Fitting Regression Models Containing Categorical Factors

Presented by

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Virginia Piedmont

Type of grass affects population of wild birds

Cool season grasses

Warm season grasses

Outline

1. Linear models with a single predictor
2. Linear models with multiple predictors
3. Logistic regression
4. Nonlinear regression models
5. Life data regression

Example: Pricing of Diamonds

- The file *diamonds.sgd* contains information on 308 diamonds. (JSE Data Archive, Singfat Chu, National University of Singapore)

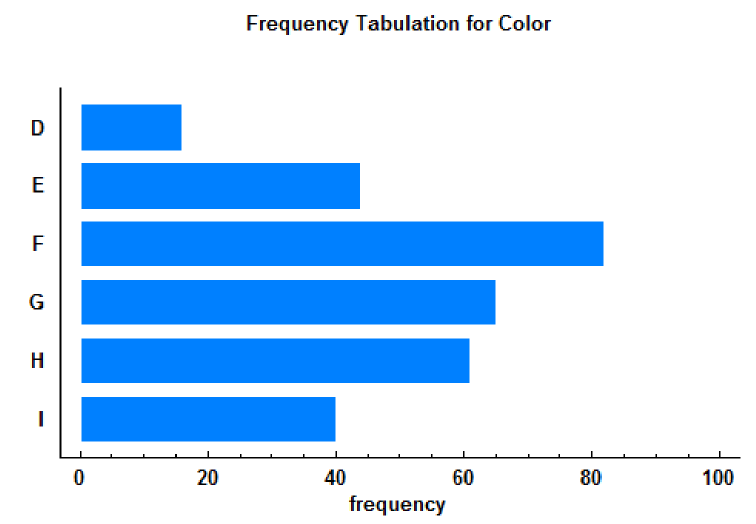
diamonds.sgd

	Carat weight	Color	Clarity	Certification body	Price
1	0.3	D	VS2	GIA	1302
2	0.3	E	VS1	GIA	1510
3	0.3	G	VVS1	GIA	1510
4	0.3	G	VS1	GIA	1260
5	0.31	D	VS1	GIA	1641
6	0.31	E	VS1	GIA	1555
7	0.31	F	VS1	GIA	1427
8	0.31	G	VVS2	GIA	1427
9	0.31	H	VS2	GIA	1126
10	0.31	I	VS1	GIA	1126
11	0.32	F	VS1	GIA	1468
12	0.32	G	VS2	GIA	1202

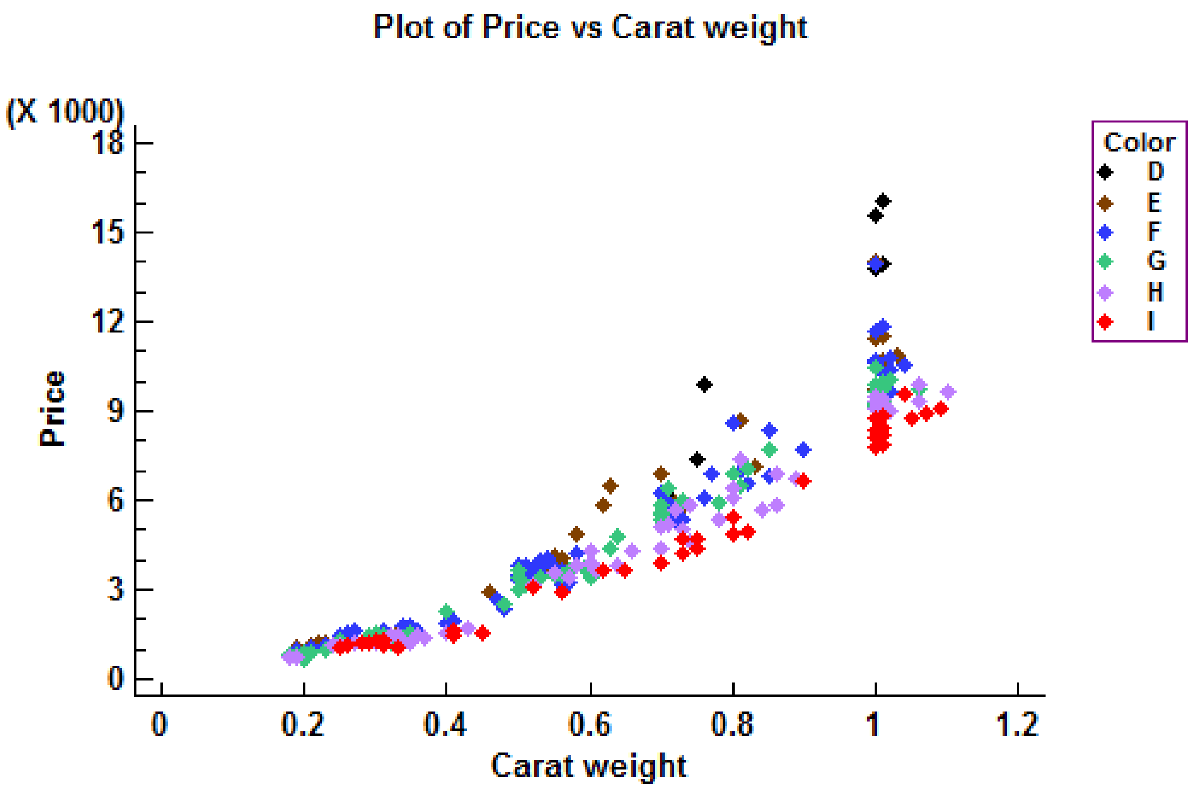
diamonds | B | C

Models with a single categorical predictor

- Dependent variable: Y = Price
- Independent variable: X = Carat weight
- Categorical variable: C = Color (6 levels)



Coded Scatterplot



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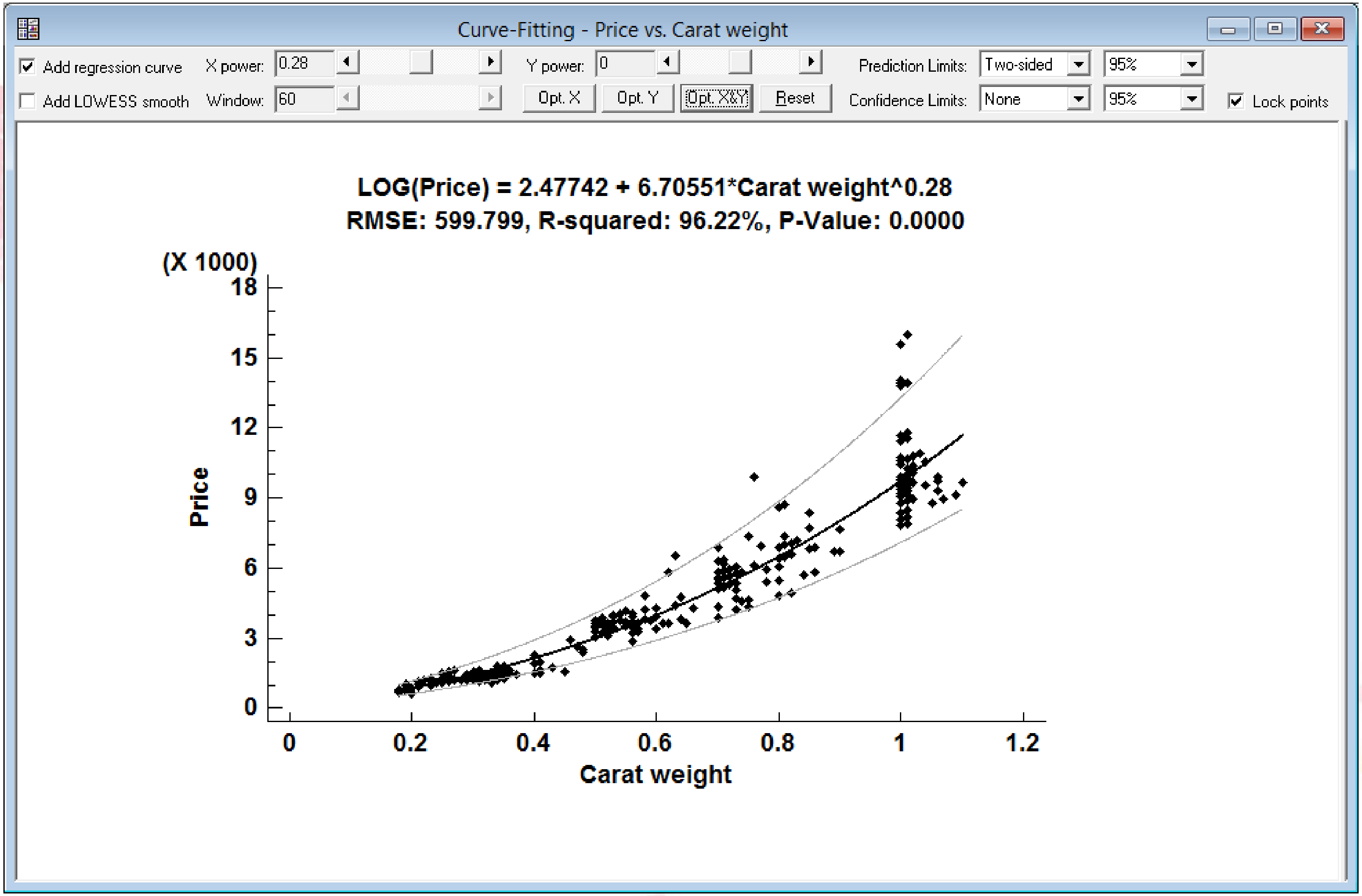
Transformation



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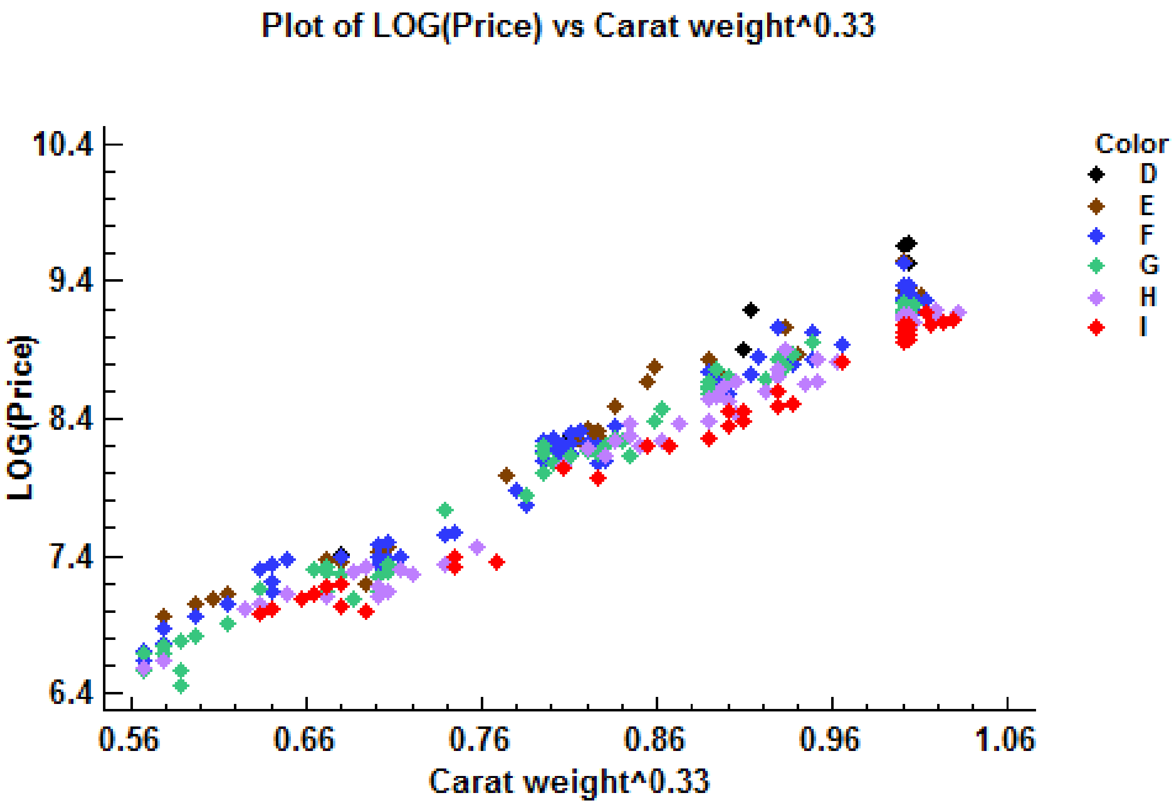
Transformed Data



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Statistical Model

$$Y = \beta_0 + \beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3 + \beta_4 I_4 + \beta_5 I_5$$
$$+ \beta_6 X + \beta_7 I_1 X + \beta_8 I_2 X + \beta_9 I_3 X + \beta_{10} I_4 X + \beta_{11} I_5 X$$

where $Y = LOG(Price)$

$$X = Carat\ weight\text{^}0.33$$

Color	I1	I2	I3	I4	I5
D	0	0	0	0	0
E	1	0	0	0	0
F	0	1	0	0	0
G	0	0	1	0	0
H	0	0	0	1	0
I	0	0	0	0	1

Model by Color

Color D: $Y = \beta_0 + \beta_6 X$

Color E: $Y = (\beta_0 + \beta_1) + (\beta_6 + \beta_7)X$

Color F: $Y = (\beta_0 + \beta_2) + (\beta_6 + \beta_8)X$

Color G: $Y = (\beta_0 + \beta_3) + (\beta_6 + \beta_9)X$

Color H: $Y = (\beta_0 + \beta_4) + (\beta_6 + \beta_{10})X$

Color I: $Y = (\beta_0 + \beta_5) + (\beta_6 + \beta_{11})X$

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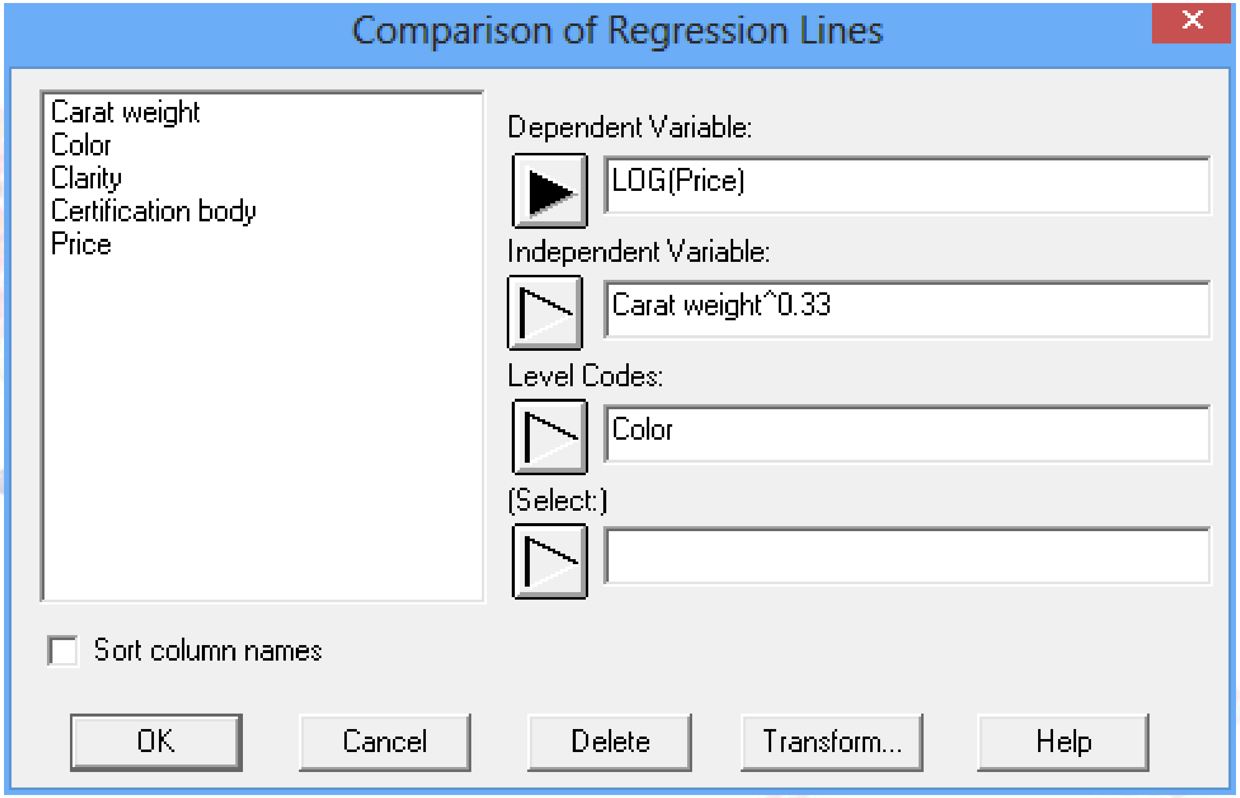
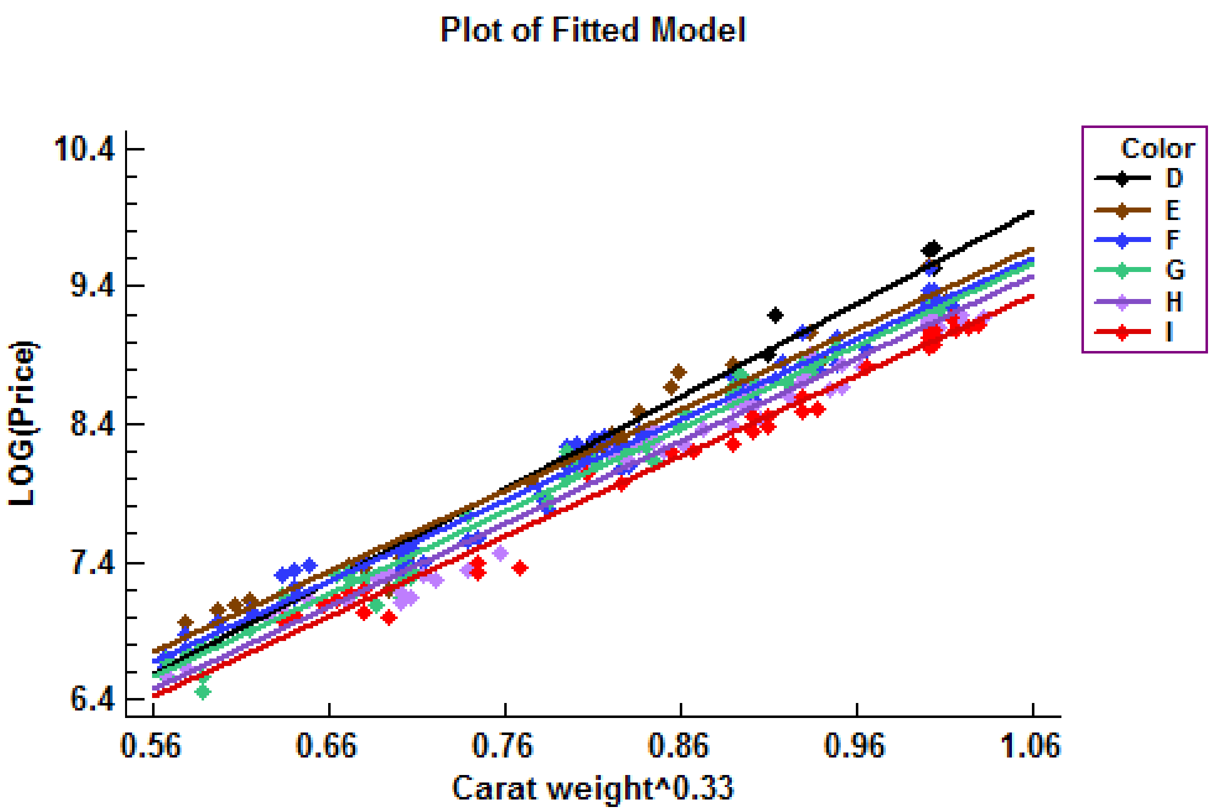
Comparison of Regression Lines



Fitted Regression Model



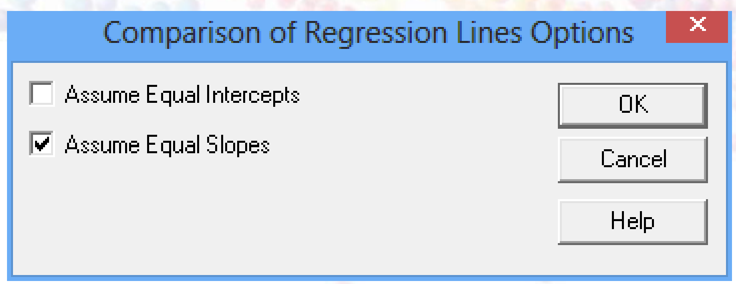
Test for Differences in the Slopes and Intercepts

- Conditional sums of squares

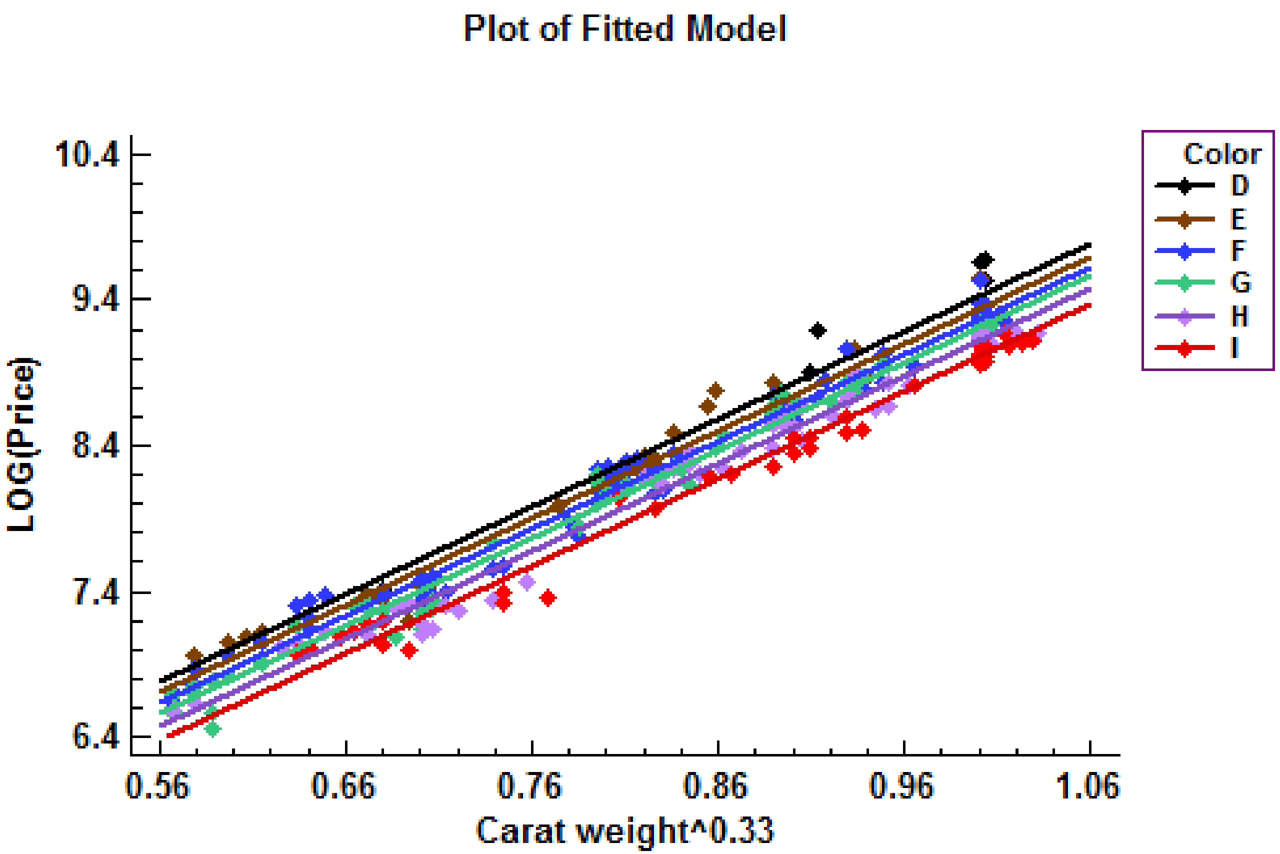
Further ANOVA for Variables in the Order Fitted

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
Carat weight^0.33	195.846	1	195.846	16521.44	0.0000
Intercepts	3.96947	5	0.793893	66.97	0.0000
Slopes	0.249657	5	0.0499314	4.21	0.0010
Model	200.065	11			

- Simplify the model using *Analysis Options*



Parallel Regression Lines



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Fitted Model

Comparison of Regression Lines - LOG(Price) versus Carat weight^0.33 by Color

Dependent variable: LOG(Price)
Independent variable: Carat weight^0.33
Level codes: Color

Number of complete cases: 308
Number of regression lines: 6

Multiple Regression Analysis

		Standard	*T*	
Parameter	*Estimate*	*Error*	*Statistic*	*P-Value*
CONSTANT	2.83694	0.147535	19.2289	0.0000
Carat weight^0.33	6.7001	0.173817	38.5469	0.0000
Color=E	0.626838	0.18309	3.42366	0.0007
Color=F	0.536887	0.166205	3.23027	0.0014
Color=G	0.339188	0.167165	2.02906	0.0433
Color=H	0.277087	0.176293	1.57174	0.1171
Color=I	0.320909	0.187515	1.71137	0.0881
Carat weight^0.33*Color=E	-0.844063	0.215945	-3.90869	0.0001
Carat weight^0.33*Color=F	-0.821593	0.196678	-4.17735	0.0000
Carat weight^0.33*Color=G	-0.661424	0.19826	-3.33614	0.0010
Carat weight^0.33*Color=H	-0.700254	0.206253	-3.39512	0.0008
Carat weight^0.33*Color=I	-0.871306	0.21686	-4.01783	0.0001

Coefficients

Color	*Intercept*	*Slope*
D	2.83694	6.7001
E	3.46378	5.85604
F	3.37383	5.87851
G	3.17613	6.03868
H	3.11403	5.99985
I	3.15785	5.8288

Analysis of Variance

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
Model	200.065	11	18.1877	1534.31	0.0000
Residual	3.5088	296	0.011854		
Total (Corr.)	203.574	307			

R-Squared = 98.2764 percent
R-Squared (adjusted for d.f.) = 98.2123 percent
Standard Error of Est. = 0.108876
Mean absolute error = 0.0833828
Durbin-Watson statistic = 1.18873 (P=0.0000)
Lag 1 residual autocorrelation = 0.401379

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Multiple Predictors

- When dealing with multiple categorical and quantitative predictors, we can use either of 2 procedures:

  – **Multiple Regression** (have to type in expressions for each indicator variable)

  – **GLM: General Linear Model** (automatically generates the indicator variables)

- Be careful: the indicator variables are set up differently in GLM (as well as the DOE procedures.)

Coding Comparison

- Comparison of Regression Lines

Color	I1	I2	I3	I4	I5
D	0	0	0	0	0
E	1	0	0	0	0
F	0	1	0	0	0
G	0	0	1	0	0
H	0	0	0	1	0
I	0	0	0	0	1

- GLM and DOE

Color	I1	I2	I3	I4	I5
D	1	0	0	0	0
E	0	1	0	0	0
F	0	0	1	0	0
G	0	0	0	1	0
H	0	0	0	0	1
I	-1	-1	-1	-1	-1

Model Comparison

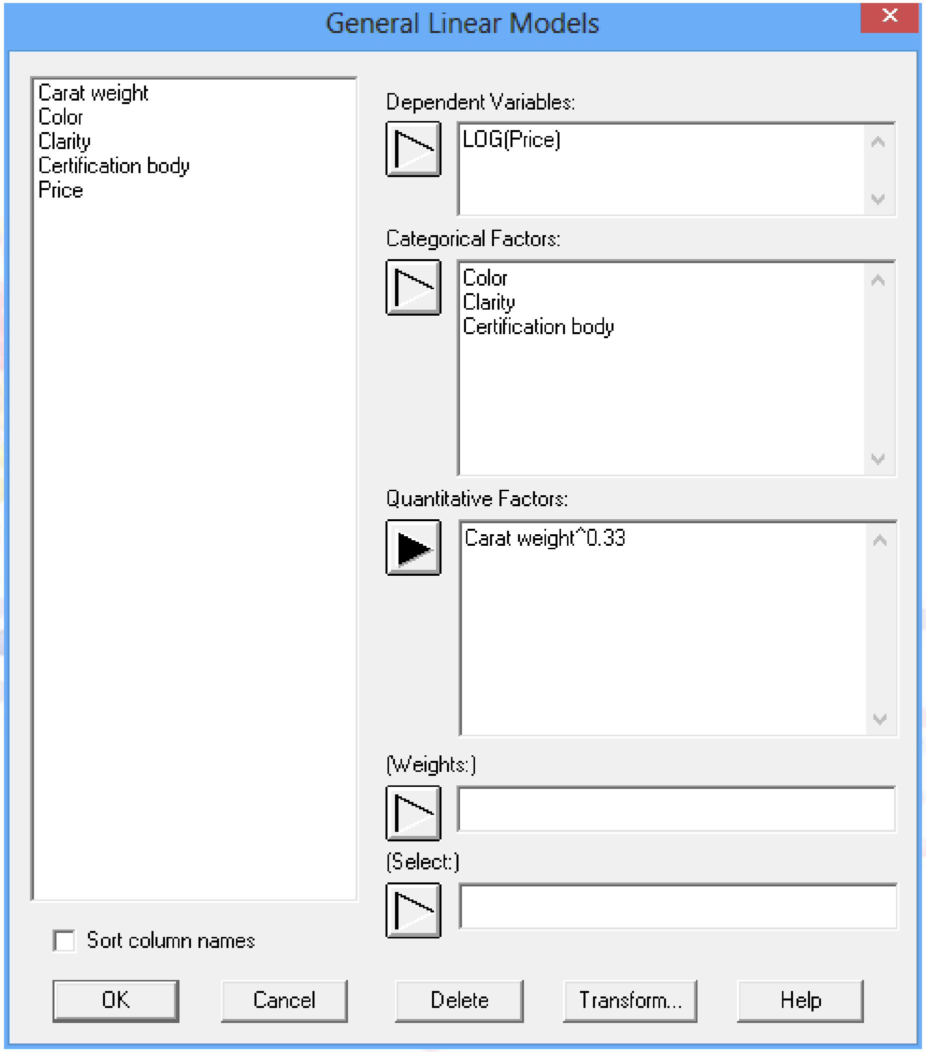
- Comparison of Regression Lines

Color	Intercept	Slope
D	β_0	β_6
E	$\beta_0 + \beta_1$	$\beta_6 + \beta_7$
F	$\beta_0 + \beta_2$	$\beta_6 + \beta_8$
G	$\beta_0 + \beta_3$	$\beta_6 + \beta_9$
H	$\beta_0 + \beta_4$	$\beta_6 + \beta_{10}$
I	$\beta_0 + \beta_5$	$\beta_6 + \beta_{11}$

- GLM and DOE

Color	Intercept	Slope
D	$\beta_0 + \beta_1$	$\beta_6 + \beta_7$
E	$\beta_0 + \beta_2$	$\beta_6 + \beta_8$
F	$\beta_0 + \beta_3$	$\beta_6 + \beta_9$
G	$\beta_0 + \beta_4$	$\beta_6 + \beta_{10}$
H	$\beta_0 + \beta_5$	$\beta_6 + \beta_{11}$
I	$\beta_0 - (\beta_1+\beta_2+\beta_3+\beta_4+\beta_5)$	$\beta_6 - (\beta_7+\beta_8+\beta_9+\beta_{10}+\beta_{11})$

Data Input Dialog Box



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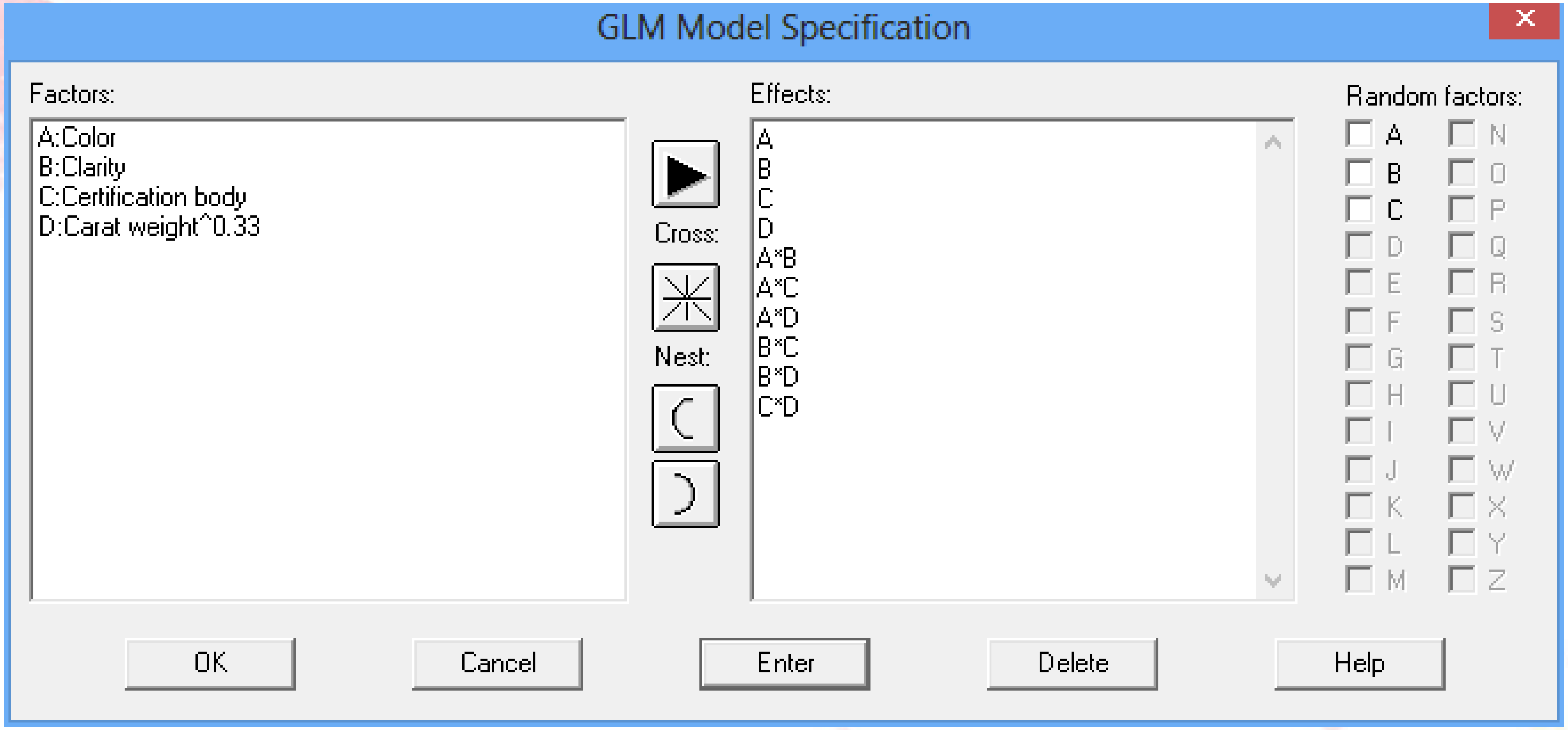
Model Specification



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Results

General Linear Models

Number of dependent variables: 1
Number of categorical factors: 3
A=Color
B=Clarity
C=Certification body
Number of quantitative factors: 1
D=Carat weight^0.33
Number of observations: 308

Analysis of Variance for LOG(Price)

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
Model	202.896	61	3.32616	1206.95	0.0000
Residual	0.677934	246	0.00275583		
Total (Corr.)	203.574	307			

Type III Sums of Squares

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
Color	0.13531	5	0.027062	9.82	0.0000
Clarity	0.0865789	4	0.0216447	7.85	0.0000
Certification body	0.0585465	2	0.0292733	10.62	0.0000
Carat weight^0.33	35.7245	1	35.7245	12963.25	0.0000
Color*Clarity	0.130609	20	0.00653043	2.37	0.0011
Color*Certification body	0.033009	10	0.0033009	1.20	0.2929
Color*Carat weight^0.33	0.0286298	5	0.00572597	2.08	0.0688
Clarity*Certification body	0.0173235	8	0.00216543	0.79	0.6156
Clarity*Carat weight^0.33	0.0320343	4	0.00800858	2.91	0.0224
Certification body*Carat weight^0.33	0.0489169	2	0.0244584	8.88	0.0002
Residual	0.677934	246	0.00275583		
Total (corrected)	203.574	307			

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Simplified Model

Type III Sums of Squares

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
Color	4.40806	5	0.881612	312.82	0.0000
Clarity	0.203955	4	0.0509888	18.09	0.0000
Certification body	0.0697858	2	0.0348929	12.38	0.0000
Carat weight^0.33	68.6518	1	68.6518	24359.17	0.0000
Color*Clarity	0.141277	20	0.00706386	2.51	0.0005
Clarity*Carat weight^0.33	0.0669007	4	0.0167252	5.93	0.0001
Certification body*Carat weight^0.33	0.052897	2	0.0264485	9.38	0.0001
Residual	0.758126	269	0.00281831		
Total (corrected)	203.574	307			

Predicting New Observations

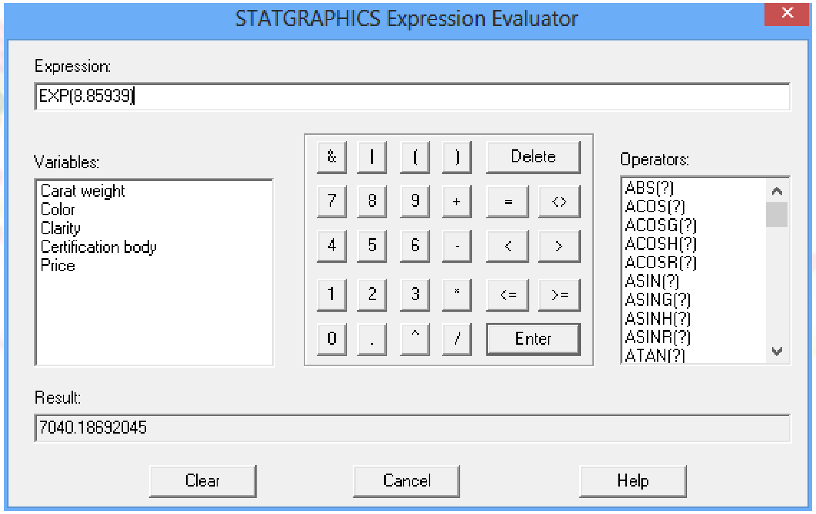
C:\Data\webinar\diamonds.sgd

	Carat weight	Color	Clarity	Certification body	Price
302	1.01	H	VS1	HRD	9153
303	1.01	I	VVS1	HRD	8873
304	1.01	I	VS1	HRD	8175
305	1.02	F	VVS2	HRD	10796
306	1.06	H	VVS2	HRD	9890
307	1.02	H	VS2	HRD	8959
308	1.09	I	VVS2	HRD	9107
309	.75	E	VVS2	HRD	

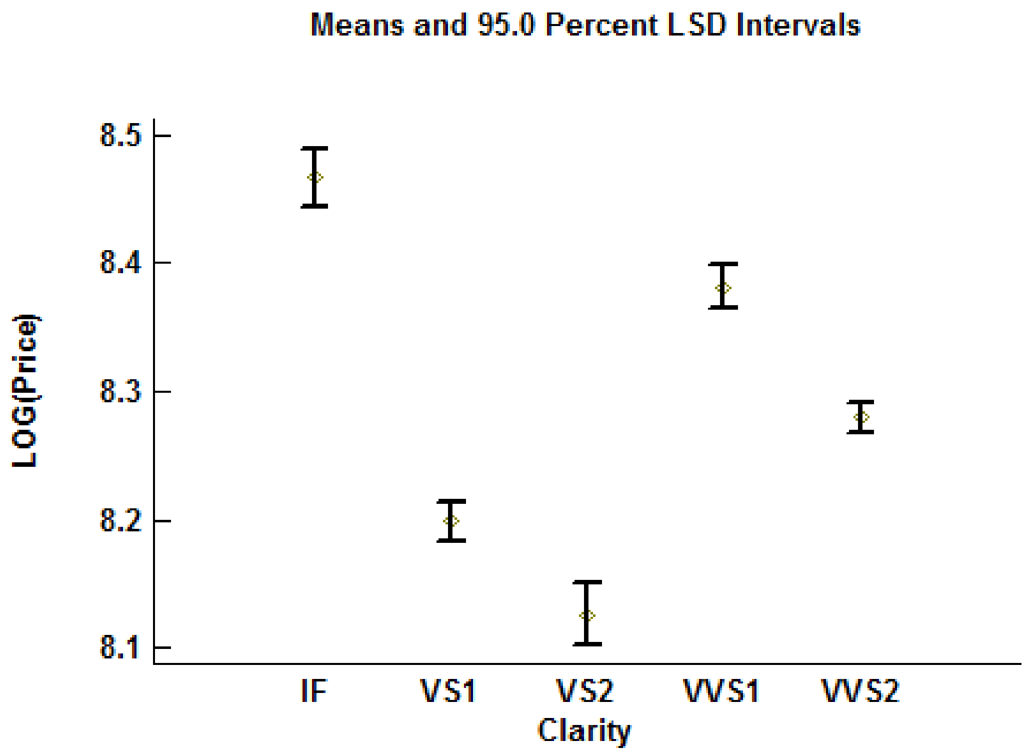
diamonds B C

Model Results for LOG(price)

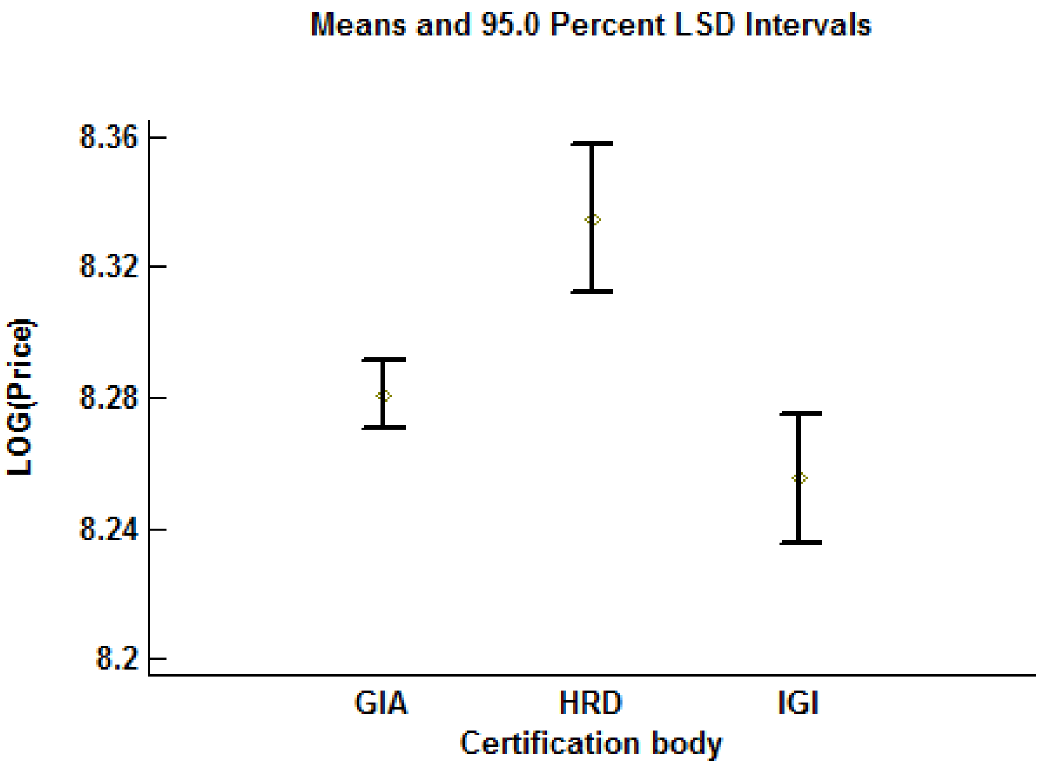
Row	Fitted Value	Stnd. Error for Forecast	Lower 95.0% CL for Forecast	Upper 95.0% CL for Forecast	Lower 95.0% CL for Mean	Upper 95.0% CL for Mean
309	8.85939	0.0564242	8.7483	8.97048	8.82176	8.89703



Least Squares Means



Least Squares Means



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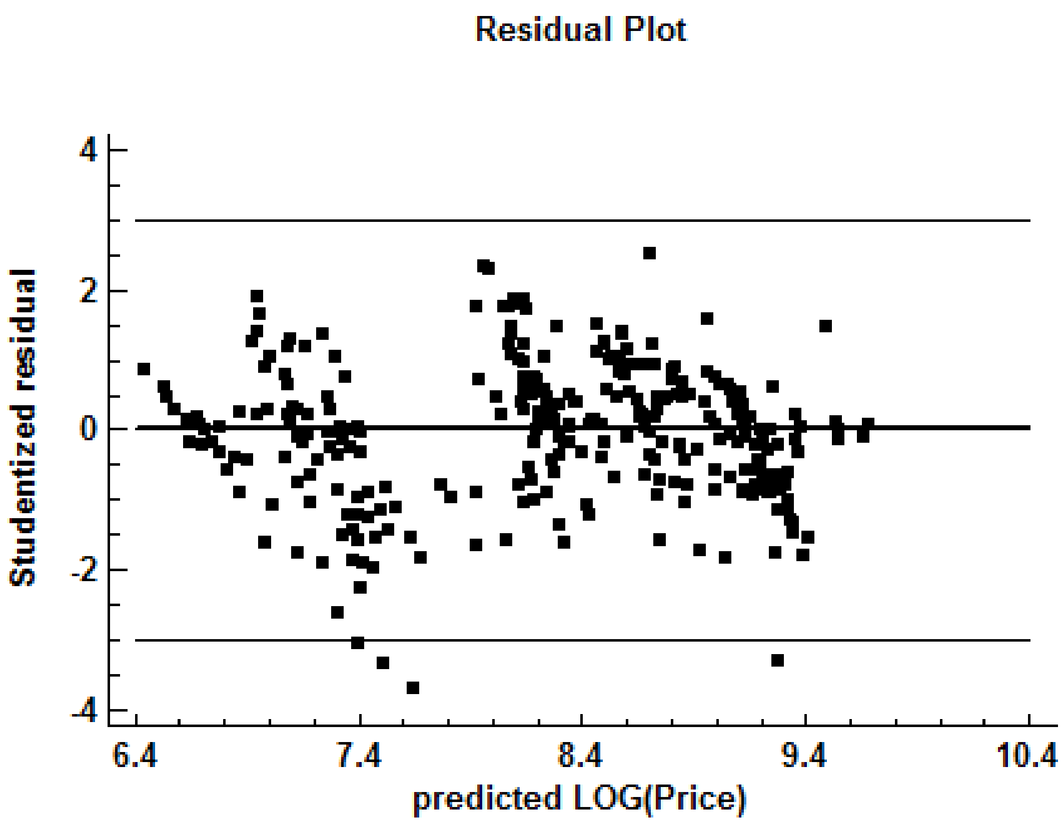
Residual Plot



Logistic Regression

Can we predict how well Barry Bonds would do when he came to bat?

Bonds Data from 2001

C:\Data\webinar\barrybonds.sgd

	Game number	At-bat	Home game	On first	On second	On third	Outs	Inning	Runs in inning	Walk	Intentional walk	Result	ERA	Giants score	Opposing team score
		within game	1=TRUE						after first pitch to Bomds			>0 if got on base	opposing pitcher	before at-bat	before at-bat
1	1	1	1	1	0	0	1	1	0	0	0	0	4.20	0	0
2	1	2	1	0	1	0	1	3	0	1	1	5	4.20	0	0
3	1	3	1	0	0	0	1	5	1	0	0	4	4.20	1	0
4	1	4	1	1	0	1	2	6	0	1	0	5	5.49	3	1
5	1	5	1	0	0	0	2	8	0	0	0	0	4.00	3	2
6	2	1	1	0	0	0	2	1	0	0	0	0	5.75	0	0
7	2	2	1	1	1	0	2	3	0	0	0	0	5.75	0	3
8	2	3	1	0	1	0	2	5	0	0	0	0	5.75	1	4
9	2	4	1	1	0	0	1	7	3	0	0	2	4.00	4	5
10	2	5	1	1	0	0	2	8	0	1	0	5	5.21	7	6
11	3	1	1	0	0	0	2	1	0	0	0	0	4.30	0	1
12	3	2	1	1	0	0	2	3	3	0	0	1	4.30	1	1
13	3	3	1	1	0	0	2	5	0	0	0	0	4.30	4	2

barrybonds | B | C

n = 648 at-bats

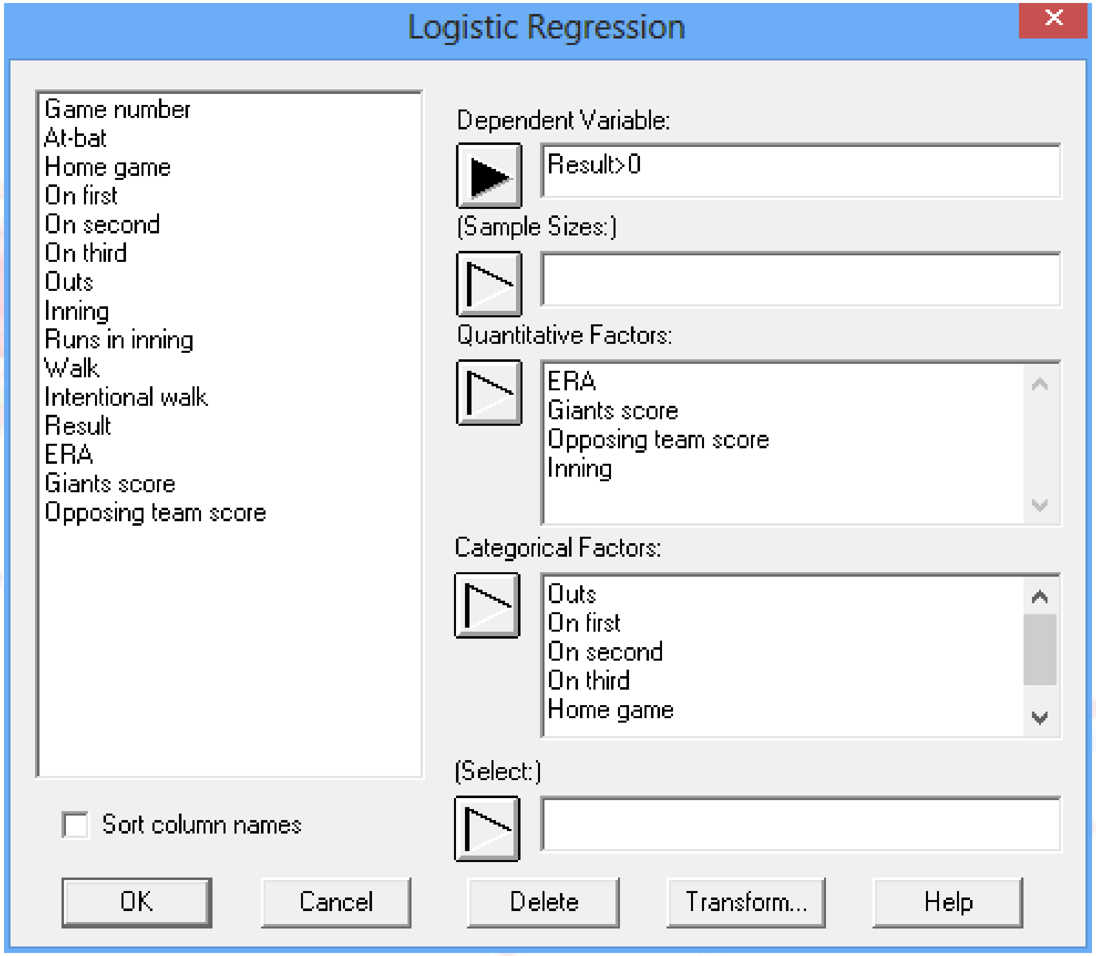
Source: JSE Data Archive, Jerome P. Reiter, Duke University

Model

- Y = 1 if Bonds reached base and 0 otherwise

- Predictors:
  – ERA of opposing pitcher
  – Runs already scored that inning
  – Opposing team's score
  – Inning
  – # of outs when he came to bat
  – Whether a runner was on first base
  – Whether a runner was on second base
  – Whether a runner was on third base
  – Whether it was a home game

Data Input Dialog Box



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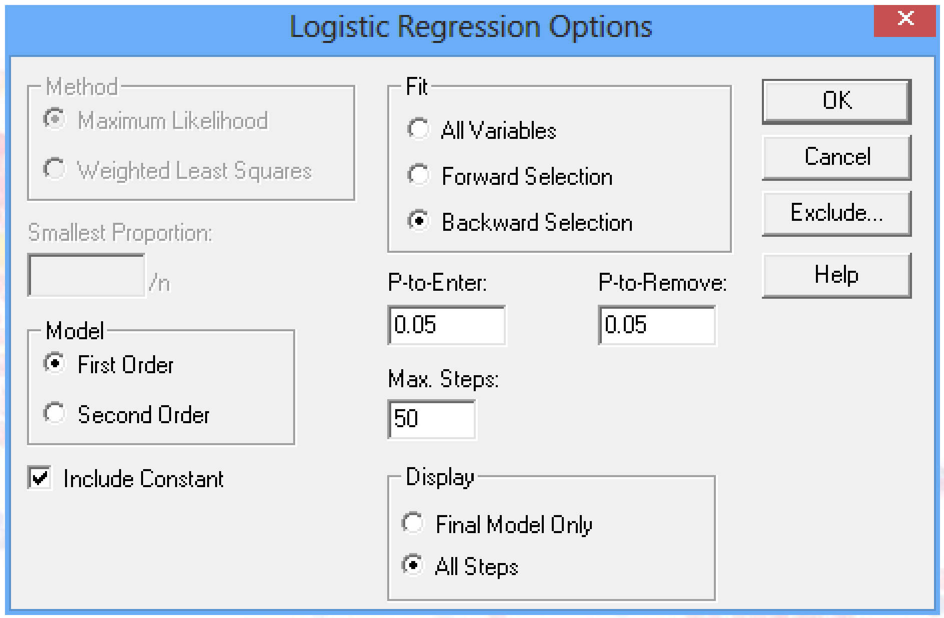
Analysis Options



Results

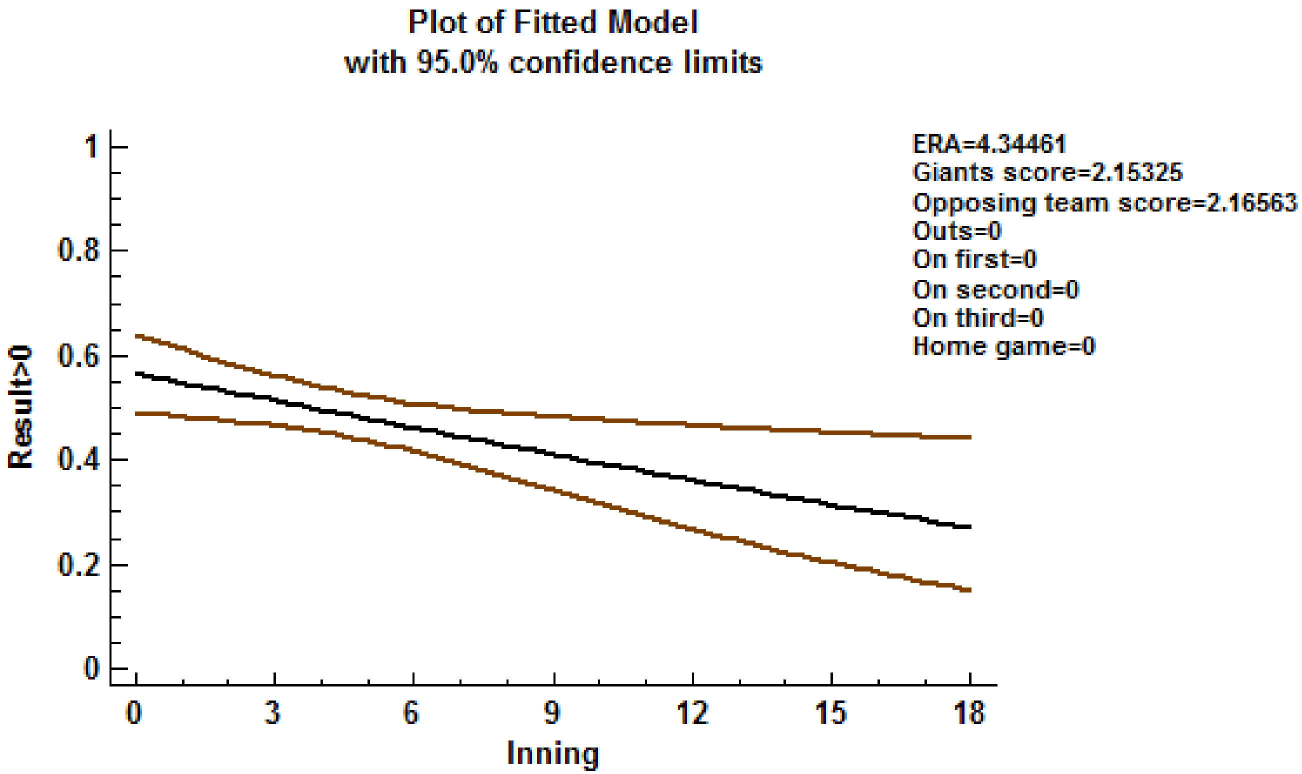
Likelihood Ratio Tests

Factor	*Chi-Square*	*Df*	*P-Value*
Inning	6.34437	1	0.0118
On second	12.3916	1	0.0004

Estimated Regression Model (Maximum Likelihood)

		Standard	*Estimated*
Parameter	*Estimate*	*Error*	*Odds Ratio*
CONSTANT	1.02656	0.250474	
Inning	-0.0695187	0.0277701	0.932843
On second=0	-0.762319	0.221567	0.466583

Plot of Probability of Reaching Base



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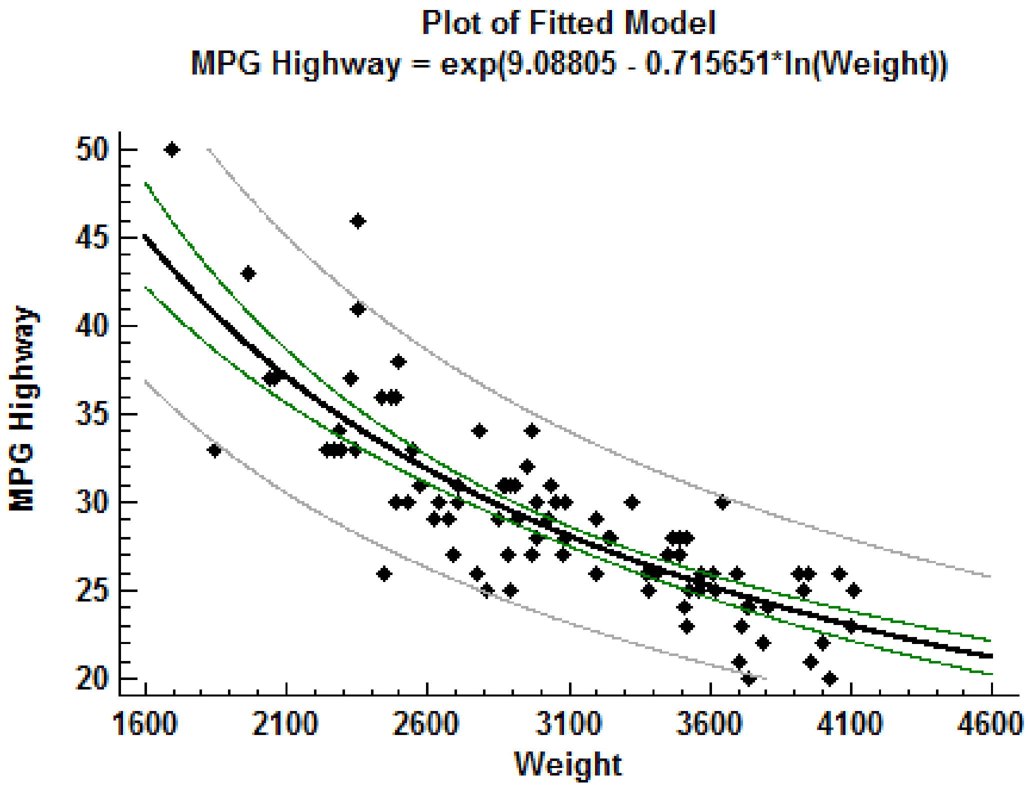
Nonlinear Models

- Data file: 93cars.sgd
  - Y: MPG Highway
  - X: weight
  - C: manual
- The relationship between Y and X is nonlinear.

Multiplicative Model



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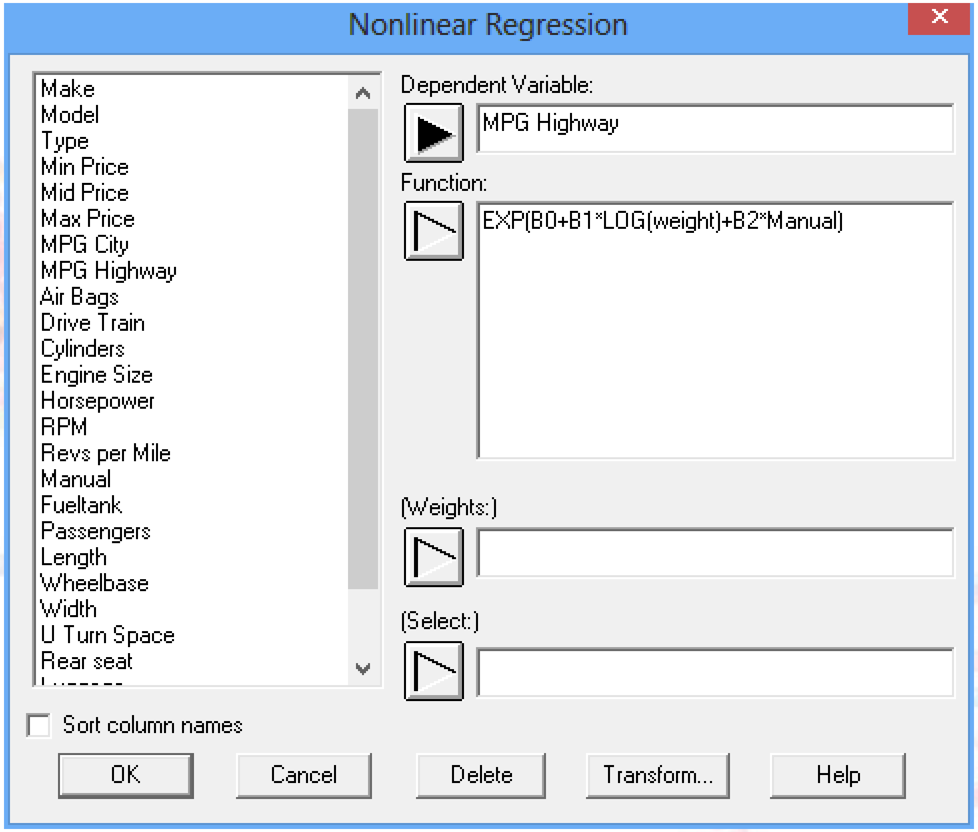
Nonlinear Regression



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Results

Nonlinear Regression - MPG Highway
Dependent variable: MPG Highway (miles per gallon in highway driving)
Independent variables:
  weight (pounds)
  manual (0=no, 1=yes)

Function to be estimated: EXP(B0+B1*LOG(weight)+B2*manual)
Initial parameter estimates:
  B0 = 0.1
  B1 = 0.1
  B2 = 0.1
Number of observations: 93

Estimation method: Marquardt
Estimation stopped due to convergence of parameter estimates.
Number of iterations: 10
Number of function calls: 45

Estimation Results

			Asymptotic	*95.0%*
		Asymptotic	*Confidence*	*Interval*
Parameter	*Estimate*	*Standard Error*	*Lower*	*Upper*
B0	9.66687	0.494256	8.68495	10.6488
B1	-0.783745	0.060318	-0.903577	-0.663912
B2	-0.0440352	0.0292223	-0.10209	0.0140201

Analysis of Variance

Source	*Sum of Squares*	*Df*	*Mean Square*
Model	80506.6	3	26835.5
Residual	786.421	90	8.73801
Total	81293.0	93	
Total (Corr.)	2615.31	92	

R-Squared = 69.9301 percent
R-Squared (adjusted for d.f.) = 69.2619 percent
Standard Error of Est. = 2.95601
Mean absolute error = 2.19093
Durbin-Watson statistic = 1.65999
Lag 1 residual autocorrelation = 0.168988

MPG Highway = EXP(9.66687
-0.783745*LOG(weight)
-0.0440352*manual)

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Life Data Regression

- Data file: methadone.sgd

methadone.sgd

	Clinic	Censored	Days in clinic	Prison?	Dose
	1 or 2	1=still in clinic		1=spent time in prison	
1	1	0	428	0	50
2	1	0	275	1	55
3	1	0	262	0	55
4	1	0	183	0	30
5	1	0	259	1	65
6	1	0	714	0	55
7	1	0	438	1	65
8	1	1	796	1	60
9	1	0	892	0	50
10	1	0	393	1	65
11	1	1	161	1	80
12	1	0	836	1	60
13	1	0	523	0	55
14	1	0	612	0	70

methadone B C

Caplehorn, J. (1991). Methadone dosage and retention of patients in maintenance treatment. *Medical Journal of Australia*.

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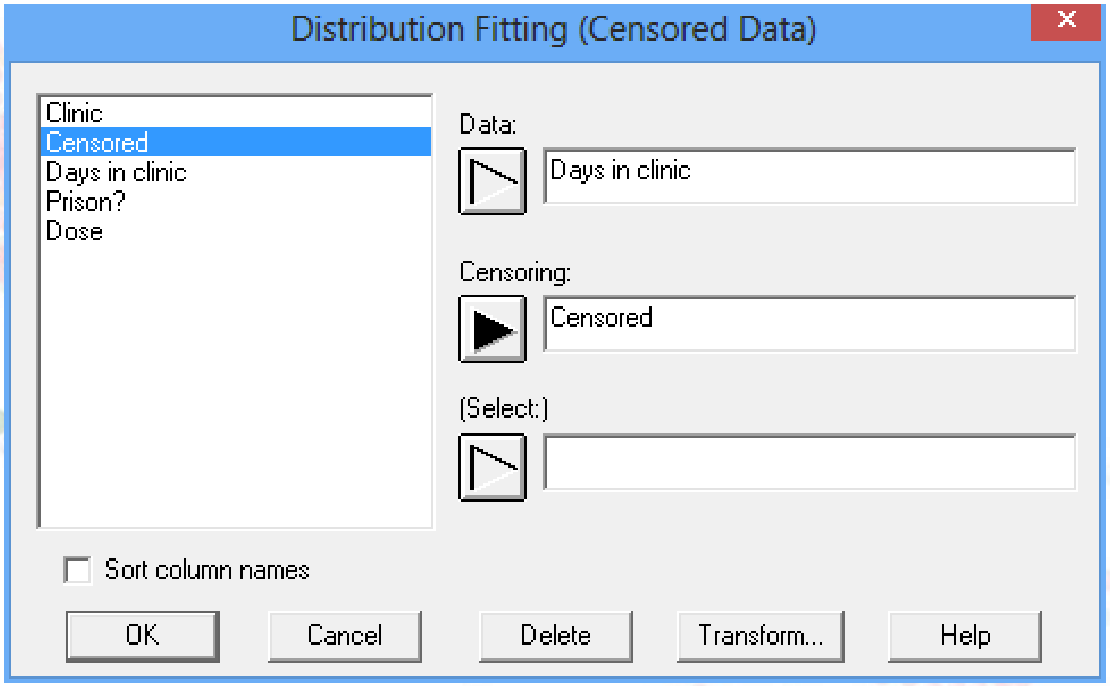
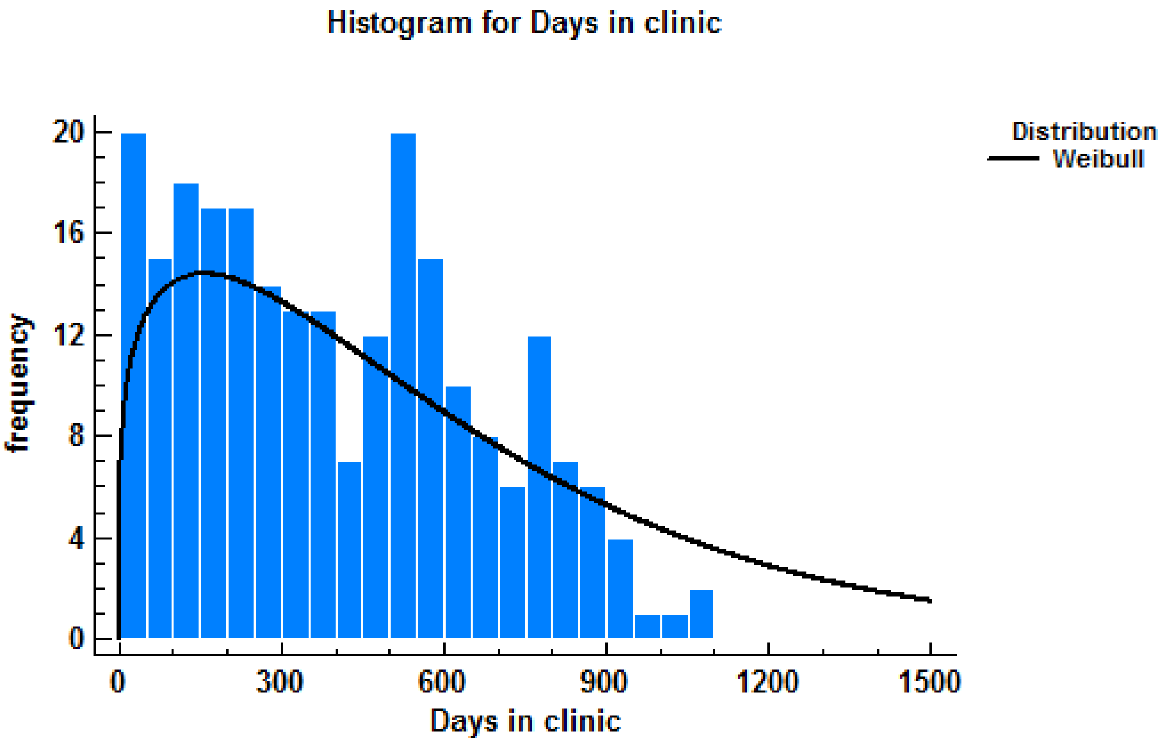
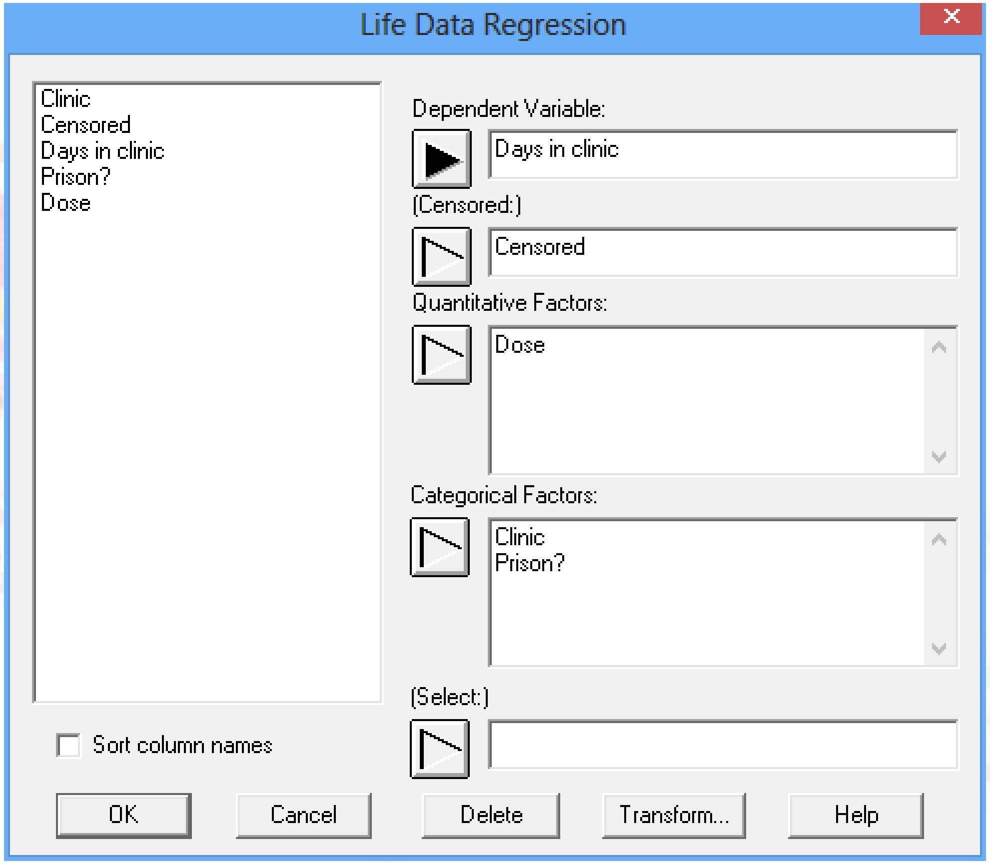
Distribution Fitting



Distribution Fitting



Data Input Dialog Box



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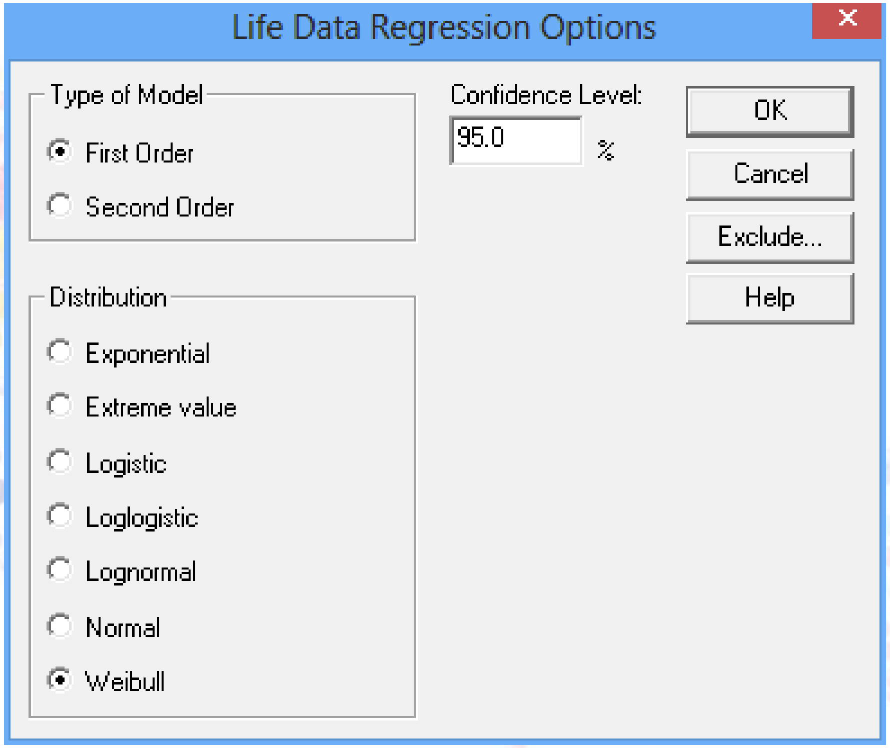
Analysis Options



Results

Life Data Regression - Days in clinic

Dependent variable: Days in clinic
Censoring: Censored (1=still in clinic)
Factors:
  Dose
  Clinic (1 or 2)
  Prison? (1=spent time in prison)

Number of uncensored values: 150
Number of right-censored values: 88

Estimated Regression Model - Weibull

		Standard	*Lower 95.0%*	*Upper 95.0%*
Parameter	*Estimate*	*Error*	*Conf. Limit*	*Conf. Limit*
CONSTANT	5.29378	0.309429	4.68731	5.90025
Dose	0.0244232	0.00459046	0.015426	0.0334203
Clinic=1	-0.709329	0.157267	-1.01757	-0.401091
Prison?=0	0.229509	0.120806	-0.00726626	0.466285
SIGMA	0.729943	0.0493229	0.6394	0.833308

Log likelihood = -1084.48

Likelihood Ratio Tests

Factor	*Chi-Square*	*Df*	*P-Value*
Dose	28.5474	1	0.0000
Clinic	24.9152	1	0.0000
Prison?	3.54961	1	0.0596

Days in clinic = exp(5.29378 + 0.0244232*Dose - 0.709329*Clinic=1 + 0.229509*Prison?=0)

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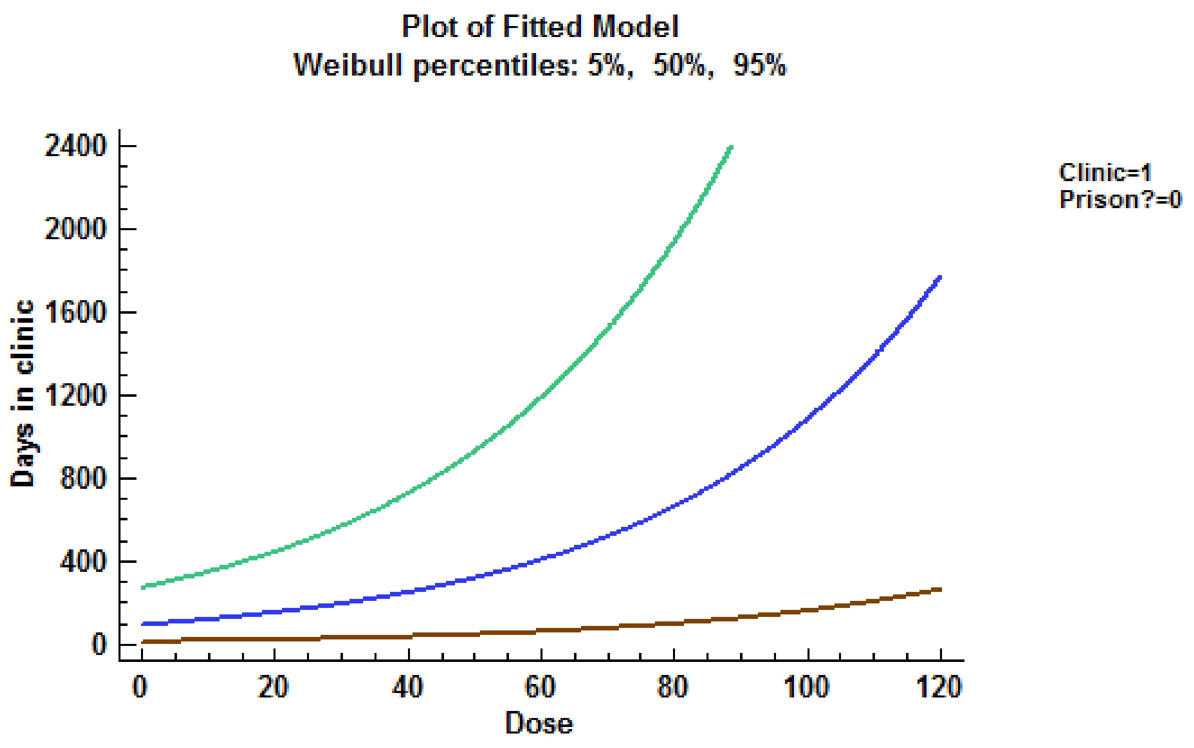
Percentile Plot



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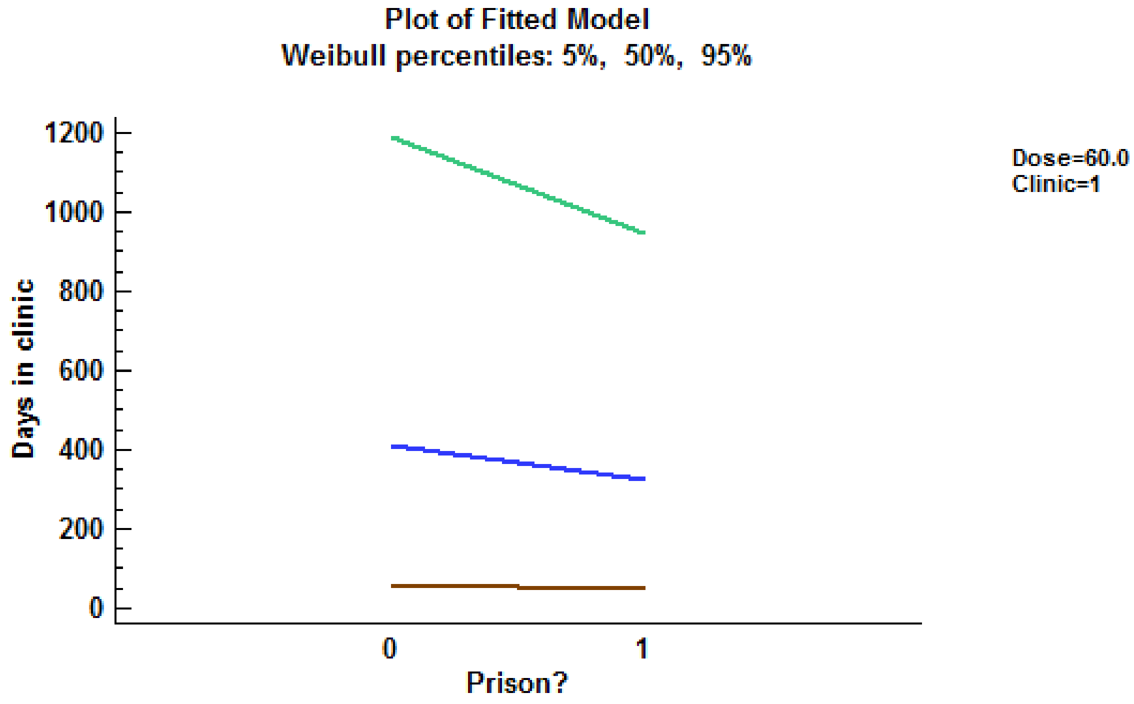
Percentile Plot



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More Information

- Video, slides and sample data may be found at www.statgraphics.com/webinars.