# Multivariate Data Analysis Using Statgraphics Centurion: Part 1

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# **Multivariate Statistical Methods**

The simultaneous observation and analysis of more than one response variable.

\*Primary Uses

- 1. Data reduction or structural simplification
- 2. Sorting and grouping
- 3. Investigation of the dependence among variables
- 4. Prediction
- 5. Hypothesis construction and testing

\*Johnson and Wichern, Applied Multivariate Statistical Analysis

# **STATGRAPHICS Contents**

- Plot Multivariate Visualization
  - Scatterplot matrix
  - Parallel coordinates plot
  - Andrews plot
  - Star glyphs and sunray plots
  - Chernoff faces
- Describe Multivariate Methods
  - Correlation analysis
  - Principal components analysis
  - Factor analysis
  - Canonical correlations
  - Cluster analysis
  - Correspondence analysis
  - Multiple correspondence analysis

#### **STATGRAPHICS** Contents

- Compare Analysis of Variance
  - General Linear Models MANOVA
- Relate Multiple Factors
  - Partial Least Squares
- Relate Classification Methods
  - Discriminant analysis
  - Neural networks Bayesian classifier
- SPC Control Charts Multivariate Control Charts
  - T-Squared Chart
  - Multivariate EWMA Chart

# **Uniwin from Sigma Plus**

Package of multivariate statistical methods that read Statgraphics data files.

- Additional features for:
  - Principal components analysis
  - Simple and multiple correspondence analysis
  - Cluster analysis
- Other procedures:
  - Factor analysis for mixed data (quantitative and qualitative)
  - Multiple factorial analysis and STATIS two methods for three-way tables
  - Classical multidimensional scaling
  - Confidence and tolerance ellipses
  - Qualitative discriminant analysis
  - Scoring
  - Principal components regression

### Dataset – countries of the world.sgd

#### <u>DataViewer</u>

Number of columns: 21 Number of rows: 227 Number of complete cases: 179

Column			Nonmissing	Unique		
Name	Comment	Type	Values	Values	Minimum	Maximum
Country		Character	227	227		
Region		Character	227	11		
Population		Numeric	227	227	7026.0	1.31397E9
Area	sq. mi.	Numeric	227	226	2.0	1.70752E7
Pop. Density	persq. km	Numeric	227	219	0.026	16271.5
Coastline	coast/area ratio	Numeric	227	151	0	870.66
Net migration	immigrants per 1000 population	Numeric	224	157	-20.99	23.06
Infant mortality	per 1000 births	Numeric	224	220	2.29	191.19
GDP	\$ per capita	Numeric	226	130	500.0	55100.0
Literacy	%	Numeric	209	140	17.6	100.0
Phones	per 1000	Numeric	223	214	0.2	1035.6
Arable	%	Numeric	225	203	0	62.11
Crops	%	Numeric	225	162	0	50.68
Other	%	Numeric	225	209	33.33	100.0
Farmable	Arable + Crops	Numeric	225	209	0	66.67
Climate		Numeric	205	6	1.0	4.0
Birthrate	per 1000	Numeric	224	220	7.29	50.73
Deathrate	per 1000	Numeric	223	201	2.29	29.74
Agriculture	proportion	Numeric	212	149	0.001	0.769
Industry	proportion	Numeric	211	155	0.02	0.906
Service	proportion	Numeric	212	167	0.062	0.954

# Multivariate Methods Webinar – Part 1

- 1. Correlation analysis
- 2. Partial correlation analysis
- 3. Parallel coordinates plot
- 4. Principal components analysis (PCA)
- 5. Factor analysis
- 6. Multivariate T-squared chart
- 7. Additional features using UNIWIN (supplementary variables for PCA and multidimensional scaling)

# Variables for this Study (circa 2006)

- 1. **Pop. Density** inhabitants per square km
- 2. **Farmable** % of land usable for growing crops
- 3. Birth rate annual births per 1,000 of population
- 4. **Literacy** % of population classified as literate
- Infant mortality deaths of infants under 1 year old per 100 live births
- 6. **GDP** Gross Domestic Product per capita
- 7. **Death rate** annual deaths per 1,000 of population
- 8. **Agriculture** proportion of GDP attributable to agriculture

# Matrix Plot – gives initial view of the data



### **Box-Cox Transformations**

Variable	Stnd. skewness	Optimal Box- Cox power	Selected transformation	Stnd. Skewness after transformation
Pop. density	50.96	0.027	Log	-1.58
Farmable	5.86	0.386	Square root	0.32
Birthrate	4.33	-0.083	Log	0.43
Literacy	-7.19	3.419	Cube	-3.97
Infant mortality	8.74	0.002	Log	-0.27
GDP	8.44	0.065	Log	-0.82
Deathrate	10.07	-0.175	Log	1.58
Agriculture	8.43	0.254	Cube root	0.81

#### Matrix Plot – transformed data



#### **Correlation Analysis**

#### Estimates linear correlations between pairs of variables.

#### Correlations

	Pop. Density	Farmable	Birthrate	Literacy	Infant mortality	GDP	Deathrate	Agriculture
Pop. Density		0.4211	-0.2694	0.1589	-0.3048	0.1647	-0.1927	-0.2712
		(225)	(224)	(209)	(224)	(226)	(223)	(212)
		0.0000	0.0000	0.0215	0.0000	0.0132	0.0039	0.0001
Farmable	0.4211		-0.1180	0.1276	-0.0530	-0.0680	0.0189	0.0518
	(225)		(222)	(209)	(222)	(224)	(221)	(211)
	0.0000		0.0795	0.0656	0.4319	0.3110	0.7802	0.4539
Birthrate	-0.2694	-0.1180		-0.7881	0.8567	-0.8214	0.2042	0.6853
	(224)	(222)		(207)	(223)	(224)	(223)	(211)
	0.0000	0.0795		0.0000	0.0000	0.0000	0.0022	0.0000
Literacy	0.1589	0.1276	-0.7881		-0.7510	0.6963	-0.3060	-0.5948
	(209)	(209)	(207)		(207)	(209)	(206)	(199)
	0.0215	0.0656	0.0000		0.0000	0.0000	0.0000	0.0000
Infant mortality	-0.3048	-0.0530	0.8567	-0.7510		-0.8619	0.4159	0.7558
	(224)	(222)	(223)	(207)		(224)	(223)	(211)
	0.0000	0.4319	0.0000	0.0000		0.0000	0.0000	0.0000
GDP	0.1647	-0.0680	-0.8214	0.6963	-0.8619		-0.3078	-0.8037
	(226)	(224)	(224)	(209)	(224)		(223)	(212)
	0.0132	0.3110	0.0000	0.0000	0.0000		0.0000	0.0000
Deathrate	-0.1927	0.0189	0.2042	-0.3060	0.4159	-0.3078		0.3520
	(223)	(221)	(223)	(206)	(223)	(223)		(210)
	0.0039	0.7802	0.0022	0.0000	0.0000	0.0000		0.0000
Agriculture	-0.2712	0.0518	0.6853	-0.5948	0.7558	-0.8037	0.3520	
	(212)	(211)	(211)	(199)	(211)	(212)	(210)	
	0.0001	0.4539	0.0000	0.0000	0.0000	0.0000	0.0000	

Correlation

(Sample Size)

P-Value

#### **Partial Correlation Analysis**

Estimates correlations between pairs of variables, adjusting for effects of the other variables.

rarual correlations	Partial	Correlations
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	Pop. Density	Farmable	Birthrate	Literacy	Infant mortality	GDP	Deathrate	Agriculture
Pop. Density		0.4245	-0.1043	-0.2020	-0.1745	-0.1884	-0.1013	-0.2123
		(225)	(224)	(209)	(224)	(226)	(223)	(212)
		0.0000	0.1248	0.0038	0.0098	0.0051	0.1370	0.0022
Farmable	0.4245		-0.1021	0.1660	0.0166	-0.1385	0.0527	0.1210
	(225)		(222)	(209)	(222)	(224)	(221)	(211)
	0.0000		0.1347	0.0179	0.8080	0.0410	0.4417	0.0840
Birthrate	-0.1043	-0.1021		-0.4143	0.4148	-0.2780	-0.3369	-0.0144
	(224)	(222)		(207)	(223)	(224)	(223)	(211)
	0.1248	0.1347		0.0000	0.0000	0.0000	0.0000	0.8379
Literacy	-0.2020	0.1660	-0.4143		-0.1439	-0.0017	-0.1654	-0.0401
	(209)	(209)	(207)		(207)	(209)	(206)	(199)
	0.0038	0.0179	0.0000		0.0416	0.9810	0.0192	0.5798
Infant mortality	-0.1745	0.0166	0.4148	-0.1439		-0.3937	0.3461	0.0931
	(224)	(222)	(223)	(207)		(224)	(223)	(211)
	0.0098	0.8080	0.0000	0.0416		0.0000	0.0000	0.1842
GDP	-0.1884	-0.1385	-0.2780	-0.0017	-0.3937		0.0441	-0.4523
	(226)	(224)	(224)	(209)	(224)		(223)	(212)
	0.0051	0.0410	0.0000	0.9810	0.0000		0.5182	0.0000
Deathrate	-0.1013	0.0527	-0.3369	-0.1654	0.3461	0.0441		0.0966
	(223)	(221)	(223)	(206)	(223)	(223)		(210)
	0.1370	0.4417	0.0000	0.0192	0.0000	0.5182		0.1695
Agriculture	-0.2123	0.1210	-0.0144	-0.0401	0.0931	-0.4523	0.0966	
	(212)	(211)	(211)	(199)	(211)	(212)	(210)	
	0.0022	0.0840	0.8379	0.5798	0.1842	0.0000	0.1695	

Correlation

(Sample Size)

P-Value







(Region="WESTERN EUROPE")+2\*(Country="United States")

# Principal Components Analysis (PCA)

- Used to explain the variance-covariance structure of a set of variables.
- Converts a set of correlated variables into a set of linearly uncorrelated variables.

$$P_{1} = a_{1,1}X_{1} + a_{1,2}X_{2} + \dots + a_{1,k}X_{k}$$
$$P_{2} = a_{2,1}X_{1} + a_{2,2}X_{2} + \dots + a_{2,k}X_{k}$$

• Primary goals are data reduction and interpretation.

# Principal Components Analysis (PCA)

- The first principal component is the linear combination of the variables that explains the largest amount of the variance.
- The second principal component is the linear combination orthogonal to the first that explains the next greatest amount of variance.
- Often helps reveal the understanding structure in the data by finding components that account for much of the variance.
- Also used to build predictive models via Principal Components Regression (PCR).

#### **Multivariate Point Cloud**

Plot of Infant mortality vs GDP and Birthrate



First principal component is dimension with maximum variance.

# PCA – Data Input

Principal Components		x
Country Region Population Area Pop. Density Coastline Net migration Infant mortality GDP Literacy Phones Arable Crops Other Farmable Climate	Data: Pop. Density Farmable Birthrate Literacy Infant mortality GDP Deathrate Agriculture	4
Birthrate Deathrate Agriculture Industry Service	(Point Labels:) Country (Select:)	
Sort column names	Delete Transform Help	

# PCA – Analysis Options

Principal Components Options		<b>—</b> ×
Missing Value Treatment	Extract By	ОК
<ul> <li>Listwise</li> </ul>	Minimum Eigenvalue	Cancel
O Pairwise	O Number of Components	Help
✓ Standardize	Minimum Eigenvalue: 1.0 Number of Components: 8	

#### **Analysis Summary**

#### Principal Components Analysis

Data variables: Pop. Density (per sq. mi.- log) Farmable (Arable + Crops - square root) Birthrate (log) Literacy (% - cubed) Infant mortality (per 1000 births - log) GDP (\$ per capita - log) Deathrate (log) Agriculture (proportion - cube root)

Data input: observations Number of complete cases: 197 Missing value treatment: listwise Standardized: yes

Number of components extracted: 2

#### Principal Components Analysis

Component		Percent of	Cumulative
Number	Eigenvalue	Variance	Percentage
1	4.4097	55.121	55.121
2	1.41423	17.678	72.799
3	0.885536	11.069	83.868
4	0.57358	7.170	91.038
5	0.309529	3.869	94.907
б	0.209147	2.614	97.522
7	0.109733	1.372	98.893
8	0.0885366	1.107	100.000

#### Scree Plot

Used to determine how many components are important.



# **Component Weights**

Shows the linear combinations defining the principal components. Helpful in interpreting their meaning.

	Fable of Co	mponent	Weigl	nts
I			<i>(</i>	

	Component	Component
	1	2
Pop. Density	0.181294	0.621948
Farmable	0.0660681	0.743694
Birthrate	-0.437552	-0.0428076
Literacy	0.392923	-0.0406149
Infant mortality	-0.45225	0.0304077
GDP	0.441459	-0.148014
Deathrate	-0.225551	0.0790226
Agriculture	-0.408214	0.165954

#### The StatAdvisor

This table shows the equations of the principal components. For example, the first principal component has the equation

0.181294\*Pop. Density + 0.0660681\*Farmable - 0.437552\*Birthrate + 0.392923\*Literacy - 0.45225\*Infant mortality + 0.441459\*GDP - 0.225551\*Deathrate - 0.408214\*Agriculture

where the values of the variables in the equation are standardized by subtracting their means and dividing by their standard deviations.

#### **2D Component Plot**

Plots the component weights for each variable.

**Plot of Component Weights** 



#### **2D Scatterplot**

Shows the value of the first 2 components for each case.

#### Scatterplot





#### Biplot



## **Save Principal Components**

_ Save	Target Variables	
<ul> <li>Eigenvalues</li> <li>Component Weights</li> </ul>	EIGENVALS CMPWGT	Cancel
Principal Components	PCOMP	
Autosave	Save comments	

#### **Factor Analysis**

Used to describe the variability amongst a set of correlated, observed variables in terms of a smaller number of unobserved variables.

The factor analysis model describes each observed variable as a linear combination of *common factors* F and *specific factors*  $\epsilon$ .

$$X_{1} - \mu_{1} = \ell_{1,1}F_{1} + \ell_{1,2}F_{2} + \dots + \ell_{1,m}F_{m} + \varepsilon_{1}$$
$$X_{2} - \mu_{2} = \ell_{2,1}F_{1} + \ell_{2,2}F_{2} + \dots + \ell_{2,m}F_{m} + \varepsilon_{2}$$
...

The coefficients are referred to as factor loadings.

# Factor Analysis – Data Input

Factor Analysis	
Country         Region         Population         Area         Pop. Density         Coastline         Net migration         Infant mortality         GDP         Literacy         Phones         Arable         Crops         Other         Farmable         Climate         Birthrate         Deathrate         Agriculture         Industry         Service	Data:   Pop. Density   Farmable   Birthrate   Literacy   Infant mortality   GDP   Deathrate   Agriculture
Sort column names	Delete Transform Help

#### Factor Analysis – Analysis Options

Factor Analysis Options	<b></b>
Missing Value Treatment	OK
<ul> <li>Listwise</li> </ul>	Cancel
O Pairwise	Estimation
✓ Standardize	Communalities
Type of Factoring	Help
Principal Components	Extract by
	Minimum Eigenvalue
Rotation	O Number of Factors
None	
C Varimax	Minimum Eigenvalue:
C Equimax	1.0
C Quartimax	Number of Factors: 8

#### **Factor Loadings**

#### **Factor Loading Matrix Before Rotation**

	Factor	Factor
	1	2
Pop. Density	0.380705	0.73963
Farmable	0.138738	0.884413
Birthrate	-0.918828	-0.0509074
Literacy	0.82511	-0.0482998
Infant mortality	-0.949693	0.0361613
GDP	0.927033	-0.17602
Deathrate	-0.473641	0.0939749
Agriculture	-0.85722	0.197355

## **Communalities and Specific Variance**

Shows relative contribution to each variable of common and specific factors.

	Estimated	Specific
Variable	Communality	Variance
Pop. Density	0.691989	0.308011
Farmable	0.801435	0.198565
Birthrate	0.846837	0.153163
Literacy	0.683139	0.316861
Infant mortality	0.903224	0.0967765
GDP	0.890373	0.109627
Deathrate	0.233167	0.766833
Agriculture	0.773775	0.226225

## **Adding Third Factor**

#### **Factor Loading Matrix Before Rotation**

	Factor	Factor	Factor
	I	2	3
Pop. Density	0.380705	0.73963	-0.266658
Farmable	0.138738	0.884413	0.141876
Birthrate	-0.918828	-0.0509074	-0.230835
Literacy	0.82511	-0.0482998	0.253663
Infant mortality	-0.949693	0.0361613	-0.0200942
GDP	0.927033	-0.17602	0.0730685
Deathrate	-0.473641	0.0939749	0.817836
Agriculture	-0.85722	0.197355	0.0455217

#### **Factor Loading Before Rotation**

#### **Plot of Factor Loadings**



#### **Factor Loading After Rotation**



Varimax rotation tries to spread out the loadings on each factor as much as possible.

#### **Scatterplot of Factor Scores**

Scatterplot



#### **T-Squared Chart**

Hotelling's  $T^2$  measures the distance of each observation from the centroid of the data, in a standardized scale.



# **T-Squared Decomposition**

#### **T-Squared Decomposition**

Observation	T-Squared	Pop. Density	Farmable	Birthrate	Literacy	Infant mortality	GDP	Deathrate	Agriculture
59	24.3257	0.033346	1.22715	2.6807	1.8167	1.25591	18.3462	0.554158	9.14817
123	32.7143	16.4965	15.4323	2.03238	0.0121135	0.000147106	3.74714	0.0523997	1.72176
139	50.4379	29.9069	24.1723	0.0442304	1.94445	0.0669598	1.71138	4.42132	15.3636
185	23.6725	7.52803	6.47165	0.0187349	0.425415	3.68599	4.87123	0.0911828	1.82684

#### Relative Contribution to T-Squared Signal

#### The StatAdvisor

This table decomposes the out-of-control signals on the T-Squared chart. It calculates the relative importance of each variable to the signal by subtracting the value of T-Squared calculated without using that variable from the full T-Squared value. Examine each row closely to determine which variable (or variables) are likely causing that signal.

Large values indicate that a particular variable has a big effect on T-Squared for that observation.

### Control Ellipse (2 variables at a time)

#### **Control Ellipse**



#### Control Ellipse (2 variables at a time)

**Control Ellipse** 



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# T-Squared chart for First 3 Principal Components





#### **Uniwin from Sigma Plus**

#### 💾 UNIWIN Plus

Fichiers Edition Graphiques Décrire Expliquer Manuel Fenêtre Aide

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	Country	Region	Pop. Density	Infant mortality	GDP	Literacy	Farmable	Birthrate	Deathrate	Agriculture	Phones 🔺
			per sq. milog	per 1000 births - log	\$ per capita - log	% - cubed	Arable + Crops - square root	per 1000 - log	per 1000 - log	proportion - cube root	per 1000 - cube
	Type = C	Type = C	Type = N	Type = N	Type = N	Type = N	Type = N	Type = N	Type = N	Type = N	Type = N
	Longueur = 227	Longueur = 227	Longueur = 227	Longueur = 227	Longueur = 227	Longueur = 227	Longueur = 227	Longueur = 227	Longueur = 227	Longueur = 227	Longueur = 2:
1	Afahanistan	ASIA (EX. NEAR EAST)	3.87120	5.09418	6.55108	46656.000	3.51426	3.84160	3.01259	0.72432	1.4
2	Albania	EASTERN EUROPE	4.82511	3.06898	8.41183	647214.625	5.05074	2.71536	1.65250	0.61446	4.1
3	Algeria	NORTHERN AFRICA	2.62467	3.43399	8.69951	343000.000	1.86279	2.84141	1.52823	0.46570	4.2
4	American Samoa	OCEANIA	5.67126	2.22678	8.98720	912673.000	5.00000	3.11174	1.18479		6.3
5	Andorra	WESTERN EUROPE	5.02454	1.39872	9.85219	1000000.000	1.48997	2.16447	1.83258		7.9
6	Angola	SUB-SAHARAN AFRICA	2.27213	5.25327	7.54961	74088.000	1.62788	3.80910	3.18635	0.45789	1.9
7	Anguilla	LATIN AMER, & CARIB	4.88356	3.04595	9.05952	857375.000	0.00000	2.65113	1.67523	0.34200	7.7
8	Antiqua & Barbuda	LATIN AMER, & CARIB	5.04986	2.96836	9.30565	704969.000	4.76760	2.82909	1.68083	0.33620	8.1
9	Argentina	LATIN AMER. & CARIB	2.66723	2.71998	9.32367	915498.611	3.57631	2.81720	2.02155	0.45629	6.0
10	Armenia	C.W. OF IND. STATES	4.60417	3.14759	8.16052	958585.256	4.45533	2.49072	2.10779	0.62058	5.8
11	Aruba	LATIN AMER, & CARIB	5.92024	1.77326	10.23996	912673.000	3.24500	2.40062	1.89912	0.15874	8.0
12	Australia	OCEANIA	0.95551	1.54543	10.27505	1000000.000	2.56710	2.49651	2.01624	0.33620	8.2
13	Austria	WESTERN EUROPE	4.58190	1.53902	10.30895	941192.000	4.21545	2.16791	2.27829	0.26207	7.6
14	Azerbaijan	C.W. OF IND. STATES	4.52070	4.40354	8.13153	912673.000	4.72652	3.03206	2.27727	0.52048	5.1
15	Bahamas, The	LATIN AMER, & CARIB	3.08191	3.22724	9.72316	873722.816	1.09545	2.86619	2.20276	0.31072	7.7
16	Bahrain	NEAR EAST	6.95702	2.84897	9.73507	707347.971	2.90689	2.87920	1.42070	0.17100	6.5
17	Bangladesh	ASIA (EX. NEAR EAST)	6.93089	4.13677	7.54961	80062.991	8.07341	3.39451	2.11263	0.58383	1.9
18	Barbados	LATIN AMER, & CARIB	6.47620	2.52573	9.66142	924010.424	6.28808	2.54239	2.15987	0.39149	7.8
19	Belarus	CW OF IND STATES	3,90399	2.59301	8,71604	988047.936	5.49090	2.41234	2.64048	0.45307	6.8
20	Belgium	WESTERN EUROPE	5.82895	1.54330	10.27849	941192.000	4.86621	2.33988	2.32923	0.21544	7.7
21	Belize	LATIN AMER & CARIB	2.52573	3.24610	8,49699	833237.621	2.13542	3.36176	1.74397	0.52171	4,8
22	Benin	SUB-SAHARAN AFRICA	4,24563	4,44265	7.00307	68417.929	4.52548	3,65971	2.50307	0.68113	2.1
23	Bermuda	NORTHERN AMERICA	7.12367	2.14359	10.49127	941192.000	4.47214	2.43361	2.04640	0.21544	9,4
24	Bhutan	ASIA (EX. NEAR EAST)	3.88156	4.60956	7.17012	75151.448	1.87617	3,51601	2.54160	0.63661	2.4
25	Bolivia	LATIN AMER, & CARIB	2.10413	3.97237	7.78322	663054.848	1.69115	3.14845	2.01890	0.50397	4.1
26	Bosnia & Herzegovina	EASTERN EUROPE	4,47734	3.04690	8,71604		4.06940	2.17134	2.11263	0.52171	5.9
27	Botswana	SUB-SAHARAN AFRICA	0.99325	3,99967	9.10498	508169.592	0.81240	3,13897	3.38439	0.28845	4.3
28	Brazil	LATIN AMER & CARIB	3.09558	3.38811	8,93590	644972.544	2.80357	2.80699	1.81970	0.43795	6.0
29	British Virgin Is.	LATIN AMER, & CARIB	5.01728	2.89315	9,68034	935441,352	5.16430	2,70069	1,48614	0.26207	7.9
30	Brupei	ASIA (EX. NEAR EAST)	4.18662	2.53449	9,83092	827936.019	1.15326	2,93332	1.23837	0.33019	6.1
31	Bulgaria	EASTERN EUROPE	4,19870	3.02286	8.93590	958585,256	6.47611	2.26696	2.65816	0.45307	6.9
32	Burkina Faso	SUB-SAHARAN AFRICA	3,92593	4,58057	7,00307	18821.096	3 82361	3,82035	2,74727	0.68541	1.9
33	Burna	ASIA (EX. NEAR EAST)	4.24563	4.20827	7,49554	620650.477	4.01995	2.88536	2.28544	0.82621	2.1 -
• •	countries for uniwin.sf6	<	1.2.000					2.0000	2.20011		•

44

# Uniwin PCA

#### Allows for supplemental variables or observations.



# Supplemental Variables are Passive (quantitative and qualitative)



#### **Correlation Circle**



47

# 2D Scatterplot with symbols proportional to squared cosines



48

# 2D Scatterplot with selection based on squares cosines



# Multidimensional Scaling (Uniwin)

Objective: to display the observations in a low dimensional coordinate system such that the distance between data points is distorted as little as possible.

Input: an N by N matrix of similarities (or dissimilarities).

Output: a map displaying the location of the points in 2 dimensions.

#### Preparing a New Data File

Step 1: removed all rows with incomplete data on the 8 variables of interest.

Step 2: used the STANDARDIZE operator to subtract the mean of each column and divide by its standard deviation.

Step 3: saved a new data file named "countries scaled for UNIWIN.sf6".

#### Load Standardized Variables

#### 💾 UNIWIN Plus

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#### 

Ð	onnées									
	Country	Region	Pop. Density	Infant mortality	GDP	Literacy	Farmable	Birthrate	Deathrate	Agriculture
			per sq. milog	per 1000 births - log	\$ per capita - log	% - cubed	Arable + Crops - square root	per 1000 - log	per 1000 - log	proportion - cube root
	Type = C	Type = C	Type = N	Type = N	Type = N	Type = N	Type = N	Type = N	Type = N	Type = N
	Longueur = 197	Longueur = 197	Longueur = 197	Longueur = 197	Longueur = 197	Longueur = 197	Longueur = 197	Longueur = 197	Longueur = 197	Longueur = 197
1	Afahanistan	ASIA (EX. NEAR EAST)	-0.22585	1.80065	-1.65397	-1.84078	-0.16251	1.66996	1.80525	1.39661
2	Albania	EASTERN EUROPE	0.38730	-0.06222	-0.10485	0.01123	0.61703	-0.54323	-0.98730	0.77961
3	Algeria	NORTHERN AFRICA	-1.02710	0.27353	0.13466	-0.92691	-1.00038	-0.29551	-1.24245	-0.05592
4	Angola	SUB-SAHARAN AFRICA	-1.25371	1.94699	-0.82267	-1.75618	-1.11957	1.60610	2.16202	-0.09982
5	Anguilla	LATIN AMER. & CARIB	0.42488	-0.08341	0.43437	0.65933	-1.94547	-0.66945	-0.94064	-0.75073
8	Antiqua & Barbuda	LATIN AMER. & CARIB	0.53177	-0.15478	0.63928	0.18934	0.47337	-0.31974	-0.92913	-0.78329
7	Argentina	LATIN AMER. & CARIB	-0.99975	-0.38325	0.65428	0.83857	-0.13103	-0.34309	-0.22957	-0.10878
8	Armenia	C.W. OF IND. STATES	0.24529	0.01009	-0.31407	0.97144	0.31494	-0.98466	-0.05250	0.81398
9	Aruba	LATIN AMER, & CARIB	1.09124	-1.25409	1.41712	0.82986	-0.29912	-1.16172	-0.48094	-1.77999
0	Australia	OCEANIA	-2.10001	-1.46365	1.44633	1.09916	-0.64305	-0.97329	-0.24047	-0.78329
1	Austria	WESTERN EUROPE	0.23097	-1.46956	1.47455	0.91780	0.19324	-1.61902	0.29758	-1.19961
2	Azerbaijan	C.W. OF IND. STATES	0.19164	1.16537	-0.33821	0.82986	0.45253	0.07914	0.29548	0.25176
3	Bahamas, The	LATIN AMER, & CARIB	-0.73320	0.08335	0.98687	0.70974	-1.38970	-0.24682	0.14251	-0.92637
4	Bahrain	NEAR EAST	1.75767	-0.26460	0.99678	0.19667	-0.47066	-0.22126	-1.46324	-1.71115
5	Bandladesh	ASIA (EX. NEAR EAST)	1.74087	0.91998	-0.82267	-1.73776	2.15058	0.79138	-0.04255	0.60754
6	Barbados	LATIN AMER, & CARIB	1.44860	-0.56193	0.93546	0.86482	1.24479	-0.88313	0.05444	-0.47275
7	Belarus	C.W. OF IND. STATES	-0.20478	-0.50004	0.14842	1.06230	0.84034	-1.13869	1.04124	-0.12689
8	Belgium	WESTERN EUROPE	1.03256	-1.46562	1.44920	0.91780	0.52340	-1.28108	0.40216	-1.46152
9	Belize	LATIN AMER, & CARIB	-1.09070	0.10070	-0.03395	0.58489	-0.86207	0.72703	-0.79949	0.25866
0	Benin	SUB-SAHARAN AFRICA	0.01483	1.20135	-1.27768	-1.77367	0.35054	1.31252	0.75911	1.15404
!1	Bermuda	NORTHERN AMERICA	1.86479	-0.91344	1.62634	0.91780	0.32347	-1.09688	-0.17854	-1.46152
22	Bhutan	ASIA (EX. NEAR EAST)	-0.21919	1.35488	-1.13861	-1.75290	-0.99360	1.03014	0.83821	0.90400
23	Bolivia	LATIN AMER, & CARIB	-1.36170	0.76875	-0.62818	0.06008	-1.08747	0.30785	-0.23501	0.15901
24	Botswana	SUB-SAHARAN AFRICA	-2.07575	0.79387	0.47222	-0.41756	-1.53330	0.28921	2.56863	-1.05147
25	Brazil	LATIN AMER, & CARIB	-0.72441	0.23133	0.33146	0.00432	-0.52308	-0.36316	-0.64400	-0.21178
26	British Virgin Is.	LATIN AMER, & CARIB	0.51083	-0.22396	0.95122	0.90007	0.67464	-0.57205	-1.32887	-1.19961
27	Brunei	ASIA (EX. NEAR EAST)	-0.02311	-0.55387	1.07658	0.56854	-1.36037	-0.11490	-1.83758	-0.81702
28	Bulgaria	EASTERN EUROPE	-0.01534	-0.10465	0.33146	0.97144	1.34019	-1.42438	1.07753	-0.12689
29	Burkina Faso	SUB-SAHARAN AFRICA	-0,19068	1,32821	-1,27768	-1,92662	-0.00556	1.62819	1,26050	1,17810
30	Burma	ASIA (EX. NEAR EAST)	0.01483	0,98575	-0,86768	-0.07069	0.09405	-0.20916	0.31226	1,96893
31	Burundi	SUB-SAHARAN AFRICA	0.93186	1.01337	-1,78231	-1.56098	1.60852	1,47599	0.95755	1.67352
32	Cambodia	ASIA (EX. NEAR EAST)	0.07542	1.04200	-0.82267	-0.95388	0 41084	0.59018	0.14478	1,28660
33	Cameroon	SUB-SAHARAN AFRICA	-0.40190	0.99960	-0.86768	-0.46422	0.04487	1 04411	0.95907	1.62608

# Create Matrix of Distances between Points (197 by 197)

💾 Créer un table	au de distances euclio	liennes		X					
Variables contenar	nt les données pour les c	bservations:							
countries scaled for uniwin.sf6:Country countries scaled for uniwin.sf6:Region									
countries scaled f countries scaled f countries scaled f countries scaled f	or uniwin.sf6:Pop. Densi or uniwin.sf6:Infant morta or uniwin.sf6:GDP or uniwin.sf6:Literacy	ty ality							
countries scaled f countries scaled f countries scaled f	countries scaled for uniwin.sf6:Farmable countries scaled for uniwin.sf6:Birthrate countries scaled for uniwin.sf6:Deathrate countries scaled for uniwin.sf6:Agriculture								
Nom de la variable tableau de distanc	pour enregistrer le es euclidiennes:	Nom du fichier dans sera enregistrée:	lequel la variab	le					
distance	6	countries scaled for	uniwin.sf6	*					
Ok	Annuler	Sélection	Aide						

#### Analyze the Distance Matrix

#### 💾 Analyse Factorielle sur Tableau de Distances ou de Dissimilarités 23 Distances ou dissimilarités: (Libellés des individus:) countries scaled for uniwin.sf6:distance 184 countries scaled for uniwin.sf6:Country countries scaled for uniwin.sf6:distance 185 countries scaled for uniwin.sf6:Country countries scaled for uniwin.sf6:distance 186 countries scaled for uniwin. S:Region countries scaled for uniwin.sf6:distance 187 countries scaled for uniwin.sf6:Pop. Density countries scaled for uniwin.sf6:distance 188 countries scaled for uniwin.sf6:Infant mortality countries scaled for uniwin.sf6:distance 189 countries scaled for uniwin.sf6:distance 190 Si valeurs propres négatives: countries scaled for uniwin.sf6:distance 191 countries scaled for uniwin.sf6:distance 192 💿 Les ignorer countries scaled for uniwin.sf6:distance\_193 countries scaled for uniwin.sf6:distance 194 Ajouter une constante countries scaled for uniwin.sf6:distance 195 countries scaled for uniwin.sf6:distance 196 Quitter la procédure countries scaled for uniwin.sf6:distance 197 0k Sélection Annuler Aide

#### **Scatterplot of Countries**



#### **Observed versus Calculated Distances**



#### **More Information**

Statgraphics Centurion: www.statgraphics.com

Uniwin: www.statgraphics.fr or www.sigmaplus.fr

Or send e-mail to info@statgraphics.com



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