Reliability and Life Data Analysis Using Statgraphics Centurion Part 1

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Life Data

The type of data considered in this webinar consists of lifetimes or times to failure.

Typical applications include:

1. Estimating product reliability
2. Estimating survival times after medical treatments

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Why Special Procedures are Needed

- The distribution of life data is rarely Gaussian.
- Interest usually centers around percentiles of the data distribution rather than the mean or standard deviation.
- The data are often censored (subjects leave the study early, or the study is halted before all experimental units fail).

3

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Statgraphics Life Data Procedures

Statgraphics includes the following procedures for analyzing life data:

Procedures with no explanatory factors

- Life tables (intervals or times)
- Distribution fitting for censored data
- Weibull analysis
- Repairable systems (intervals or times)

Procedures with explanatory factors

- Arrhenius plot
- Cox proportional hazards
- Life data regression

4

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Outline – Part 1

Example #1: estimation of survival function with censored data

Example #2: comparison of survival functions for grouped data

Example #3: analysis of repairable systems

5

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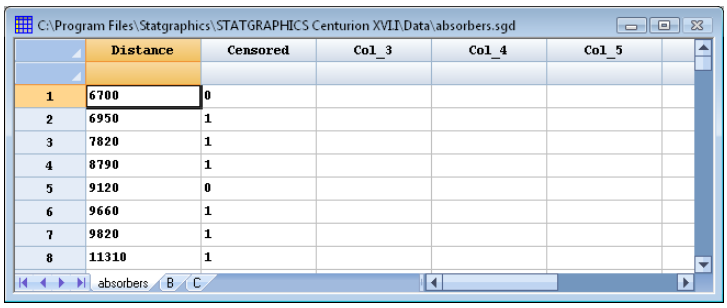
Example #1 – Distance to failure for 38 shock absorbers

Source: Statistical Methods for Reliability Data (Meeker and Escobar)



6

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Censoring indicator

Statgraphics uses an indicator variable to represent the type of censoring:

0 indicates that the value is not censored

+1 indicates a right censored value (could be larger)

-1 indicates a left censored value (could be smaller)

7

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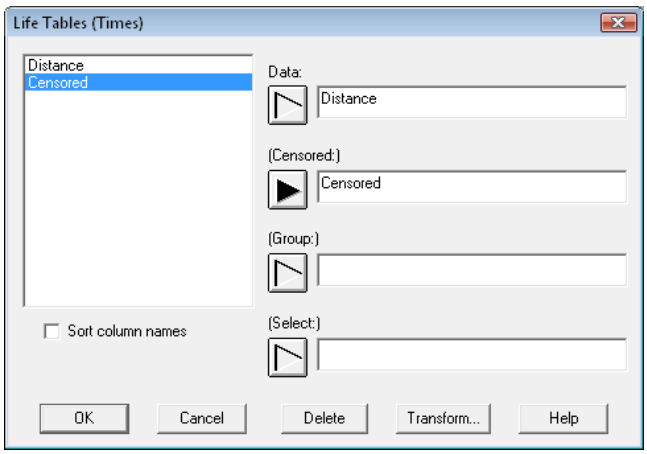
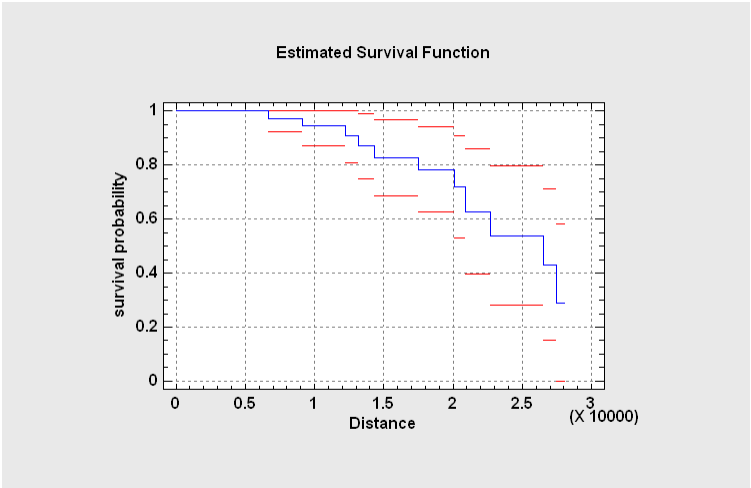
Life Tables (Times)



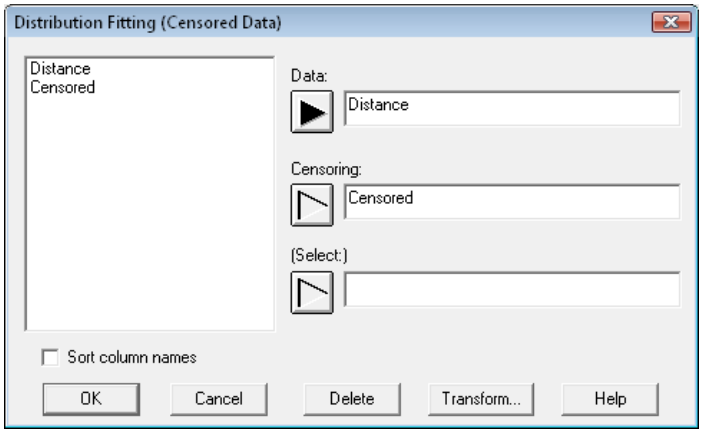
8

Survival Function – Probability of surviving until X



9

Distribution Fitting (Censored Data)



10

Comparison of Alternative Distributions

Comparison of Alternative Distributions

Distribution	*Est. Parameters*	*Log Likelihood*	*KS D*
Weibull	2	-404.991	0.0901357
Normal	2	-406.4	0.0903629
Logistic	2	-408.408	0.103344
Laplace	2	-413.516	0.108477
Smallest Extreme Value	2	-409.469	0.122783
Largest Extreme Value	2	-405.653	0.128409
Gamma	2	-404.845	0.128419
Loglogistic	2	-406.131	0.131113
Lognormal	2	-405.125	0.155015
Birnbaum-Saunders	2	-404.725	0.159099
Uniform	2	-400.338	0.159942
Inverse Gaussian	2	-404.796	0.16054
Exponential	1	-427.009	0.329046
Pareto	1	-510.249	0.448162

The StatAdvisor
This table compares the goodness-of-fit when various distributions are fit to Distance. You can select other distributions using Pane Options.

According to the Kolmogorov-Smirnov D statistic, the best fitting distribution is the Weibull distribution. To fit this distribution, press the alternate mouse button and select Analysis Options.

11

Goodness-of-Fit Test

Goodness-of-Fit Tests for Distance
Kolmogorov-Smirnov Test

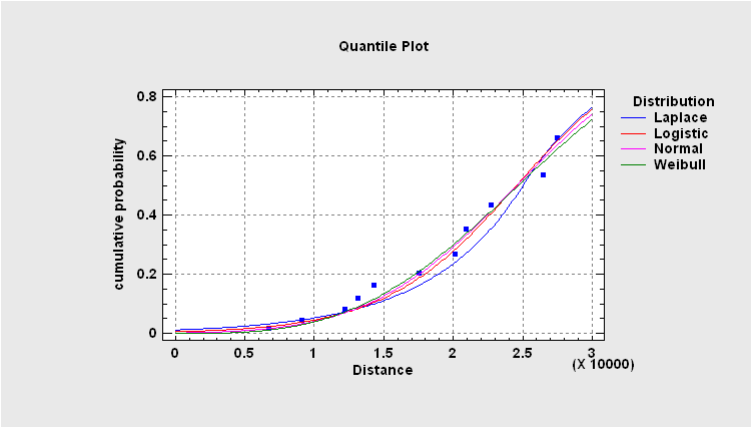
	Laplace	*Logistic*	*Normal*	*Weibull*
DPLUS	0.108477	0.0683565	0.0760417	0.0901357
DMINUS	0.108138	0.103344	0.0903629	0.087023
DN	0.108477	0.103344	0.0903629	0.0901357
P-Value	0.762511	0.811755	0.915578	0.917047

The StatAdvisor
This pane shows the results of tests run to determine whether Distance can be adequately modeled by various distributions.

P-values less than 0.05 would indicate that Distance does not come from the selected distribution with 95% confidence.

12

Quantile Plot



13

Critical Values (Percentiles)

Critical Values for Distance

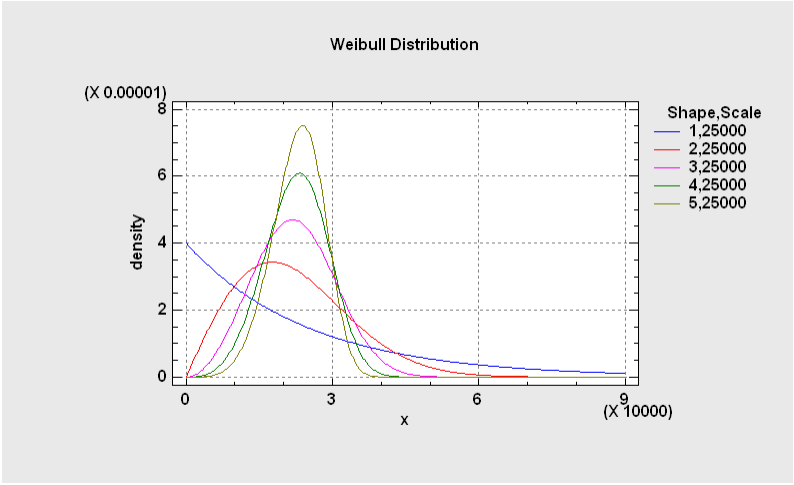
Lower Tail Area (<=)	*Laplace*	*Logistic*	*Normal*	*Weibull*
0.01	-1007.98	2647.43	5131.13	6466.15
0.1	14313.2	14074.0	13861.8	13600.0
0.5	25022.3	24544.4	24570.9	24683.6
0.9	35731.3	35014.8	35279.9	36089.5
0.99	51052.5	46441.4	44010.6	44939.6

The StatAdvisor

This pane calculates critical values for the fitted distributions. It will calculate the critical values for up to 5 lower tail areas, which you may specify by pressing the alternate mouse button and selecting Pane Options. For example, the output indicates that the value of the fitted Laplace distribution below which you would find an area equal to 0.01 is -1007.98.

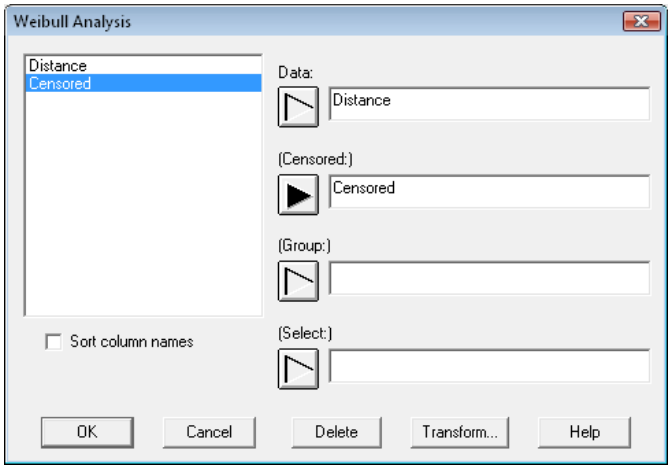
14

Weibull Distribution – defined by a shape parameter and a scale parameter



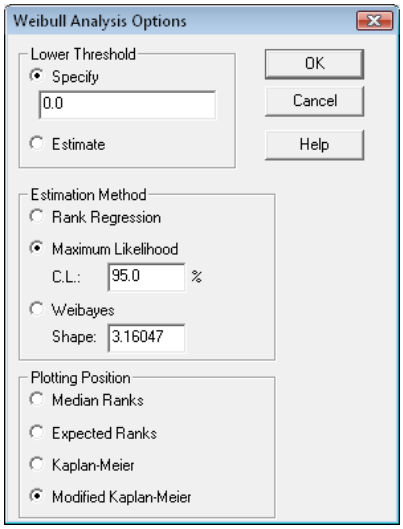
15

Weibull Analysis



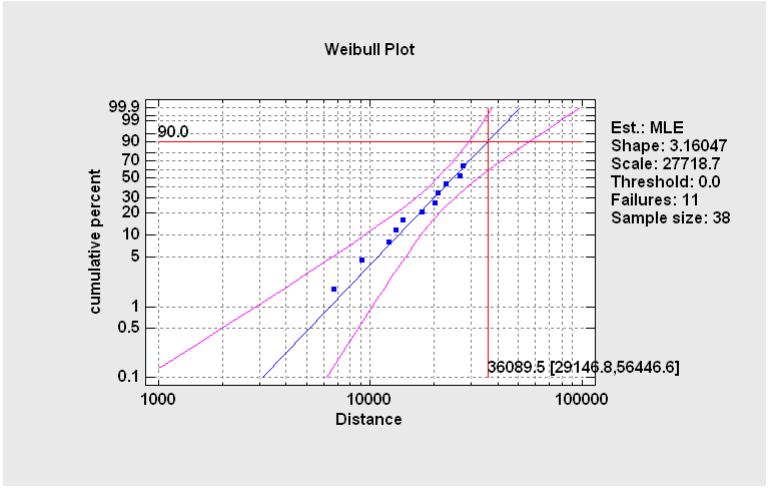
16

Analysis Options



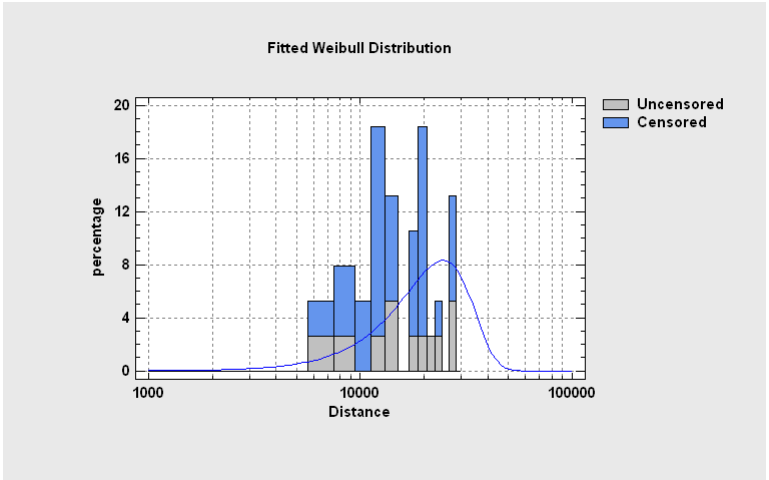
17

Weibull Plot



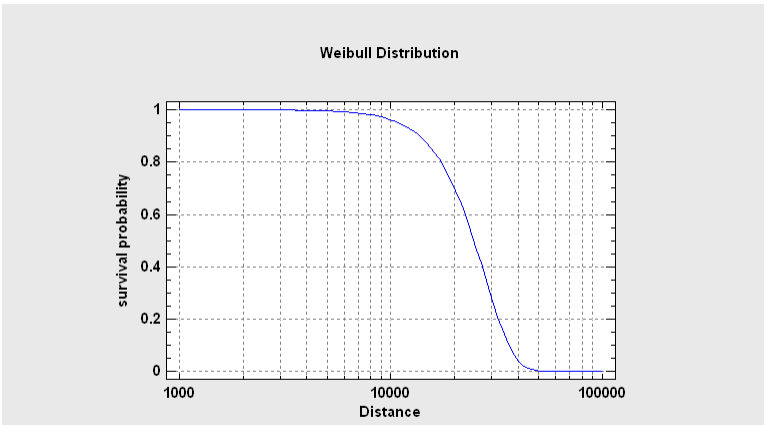
18

Fitted Weibull Distribution



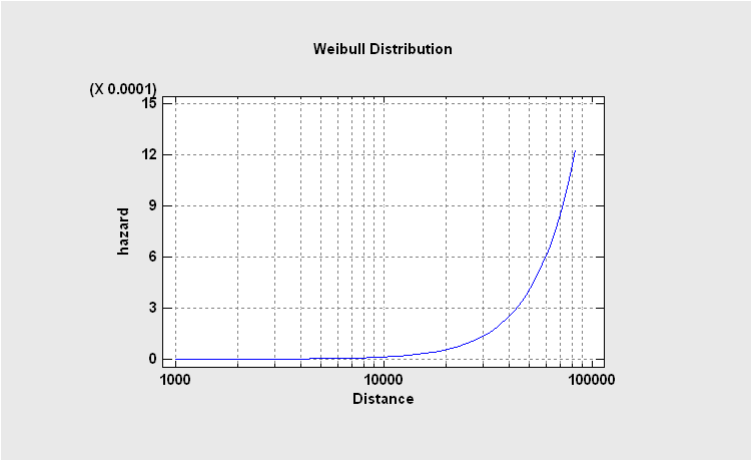
19

Survivor Function – Probability of surviving until X



20

Hazard Function – Propensity to fail given survival to X



21

Example #2 – Days to cancer mortality in rats

Source: The Statistical Analysis of Failure Time Data (Kalbfleisch and Prentice)

rats.sgd

	Group	Days	Censored	Col_4
14	1	234	0	
15	1	246	0	
16	1	265	0	
17	1	304	0	
18	1	216	1	
19	1	244	1	
20	2	142	0	
21	2	156	0	
22	2	163	0	
23	2	198	0	

rats B C

22

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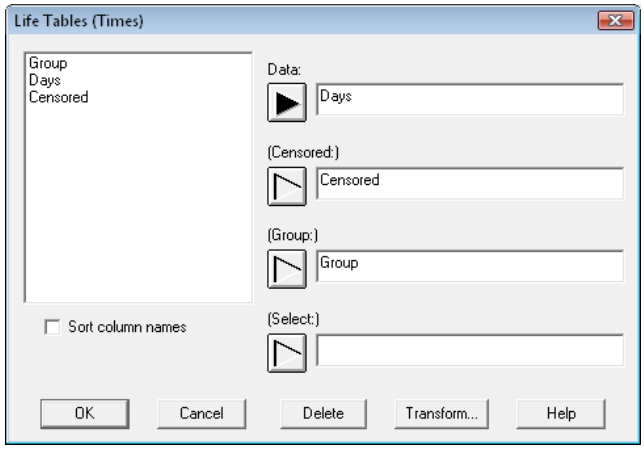
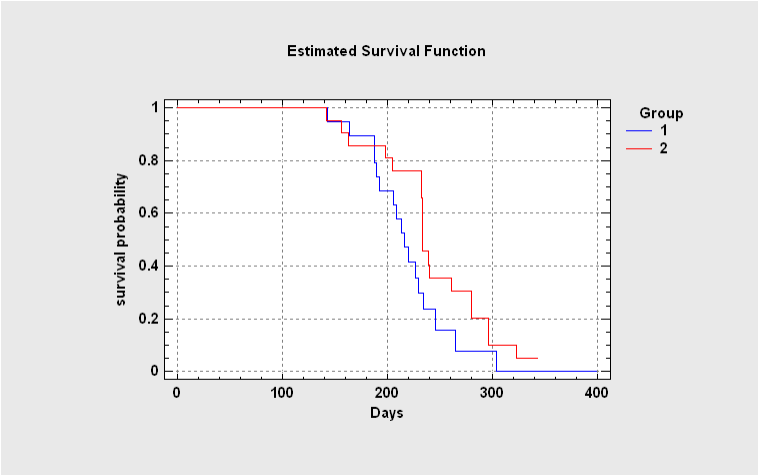
Life Tables (Times)



23

Survival Functions



24

Group Comparisons

Comparison of Groups

				Proportion
Group	*Total*	*Failed*	*Withdrawn*	*Withdrawn*
1	19	17	2	0.1053
2	21	19	2	0.0952
Total	40	36	4	0.1000

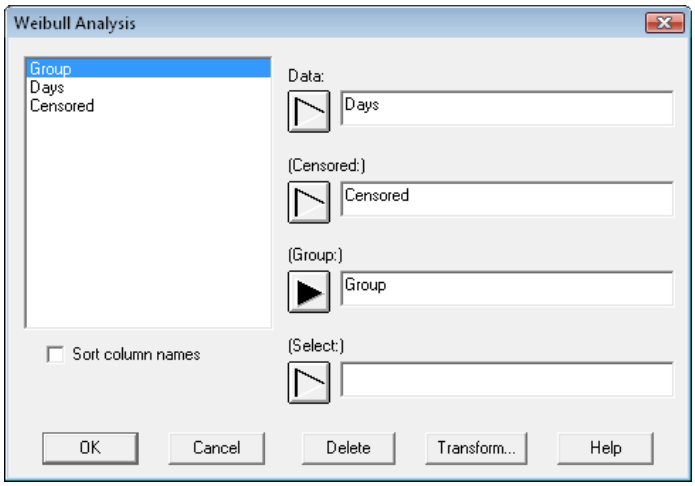
Logrank test
Chi-square = 3.12271
P-value = 0.0772045

Wilcoxon test
Chi-square = 2.65104
P-value = 0.103478

The StatAdvisor
This table displays information regarding each group of data values. It shows the total number of items tabulated, the number of items which failed, the number withdrawn or censored, and the proportion of censored items. Two tests have also been performed to determine whether there is a statistically significant difference between the survival probabilities of the 2 groups. Since the smallest P-value is greater than or equal to 0.05, there is not a statistically significant difference between the groups at the 95% confidence level.

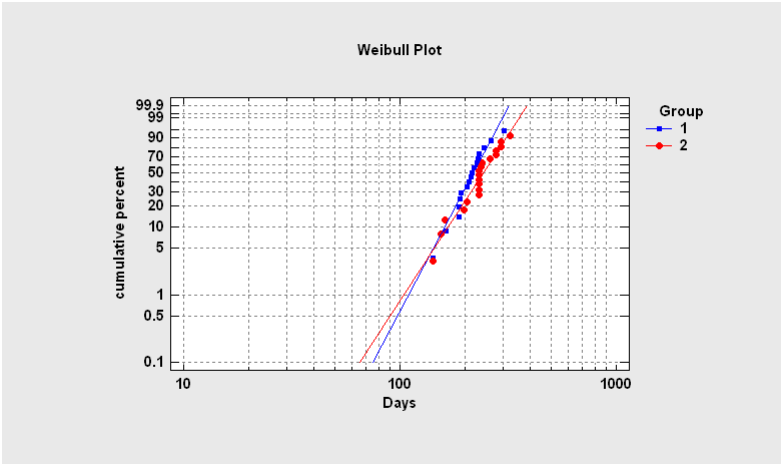
25

Weibull Analysis



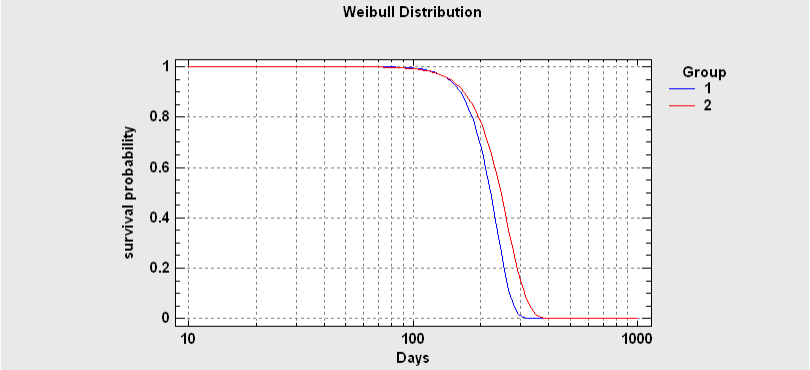
26

Weibull Plots



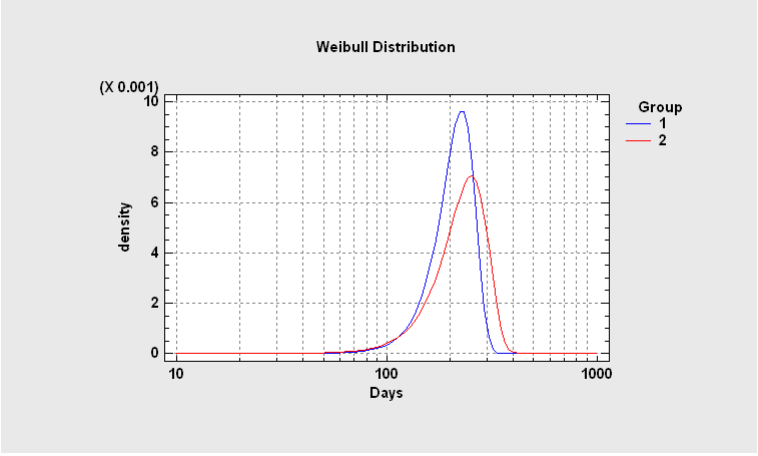
27

Survival Functions



28

Density Functions



29

Test for Significant Differences

Postulate a loglinear model for the percentiles of the lifetime distribution for group j:

$$F(t_j) = \Phi\left(\frac{\log(t) - \mu_j}{\sigma}\right)$$

Let μ_j be a function of an indicator variable I_j that takes the value 0 for one group and 1 for the other group:

$$\mu_j = \beta_0 + \beta_1 I_j$$

30

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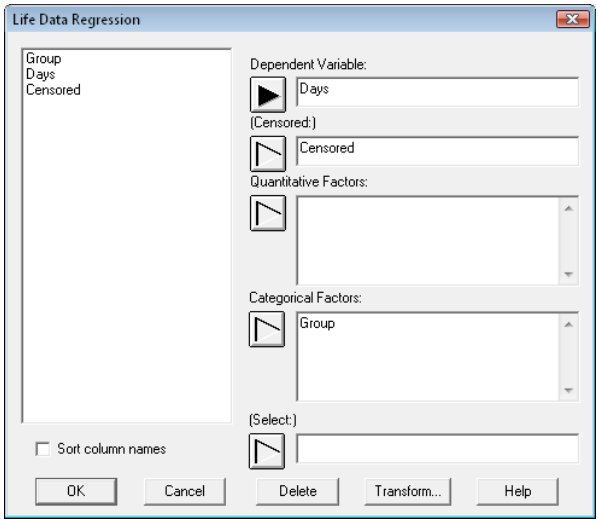
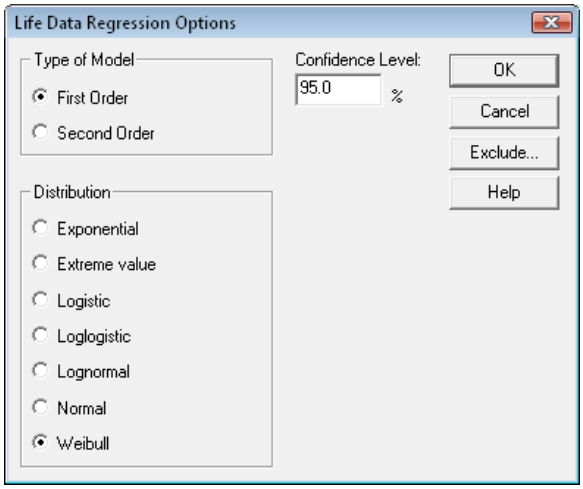
Life Data Regression



31

Analysis Options



32

Likelihood Ratio Test

Life Data Regression - Days
Dependent variable: Days
Censoring: Censored
Factors:
Group

Number of uncensored values: 36
Number of right-censored values: 4

Estimated Regression Model - Weibull

		Standard	*Lower 95.0%*	*Upper 95.0%*
Parameter	*Estimate*	*Error*	*Conf. Limit*	*Conf. Limit*
CONSTANT	5.58282	0.0429976	5.49855	5.6671
Group=1	-0.131956	0.0612686	-0.25204	-0.0118717
SIGMA	0.183325	0.0230552	0.143276	0.234569

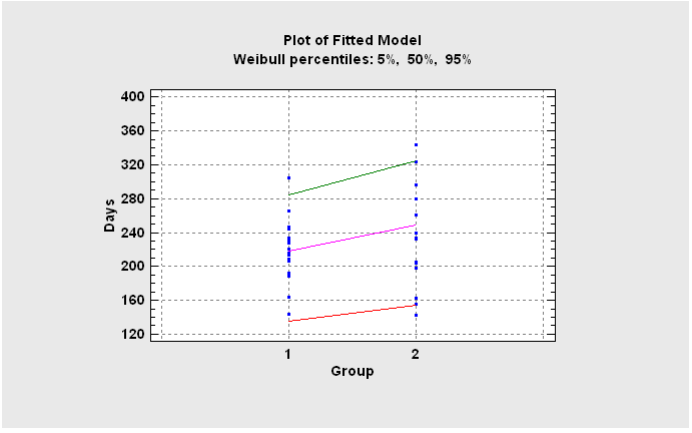
Log likelihood = -193.352

Likelihood Ratio Tests

Factor	*Chi-Square*	*Df*	*P-Value*
Group	4.1272	1	0.0422

33

Percentiles



34

Example #3 – Repairable systems

Source: The Statistical Analysis of Series of Events (Cox and Lewis)

C:\DocData16Hold\failures.sgd

	Aircraft 1	Aircraft 2	Aircraft 3	Aircraft 4	Aircraft 5	Aircraft 6
1	194	413	90	74	55	23
2	209	427	100	131	375	284
3	250	485	160	179	431	371
4	279	522	346	208	535	378
5	312	622	407	710	755	498
6	493	687	456	722	994	512
7		696	470	792	1041	574
8		865	494	813	1287	621
9		1312	550	842	1463	846
10		1496	570	1228	1645	917
11		1532	649	1287	1678	1163
12		1733	733	1314	1693	1184

failures B C

35

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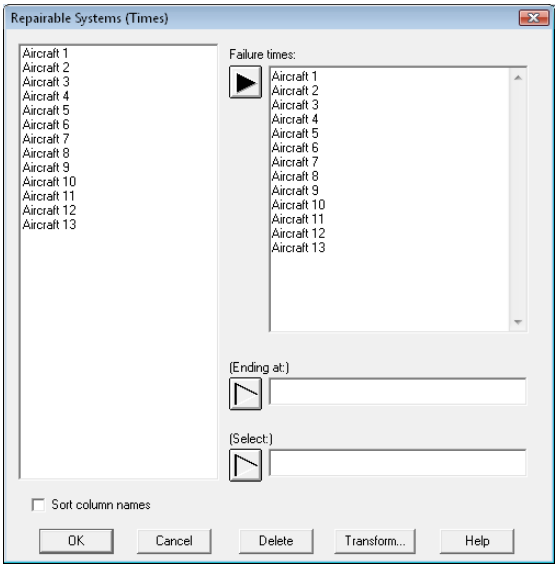
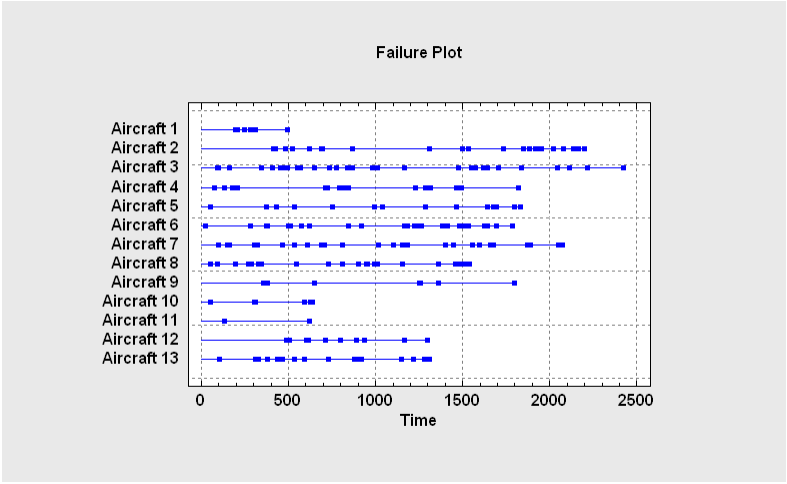
Repairable Systems (Times)



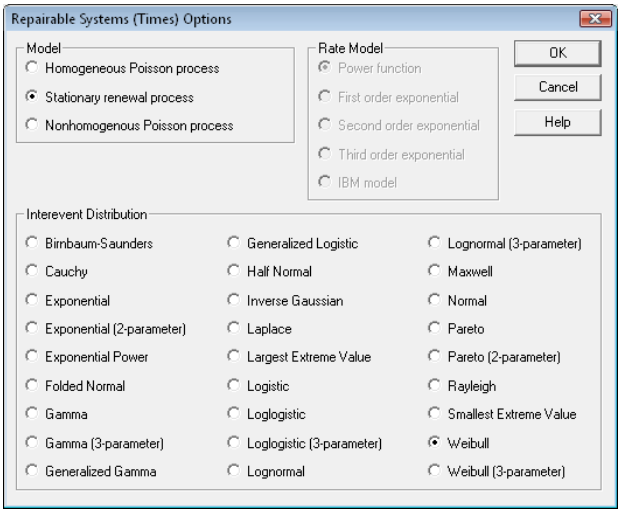
36

Failure Plot



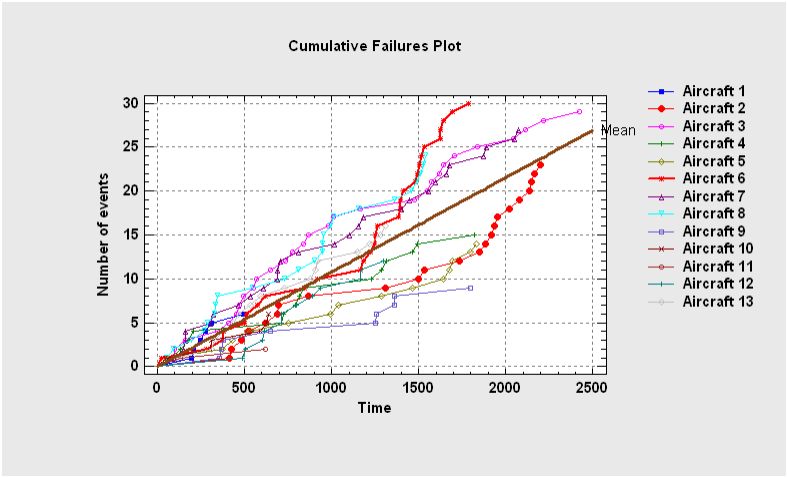
37

Analysis Options



38

Cumulative Failures Plot (Parametric)



39

Point Process Model

Point Process Model
Model: stationary renewal process
Rate model: 0.0107646

Mean cumulative events model: 0.0107646*t

Interevent distribution: Weibull
shape = 0.924492
scale = 89.5547
(Mean = 92.8974)

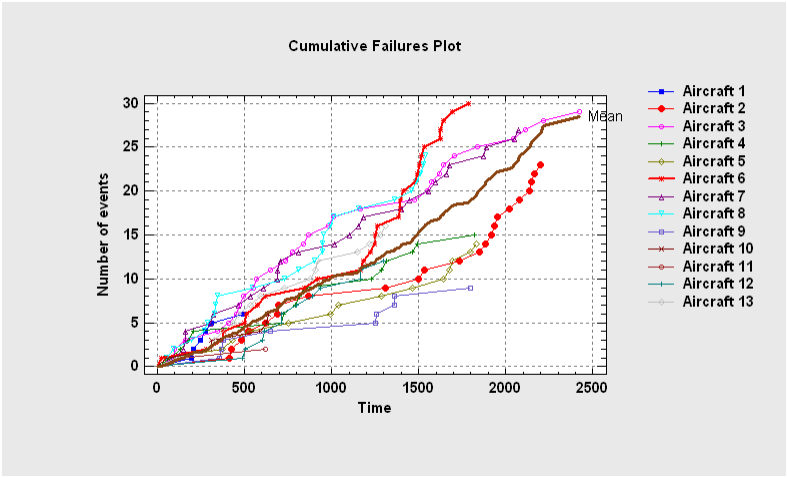
t	*Rate*	*Mean cum events*	*Mean interevent time*
0	0.0107646	0.0	92.8974
500	0.0107646	5.38228	92.8974
1000	0.0107646	10.7646	92.8974
1500	0.0107646	16.1468	92.8974
2000	0.0107646	21.5291	92.8974
2500	0.0107646	26.9114	92.8974

Goodness-of-Fit Test

Kolmogorov-Smirnov D	*P-value*
0.0519366	0.613734

40

Cumulative Failures Plot (Nonparametric)



41

Trend Test

Trend Test

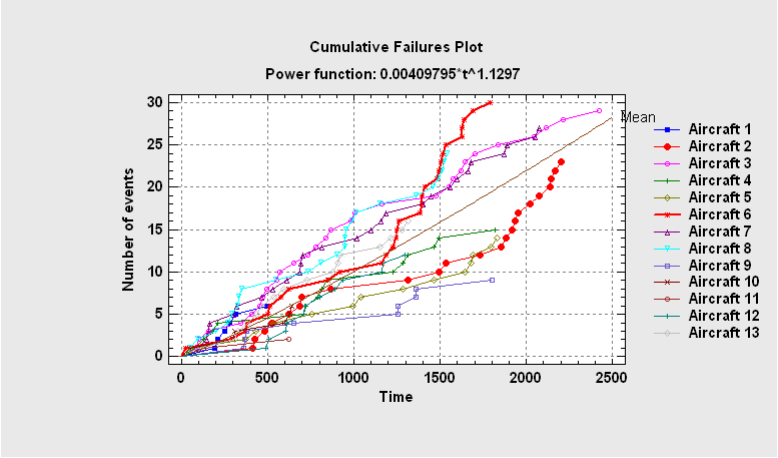
Laplace Centroid Test

	Test statistic	*P-Value*
Aircraft 1	0.0361374	0.971167
Aircraft 2	2.23544	0.025388
Aircraft 3	-1.4804	0.138765
Aircraft 4	-0.749716	0.453424
Aircraft 5	0.965489	0.334299
Aircraft 6	2.20647	0.0273513
Aircraft 7	-0.544738	0.585931
Aircraft 8	0.256495	0.797565
Aircraft 9	-0.144248	0.8853
Aircraft 10	0.689743	0.490354
Aircraft 11	-1.0092	0.312875
Aircraft 12	1.17291	0.240831
Aircraft 13	0.269283	0.787708
COMBINED	1.09474	0.273628

The StatAdvisor
The above table displays a test for trend. The Laplace Centroid Test tests whether the event times are uniformly distributed over the sampling interval. A small P-value indicates the presence of a trend. For example, since the P-value calculated when combining all of the variables is greater than or equal to 0.05, there is not a statistically significant trend at the 5.0% significance level.

42

Nonhomogeneous Poisson Process



43

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44

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