Reliability and Life Data Analysis Using Statgraphics Centurion Part 2

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Life Data

The type of data considered in this webinar consists of lifetimes or times to failure.

Typical applications include:

1. Estimating product reliability
2. Estimating survival times after medical treatments

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Statgraphics Life Data Procedures

Statgraphics includes the following procedures for analyzing life data:

Procedures with no explanatory factors

- Life tables (intervals or times)
- Distribution fitting for censored data
- Weibull analysis
- Repairable systems (intervals or times)

Procedures with explanatory factors

- Arrhenius plot
- Cox proportional hazards
- Life data regression

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Outline – Part 2

Example #4: accelerated life test for device reliability employing an Arrhenius model

Example #5: Cox proportional hazards model for cancer patient survival times

Example #6: general life data regression for capacitor reliability

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Example #4 – Accelerated life test

Source: Statistical Methods for Reliability Data (Meeker and Escobar)

device-A.sgd

	temperature degrees C	hours	devices	censored
1	40	1298	1	0
2	40	1390	1	0
3	40	3187	1	0
4	40	3241	1	0
5	40	3261	1	0
6	40	3313	1	0
7	40	4501	1	0
8	40	4568	1	0
9	40	4841	1	0
10	40	4982	1	0
11	40	5000	90	1
12	60	581	1	0
13	60	925	1	0
14	60	1432	1	0

device-A | device P50 | C

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Arrhenius models

The percentiles *P* of the failure time distribution are assumed to change with temperature *T* according to the model

$$P = A\exp\left(-\frac{E}{kT}\right)$$

T is temperature in degrees Kelvin (°C + 273.15)
k = 1/11605 (Boltzmann's constant)
A and *E* are two unknown parameters

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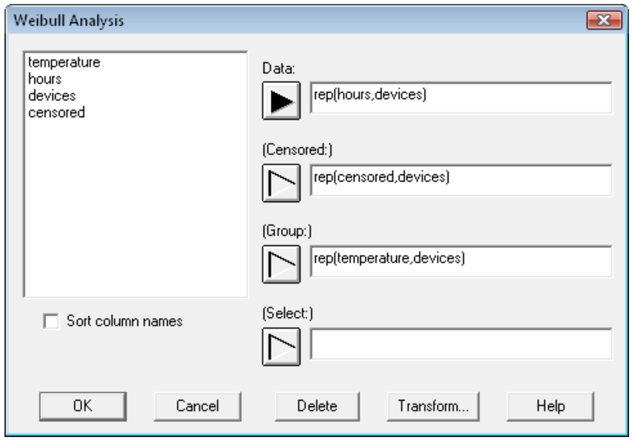
Step #1: Fit a Weibull distribution at each temperature



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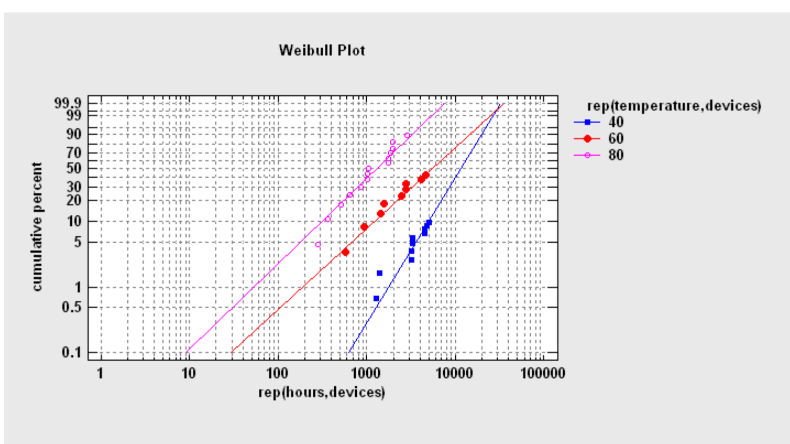
Weibull Plot



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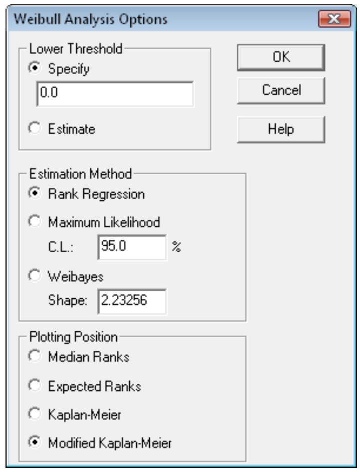
Analysis Options



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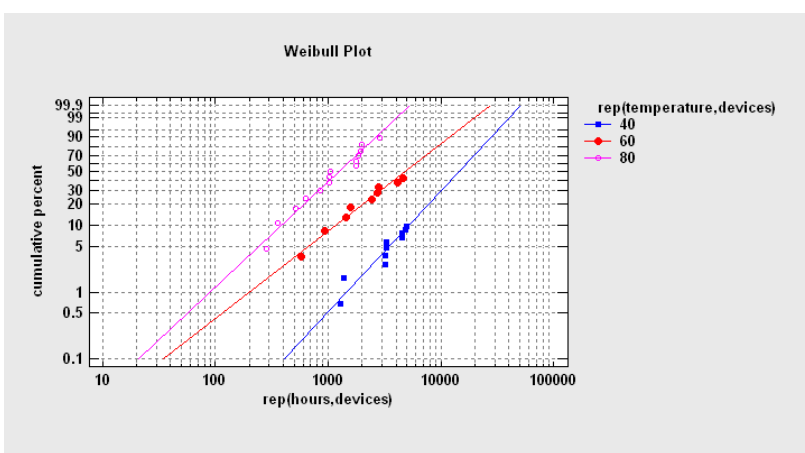
Weibull Plot



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Step #2: Estimate the percentiles

Critical Values for rep(hours,devices)

rep(temperature,devices)	*X*	*Lower Tail Area (<)*	*Upper Tail Area (>)*
40	14639.9	0.5	0.5
60	4846.64	0.5	0.5
80	1261.06	0.5	0.5

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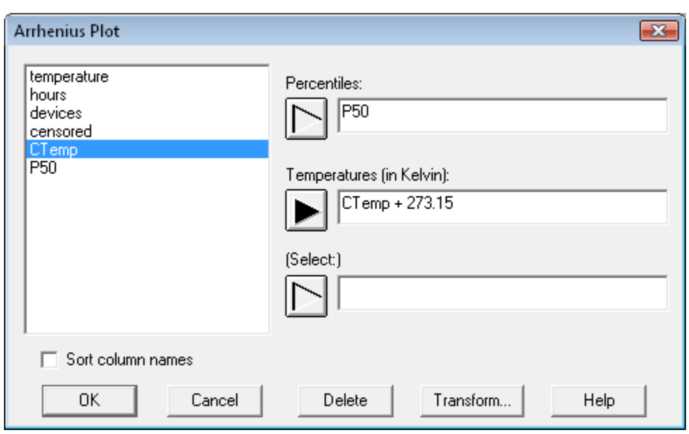
Step #3: Fit the Arrhenius model to the 50th percentiles



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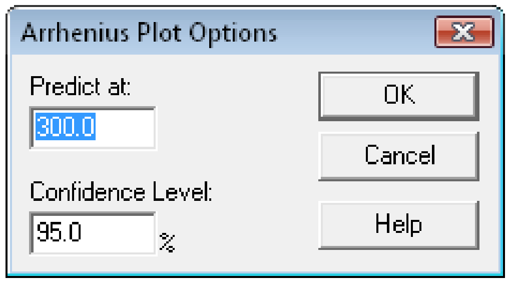
Analysis Options



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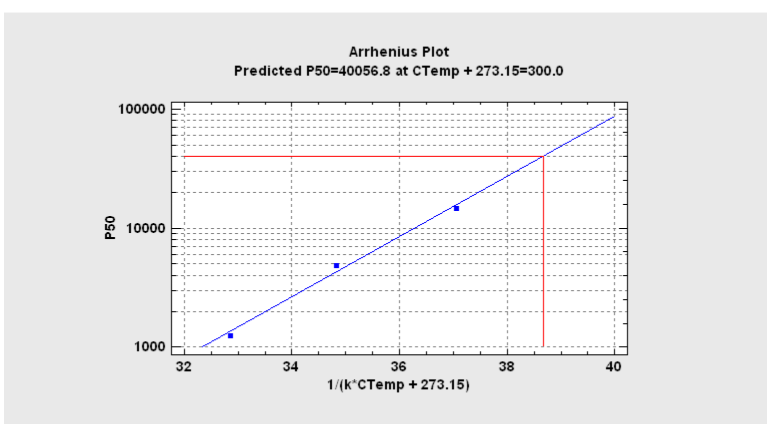
Step #4: Extrapolate to normal operating temperature



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Example #5 – Cox Proportional Hazards

Source: Modeling Survival Data in Medical Research (Collett)

C:\Webinar\nephrectomy.sgd

	Survival time	Censored	Nephrectomy	Age
1	9	0	0	1
2	6	0	0	1
3	21	0	0	1
4	15	0	0	2
5	8	0	0	2
6	17	0	0	2
7	12	0	0	3
8	104	1	1	1
9	9	0	1	1
10	56	0	1	1
11	35	0	1	1
12	52	0	1	1
13	68	0	1	1
14	77	1	1	1
15	84	0	1	1

nephrectomy / B / C

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Cox PH Model

The hazard function at any combination of the X's is assumed to be proportional to the hazard at a baseline set of conditions.

$$h_x(t) = \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k) h_0(t)$$

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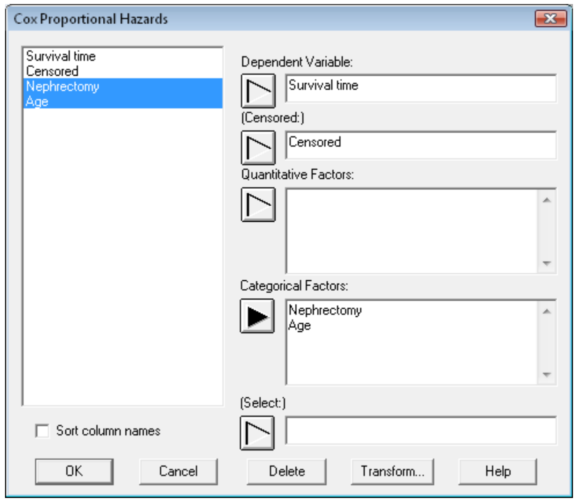
Data Input



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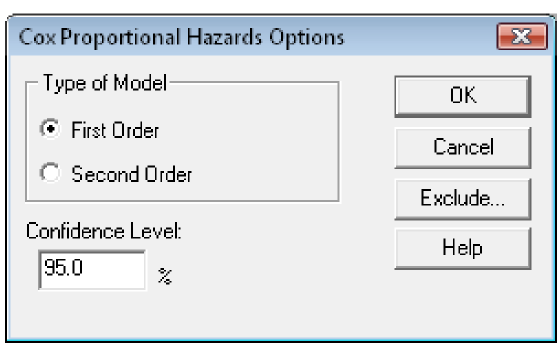
Analysis Options



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Analysis Summary

Cox Proportional Hazards - Survival time
Dependent variable: Survival time
Censoring: Censored
Factors:
Nephrectomy
Age

Number of uncensored values: 32
Number of right-censored values: 4

Estimated Regression Model

		Standard	Lower 95.0%	Upper 95.0%
Parameter	*Estimate*	*Error*	*Conf. Limit*	*Conf. Limit*
Nephrectomy=1	-1.41108	0.377288	-2.15056	-0.67161
Age=2	0.0124456	0.330352	-0.635033	0.659924
Age=3	1.34132	0.448089	0.463082	2.21956

Log likelihood = -82.7542

Likelihood Ratio Tests

Factor	*Chi-Square*	*Df*	*P-Value*
Nephrectomy	6.66386	1	0.0098
Age	4.73827	2	0.0936

The StatAdvisor
The output shows the results of fitting a failure-time regression model to describe the relationship between Survival time and 2 independent variable(s). The hazard function at a selected combination of the input factors x is a multiple of the baseline hazard function h(t|0), as shown below:

h(t|x)=h(t|0)*exp(-1.41108*Nephrectomy=1 + 0.0124456*Age=2 + 1.34132*Age=3)

In determining whether the model can be simplified, notice that the highest P-value for the likelihood ratio tests is 0.0936, belonging to Age. Because the P-value is greater or equal to 0.05, that term is not statistically significant at the 95.0% or higher confidence level. Consequently, you should consider removing Age from the model.

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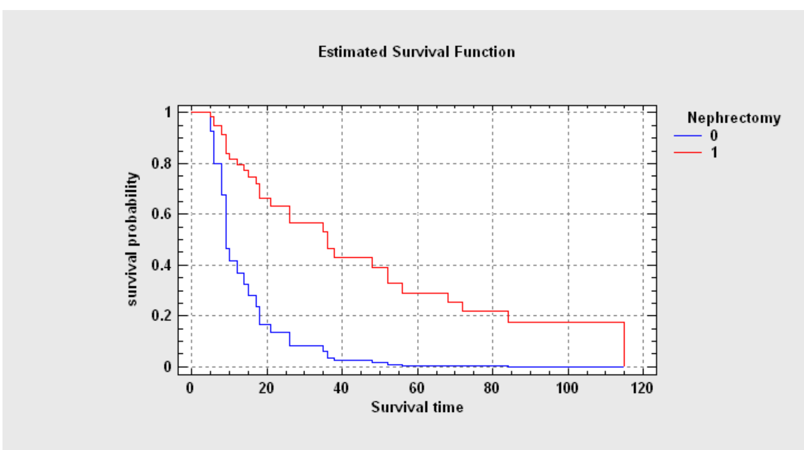
Survival Function



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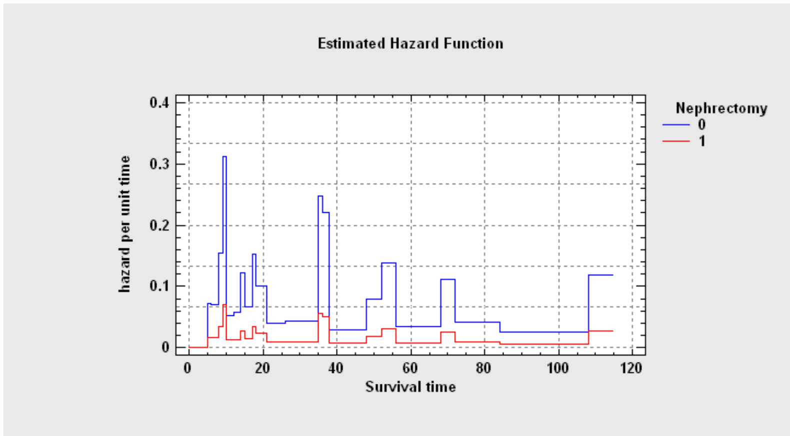
Hazard Function



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Example #6 – Life data regression

Source: <u>Statistical Methods for Reliability Data</u> (Meeker and Escobar)

C:\Program Files\Statgraphics\STATGRAPHICS Centurion XVI\Data\capacitors.sgd

	voltage	temperature	hours	Col_4	Col_5	Col_6
12	300	170	628			
13	350	170	258			
14	350	170	258			
15	350	170	347			
16	350	170	588			
17	200	180	959			
18	200	180	1065			
19	200	180	1065			
20	200	180	1087			
21	250	180	216			
22	250	180	315			
23	250	180	455			
24	250	180	473			
25	300	180	241			

capacitors B C

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General failure time regression models

- Location-scale models (normal, logistic, smallest E.V.)
- Log-location-scale models (lognormal, loglogistic, Weibull, exponential):

$$P(T \leq t) = \Phi\left(\frac{\log(t) - \mu}{\sigma}\right)$$

$$\mu = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots$$

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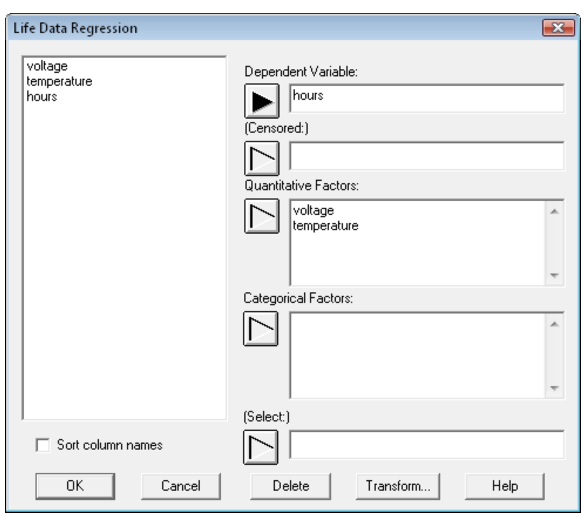
Data input



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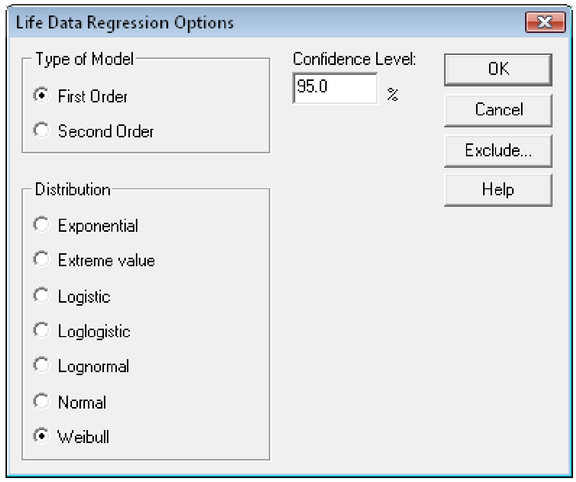
Analysis Options



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Analysis Summary

Life Data Regression - hours

Dependent variable: hours
Factors:
voltage
temperature

Number of uncensored values: 32
Number of right-censored values: 0

Estimated Regression Model - Weibull

		Standard	*Lower 95.0%*	*Upper 95.0%*
Parameter	*Estimate*	*Error*	*Conf. Limit*	*Conf. Limit*
CONSTANT	11.6981	1.96481	7.84716	15.5491
voltage	-0.00660564	0.000883368	-0.00833701	-0.00487426
temperature	-0.0200546	0.0110668	-0.0417451	0.00163591
SIGMA	0.312591	0.0432654	0.238321	0.410007

Log likelihood = -211.019

Likelihood Ratio Tests

Factor	*Chi-Square*	*Df*	*P-Value*
voltage	29.3505	1	0.0000
temperature	3.06457	1	0.0800

The StatAdvisor

The output shows the results of fitting a failure-time regression model to describe the relationship between hours and 2 independent variable(s). The equation of the fitted model is

hours = exp(11.6981 - 0.00660564*voltage - 0.0200546*temperature)

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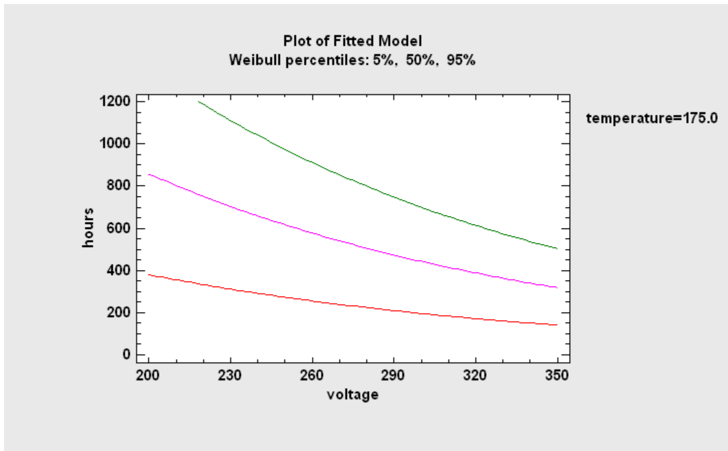
Plot of Fitted Model



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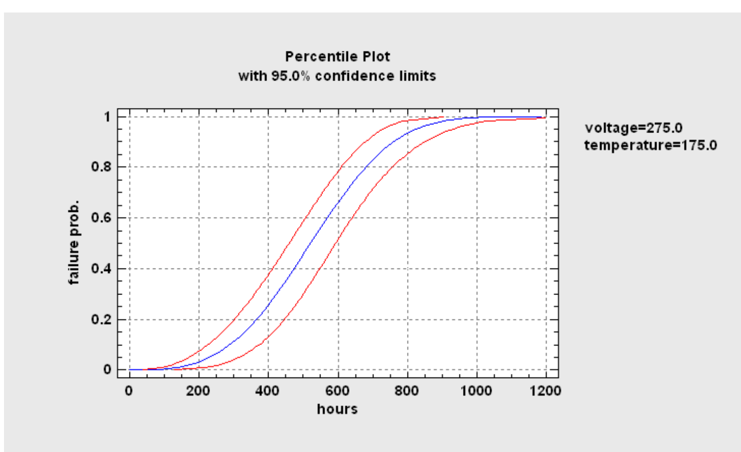
Percentile Plot



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Percentiles Table

Table of Inverse Predictions for hours
voltage=275.0
temperature=175.0

		Standard	Lower 95.0%	Upper 95.0%
Percent	Percentile	Error	Conf. Limit	Conf. Limit
0.1	67.5506	21.7934	35.8931	127.13
0.5	111.788	28.418	67.9212	183.985
1.0	138.942	31.2525	89.4071	215.922
2.0	172.831	33.8385	117.751	253.675
3.0	196.498	35.1376	138.404	278.978
4.0	215.334	35.921	155.281	298.612
5.0	231.266	36.4308	169.834	314.92
6.0	245.231	36.7737	182.781	329.016
7.0	257.762	37.0057	194.543	341.525
8.0	269.198	37.1599	205.387	352.835
9.0	279.764	37.2571	215.494	363.203
10.0	289.623	37.3114	224.996	372.813
15.0	331.643	37.2028	266.186	413.197
20.0	366.192	36.7907	300.739	445.89
25.0	396.457	36.2715	331.376	474.321
30.0	424.014	35.7291	359.463	500.156
35.0	449.788	35.2101	385.811	524.374
40.0	474.4	34.7469	410.959	547.633
45.0	498.307	34.3677	435.302	570.432
50.0	521.89	34.1008	459.156	593.195
55.0	545.493	33.9785	482.801	616.325
60.0	569.466	34.0404	506.508	640.25
65.0	[illegible]	[illegible]	[illegible]	[illegible]

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Second Order Model

Life Data Regression - hours
Dependent variable: hours
Factors:
voltage
temperature

Number of uncensored values: 32
Number of right-censored values: 0

Estimated Regression Model - Weibull

		Standard	Lower 95.0%	Upper 95.0%
Parameter	Estimate	Error	Conf. Limit	Conf. Limit
CONSTANT	17.5945	3.18682	11.3484	23.8406
voltage	-0.0359453	0.0128568	-0.0611443	-0.0107464
temperature	-0.0315621	0.0115497	-0.0541991	-0.00892516
voltage^2	0.0000529916	0.0000231238	0.00000766959	0.0000983135
SIGMA	0.289643	0.039891	0.221121	0.379397

Log likelihood = -208.593

Likelihood Ratio Tests

Factor	Chi-Square	Df	P-Value
voltage	7.01463	1	0.0081
temperature	6.42111	1	0.0113
voltage^2	4.85271	1	0.0276

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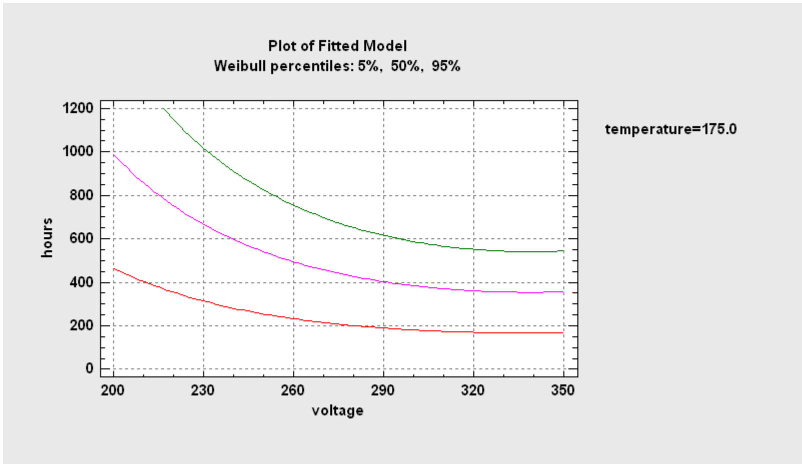
Second Order Model



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