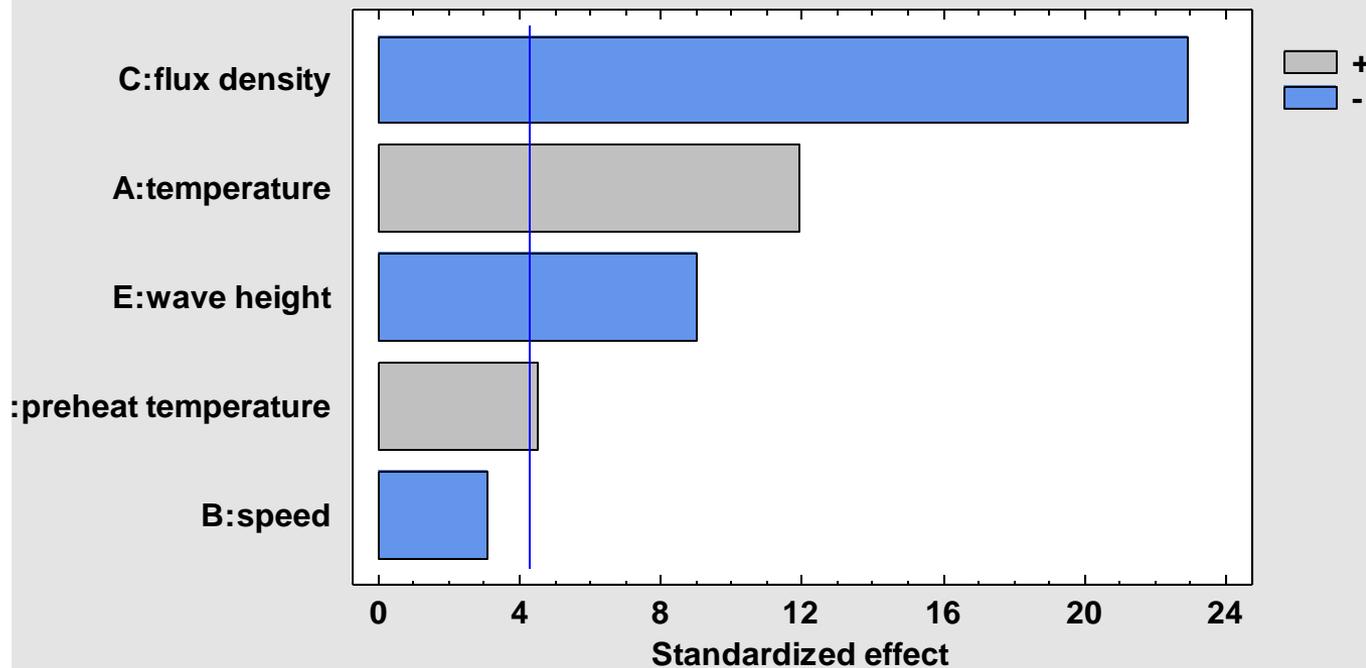
n = number of rows in outer array

$$\sum_{i=1}^n \left( \frac{y_i^2}{n} \right) = \bar{y}^2 + \left( 1 - \frac{1}{n} \right) s^2$$

Filename: solder2.sgx

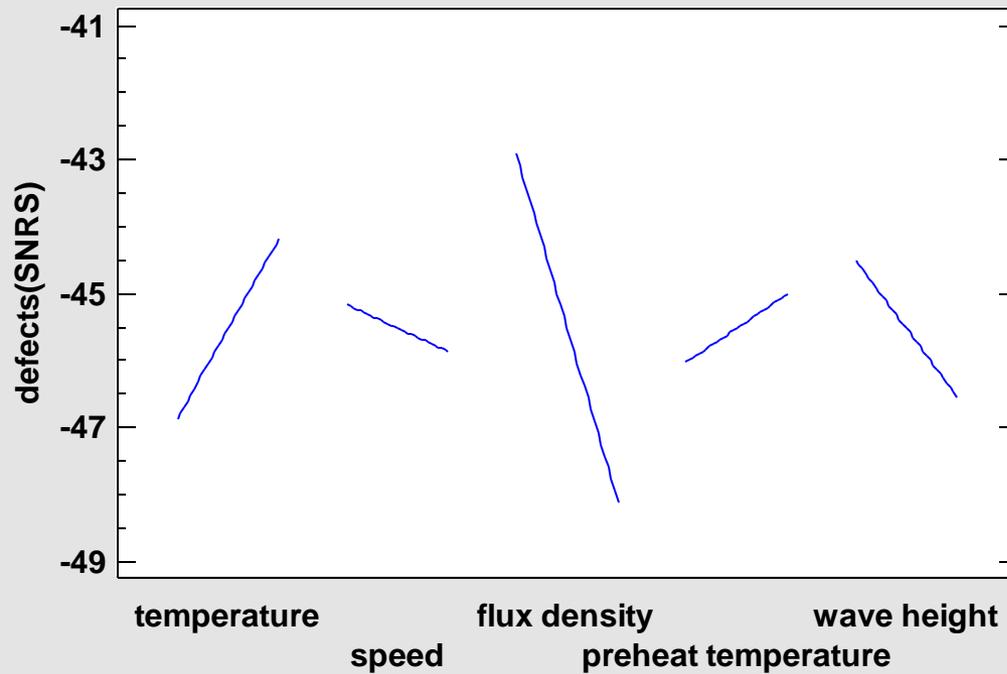
# Step 8: Analyze data (cont).

Standardized Pareto Chart for defects(SNRS)



# Step 8: Analyze data (cont.)

Main Effects Plot for defects(SNRS)



# Step 9: Optimize response

## Step 9: Optimize the responses

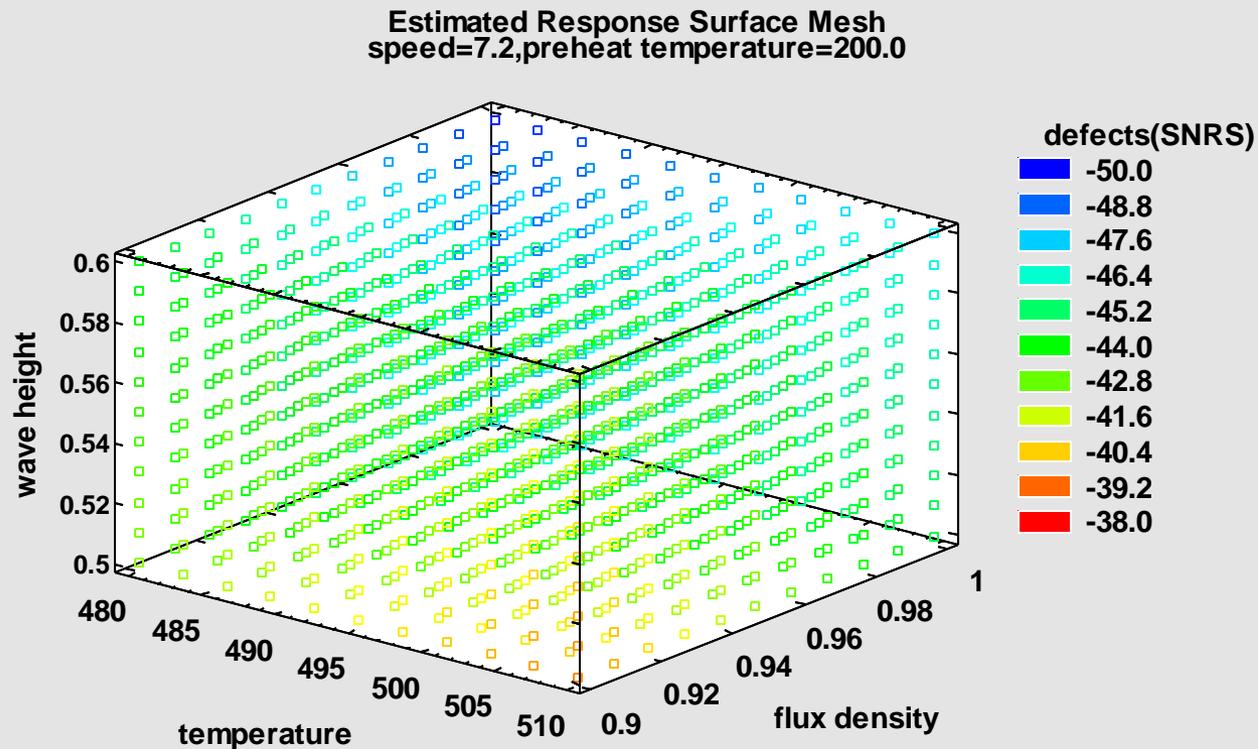
### Response Values at Optimum

<i>Response</i>	<i>Prediction</i>	<i>Lower 95.0% Limit</i>	<i>Upper 95.0% Limit</i>	<i>Desirability</i>
defects(SNRS)	-39.6893	-40.8836	-38.4951	0.984796

### Factor Settings at Optimum

<i>Factor</i>	<i>Setting</i>
temperature	510.0
speed	7.2
flux density	0.9
preheat temperature	200.0
wave height	0.5

# Step 9: Optimize response (Cont.)



# Example #2 – Combined Array

- Myers, Montgomery, and Anderson-Cook (2009) provide an example optimizing the quality of color television images.
- Response:  $Y$  = reception quality in decibels
- Controllable factors:
  - $X_1$  = filter tabs
  - $X_2$  = sampling frequency
- Noise factors:
  - $Z_1$  = image bits
  - $Z_2$  = voltage

# Step 1: Specify response

Design of Experiments Wizard - Define Responses

Design file: <untitled>

Comment: Color television signal study

Number of responses: 1

Response Name	Units	Analyze	Goal	Target	Impact (1-5)	Sensitivity	Minimum	Maximum
1 reception quality	decibels	Mean	Maximize	0.5	3.0	Medium	0	40
2 Var_2		Mean	Maximize	0.5	3.0	Medium		
3 Var_3		Mean	Maximize	0.5	3.0	Medium		
4 Var_4		Mean	Maximize	0.5	3.0	Medium		
5 Var_5		Mean	Maximize	0.5	3.0	Medium		
6 Var_6		Mean	Maximize	0.5	3.0	Medium		
7 Var_7		Mean	Maximize	0.5	3.0	Medium		
8 Var_8		Mean	Maximize	0.5	3.0	Medium		
9 Var_9		Mean	Maximize	0.5	3.0	Medium		
10 Var_10		Mean	Maximize	0.5	3.0	Medium		
11 Var_11		Mean	Maximize	0.5	3.0	Medium		
12 Var_12		Mean	Maximize	0.5	3.0	Medium		
13 Var_13		Mean	Maximize	0.5	3.0	Medium		
14 Var_14		Mean	Maximize	0.5	3.0	Medium		
15 Var_15		Mean	Maximize	0.5	3.0	Medium		
16 Var_16		Mean	Maximize	0.5	3.0	Medium		

OK Cancel Help

# Step 2: Specify factors

Design of Experiments Wizard - Define Factors

Design file: C:\DocData16\signal.sgx  
Comment: Color television signal study

Number of controllable process factors: 2    Number of controllable mixture components: 0    Number of noise factors: 2

Factor	Name	Units	Type	Role	Low	High	Levels
A	filter tabs		Continuous	Controllable	5.0	21.0	1,2
B	sampling frequency	MHz	Continuous	Controllable	6.25	13.5	1,2
C	image bits		Continuous	Noise	256.0	512.0	1,2
D	voltage	volts	Continuous	Noise	100.0	200.0	1,2
E	Factor_E		Continuous		-1.0	1.0	1,2,3,4
F	Factor_F		Continuous		-1.0	1.0	1,2,3,4
G	Factor_G		Continuous		-1.0	1.0	1,2,3,4
H	Factor_H		Continuous		-1.0	1.0	1,2,3,4
I	Factor_I		Continuous		-1.0	1.0	1,2,3,4
J	Factor_J		Continuous		-1.0	1.0	1,2,3,4
K	Factor_K		Continuous		-1.0	1.0	1,2,3,4
L	Factor_L		Continuous		-1.0	1.0	1,2,3,4
M	Factor_M		Continuous		-1.0	1.0	1,2,3,4

Total for controllable mixture components: 100.0

Factors A-M    Factors N-Z

OK    Back    Cancel    Help

# Step 3: Select design

Design of Experiments Wizard - Select Design

Design file: C:\DocData16\lvsignal.sgx

Comment: Color television signal study

Robust Parameter Design

- Combined array
- Crossed array

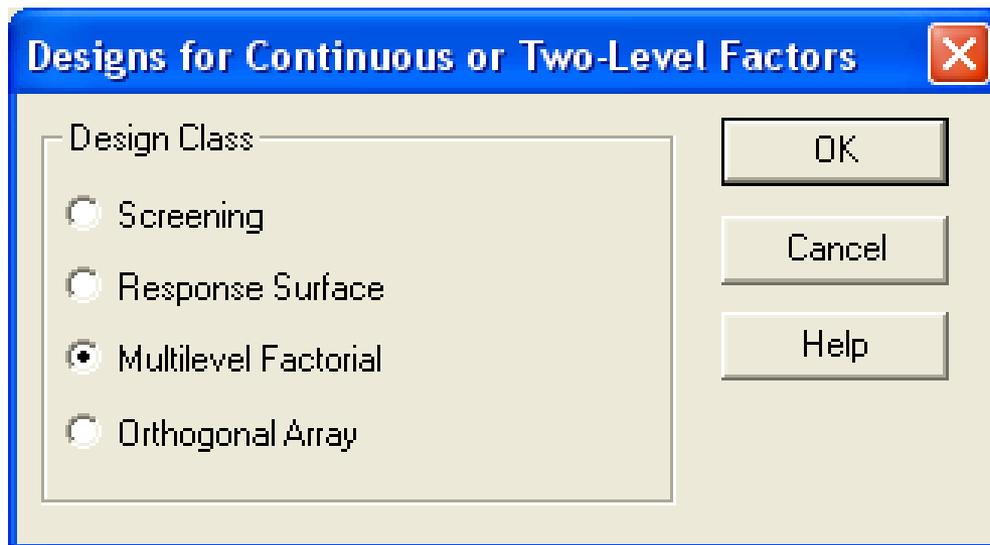
Segment	Factors	Runs	Blocks	Design	
Options...	Process factors	4	0	0	Press the Options button.
Options...	Mixture components	0	0	0	
Options...		0	0	0	
	COMBINED	4	0	0	Samples per run: 1

BLOCK	filter tabs	sampling frequency MHz	image bits	voltage volts
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OK Cancel Rerandomize Help

# Step 3: Select design (cont.)

Select a single design for both the controllable and noise factors.



# Step 3: Select design (cont.)

Multilevel Factorial Design Options

Factor	Levels
filter tabs	3
sampling frequency	3
image bits	2
voltage	2

Runs: 36      Error d.f.: 23

Replicate Design

Number: 0

Randomize

OK  
Cancel  
Back  
Help

# Step 3: Select design (cont.)

Design of Experiments Wizard - Select Design

Design file: C:\DocData16\vtvsignal.sgx  
Comment: Color television signal study

Robust Parameter Design  
 Combined array  
 Crossed array

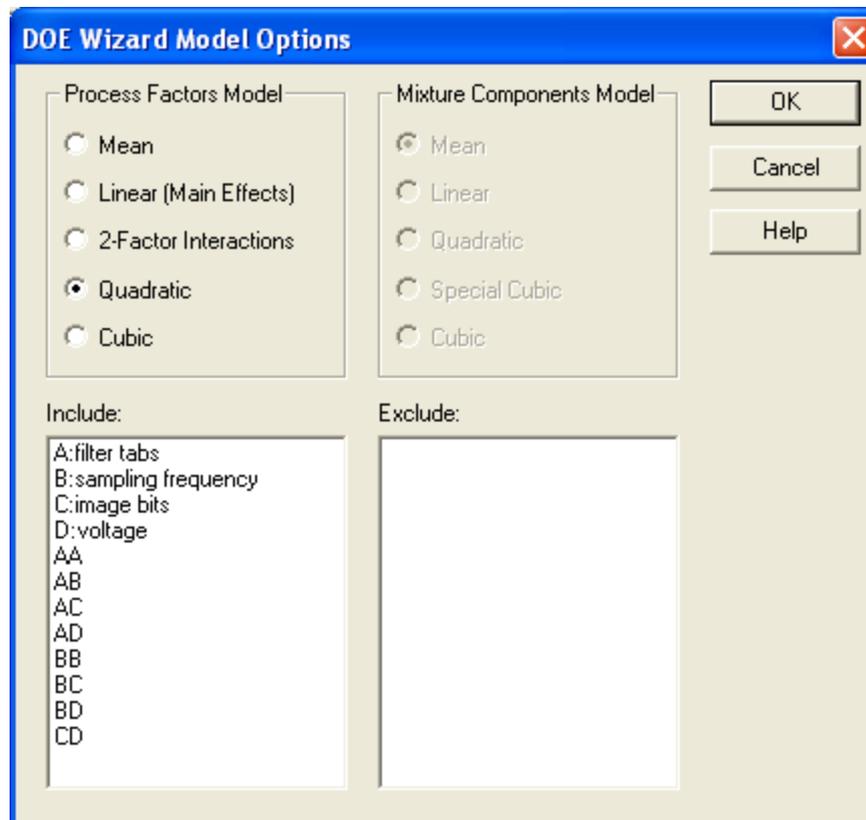
Options...	Segment	Factors	Runs	Blocks	Design
Options...	Process factors	4	36	1	Multilevel Factorial
Options...	Mixture components	0	0	0	
Options...		0	0	0	
	COMBINED	4	36	1	Samples per run: 1

	BLOCK	filter tabs	sampling frequency MHz	image bits	voltage volts
1	1	5.0	6.25	256.0	100.0
2	1	13.0	6.25	256.0	100.0
3	1	21.0	6.25	256.0	100.0
4	1	5.0	9.875	256.0	100.0
5	1	13.0	9.875	256.0	100.0
6	1	21.0	9.875	256.0	100.0
7	1	5.0	13.5	256.0	100.0
8	1	13.0	13.5	256.0	100.0
9	1	21.0	13.5	256.0	100.0
10	1	5.0	6.25	512.0	100.0
11	1	13.0	6.25	512.0	100.0
12	1	21.0	6.25	512.0	100.0
13	1	5.0	9.875	512.0	100.0
14	1	13.0	9.875	512.0	100.0
15	1	21.0	9.875	512.0	100.0

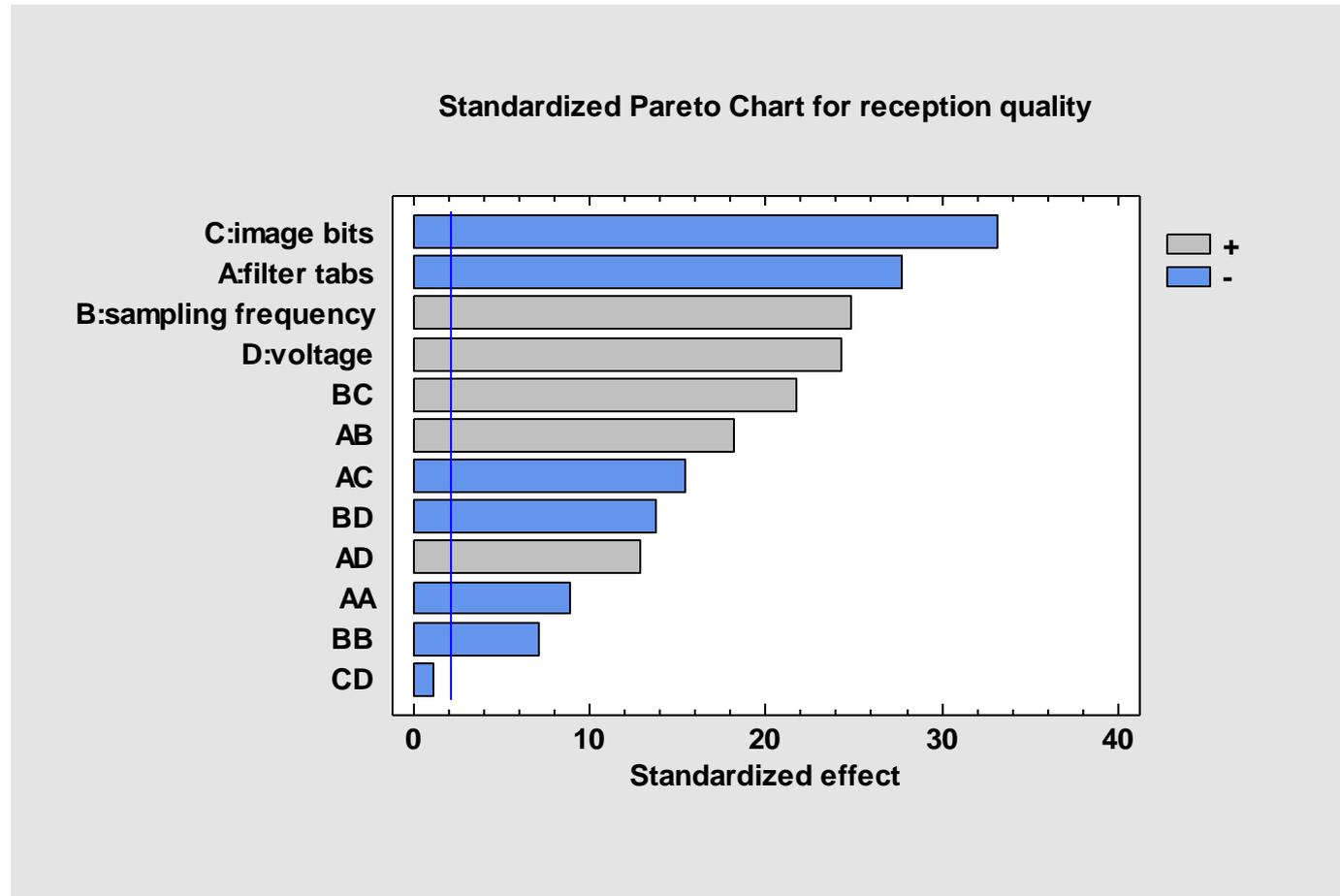
OK Cancel Rerandomize Help

# Step 4: Select model

The default model has two-factor interactions and quadratic effects for the 3-level factors.

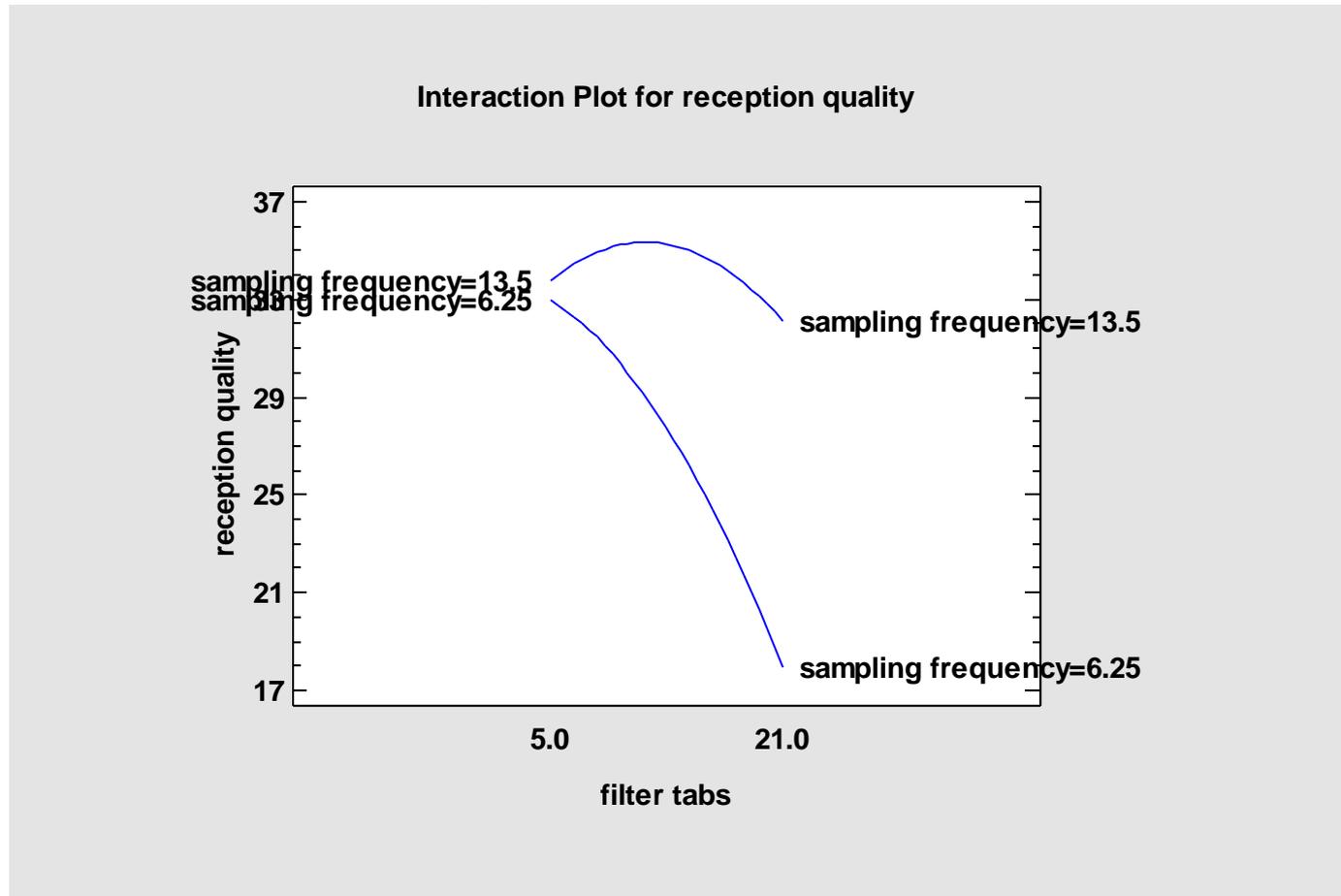


# Step 8: Analyze data



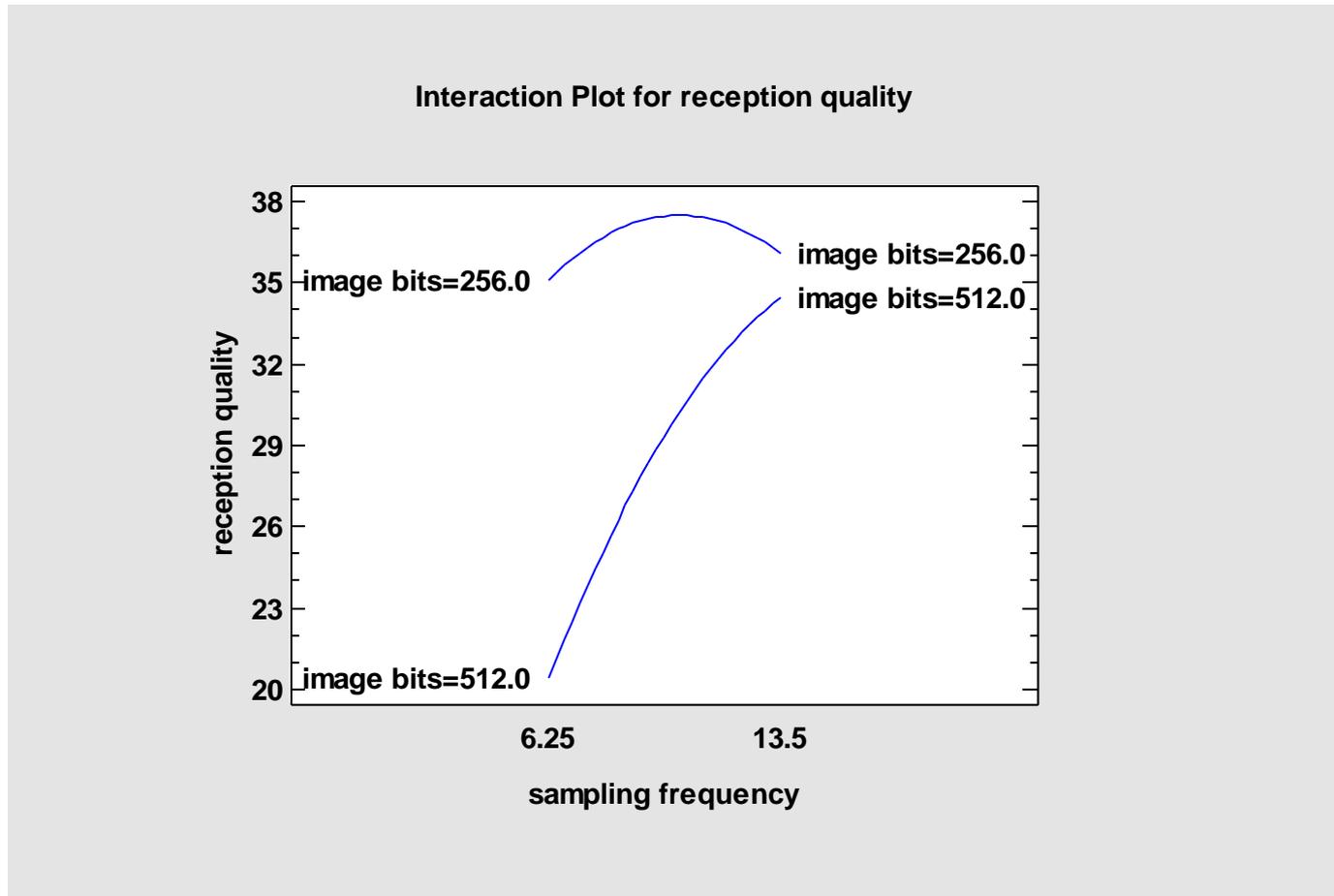
Filename: tvsignal.sgx

# Step 8: Analyze data (cont.)



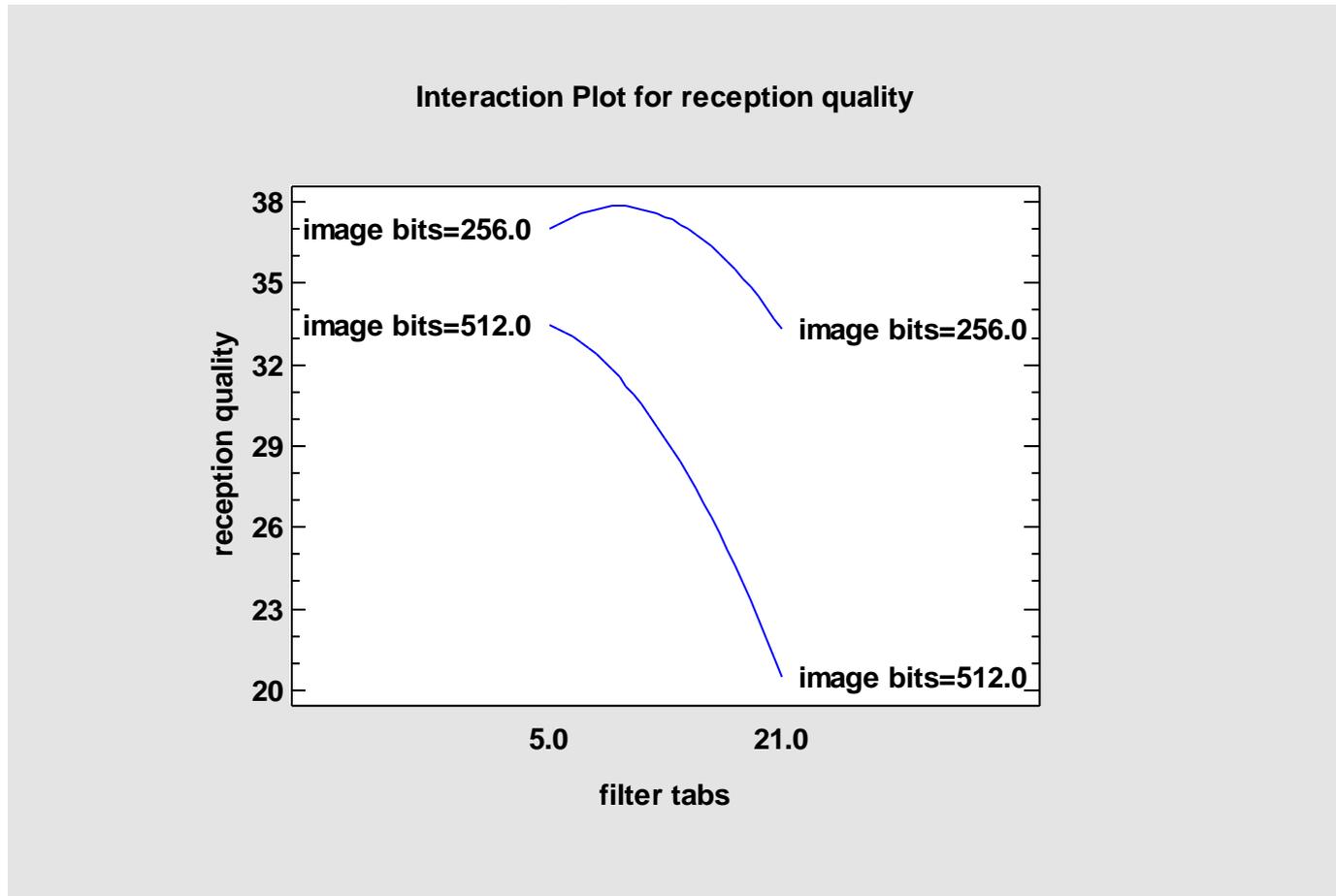
Reception quality is good at high sampling frequency, or at low sampling frequency and low filter tabs.

# Step 8: Analyze data (cont.)



Reception quality is less affected by changes in image bits at high sampling frequency.

# Step 8: Analyze data (cont.)



Reception quality is less affected by changes in image bits at low filter tabs.

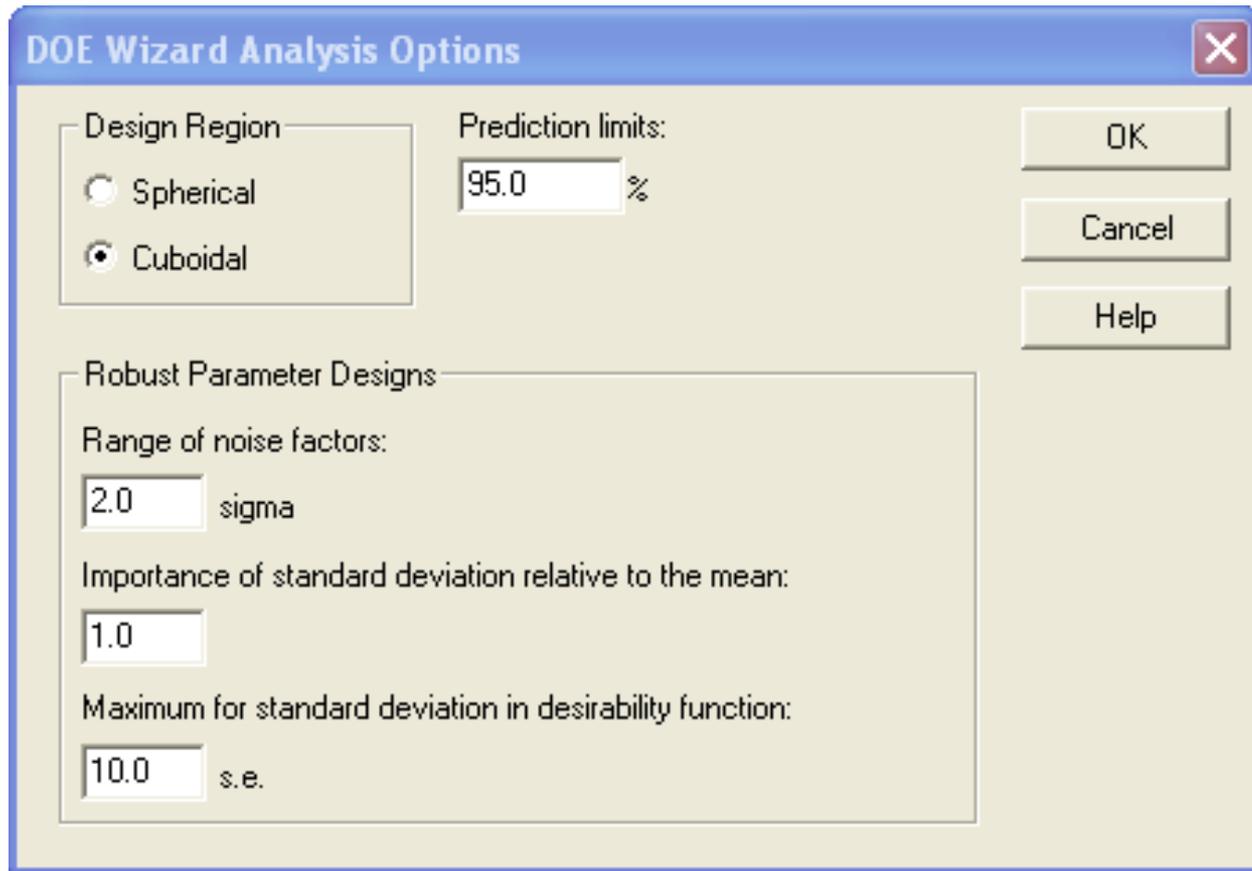
# Step 9: Optimize response

“Desirability “– on a scale of 0 to 1, how desirable are the values of the response variables at a selected combination of the controllable factors.

In this case, desirability is a combination of the mean response and the standard deviation of the response at that combination. The transmission of error formula is used to estimate the std. deviation:

$$se = \sqrt{\sum_{i=1}^r \left[ \frac{\partial y(x, z)}{\partial z_i} \sigma_z \right]^2} + \sigma^2$$

# Analysis Options



The image shows a software dialog box titled "DOE Wizard Analysis Options". It has a blue title bar with a close button (X) in the top right corner. The dialog is divided into several sections:

- Design Region:** A group box containing two radio buttons: "Spherical" (unselected) and "Cuboidal" (selected).
- Prediction limits:** A text input field containing "95.0" followed by a percentage symbol (%).
- Robust Parameter Designs:** A large group box containing:
  - Range of noise factors:** A text input field with "2.0" followed by the text "sigma".
  - Importance of standard deviation relative to the mean:** A text input field with "1.0".
  - Maximum for standard deviation in desirability function:** A text input field with "10.0" followed by the text "s.e."

On the right side of the dialog, there are three buttons: "OK", "Cancel", and "Help".

# Step 9: Optimize response (cont.)

## Step 9: Optimize the responses

### Response Values at Optimum

<i>Response</i>	<i>Optimized</i>	<i>Prediction</i>	<i>Lower 95.0% Limit</i>	<i>Upper 95.0% Limit</i>
reception quality	yes	35.3544	34.5018	36.207

<i>Standard Deviation</i>	<i>Desirability</i>
0.738624	0.883626

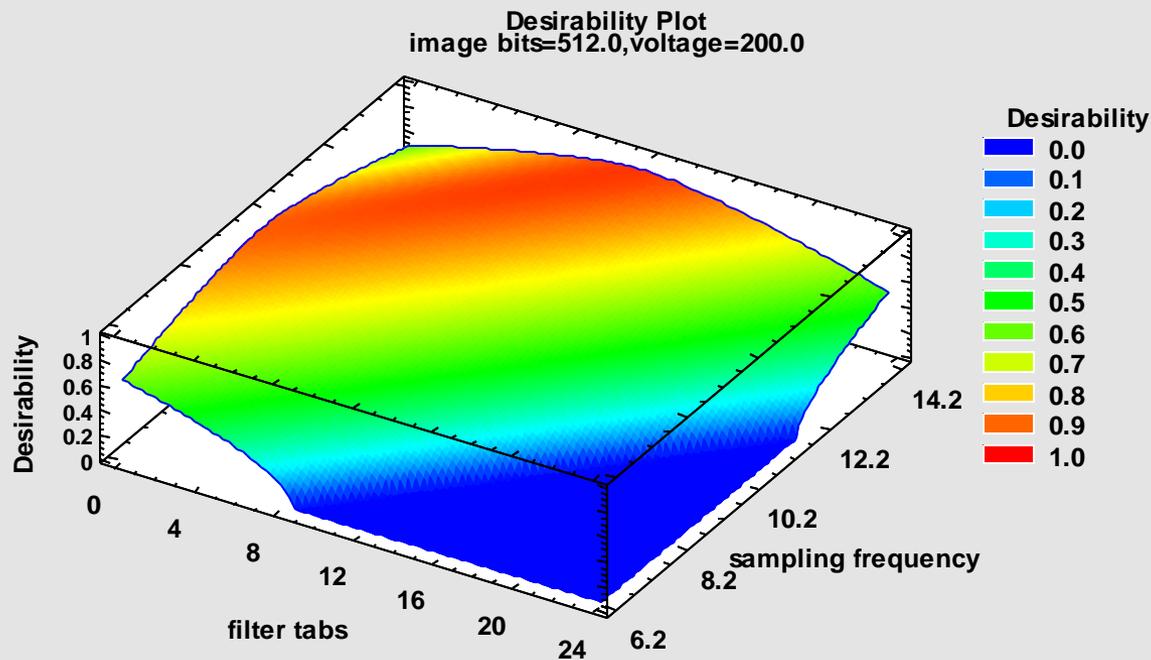
### Factor Settings at Optimum

<i>Factor</i>	<i>Setting</i>
filter tabs	9.83853
sampling frequency	13.5

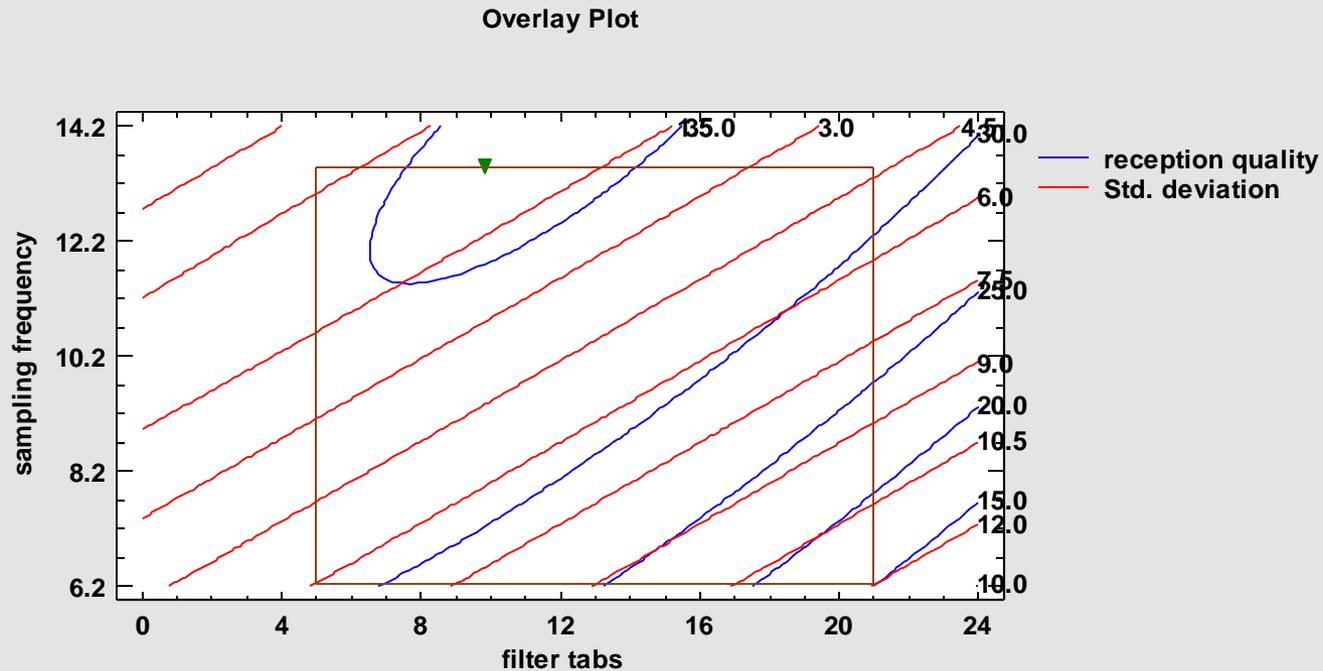
Transmission of error formula used to estimate the std. deviation of the response at the optimal conditions.

$$se = \sqrt{\sum_{i=1}^r \left[ \frac{\partial y(x, z)}{\partial z_i} \sigma_z \right]^2} + \sigma^2$$

# Step 9: Optimize response (cont.)



# Step 9: Optimize response (cont.)



# More Information

(1) Myers, R. H., Montgomery, D. C. and Anderson-Cook, C. M. (2009). Response Surface Methodology: Process and Product Optimization Using Designed Experiments, 3<sup>rd</sup> edition. New York: John Wiley and Sons.

(2) Statgraphics Centurion PDF file: Doe Wizard – Inner-Outer Arrays

(3) Statgraphics Centurion PDF file: Doe Wizard – Robust Parameter Designs

(4) [www.statgraphics.com](http://www.statgraphics.com)