Monte Carlo Simulation

Presented by

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Monte Carlo Simulation

"Monte Carlo simulations are used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables." *Investopedia*

Monte Carlo Simulation

“The world … is full of more complicated systems …. the complex interaction of many variables — or the inherently probabilistic nature of certain phenomena — rules out a definitive prediction. So a Monte Carlo simulation uses essentially random inputs (within realistic limits) to model the system and produce probable outcomes.” *MIT News*

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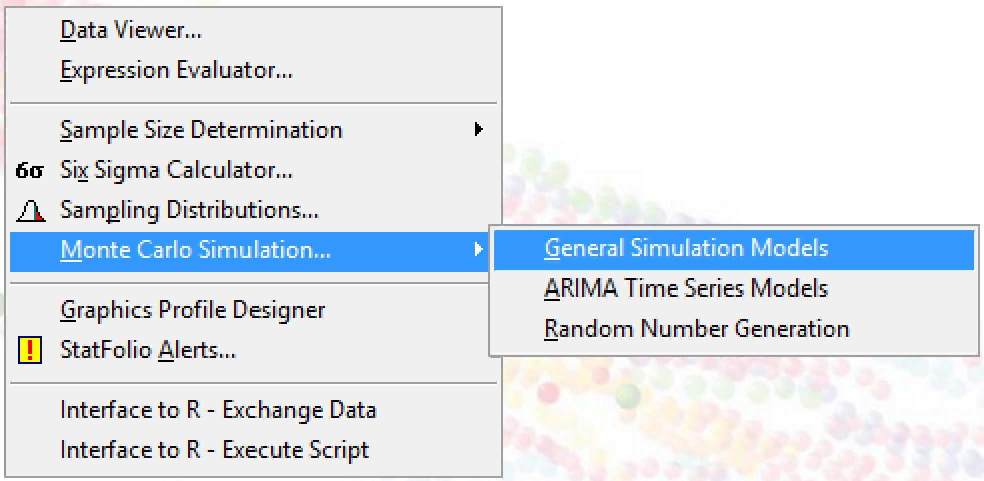
Tools menu has a section for *Monte Carlo Simulation*:



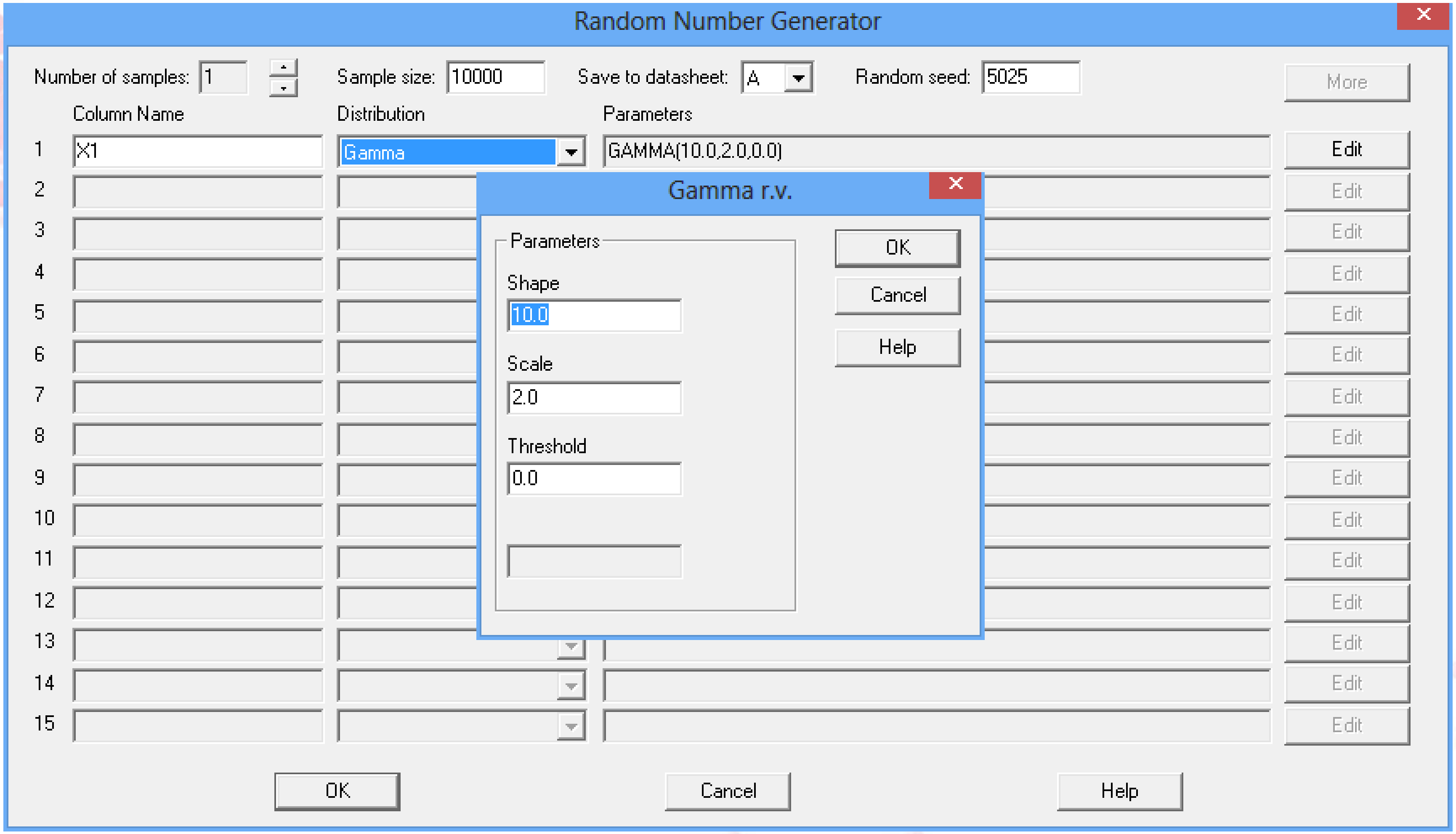
Outline

1. Random number generators
2. Example #1: Estimating the amount of oil available in an oil reservoir
3. Example #2: Simulating the consumer price index using an ARIMA time series model

Random number generators

- Includes:
  - 7 discrete distributions
  - 39 continuous distributions
  - user-specified discrete distribution
    - user specifies values for x and p(x)
  - user-specified continuous distribution
    - user specifies values for x and F(x)

Example – Gamma Distribution



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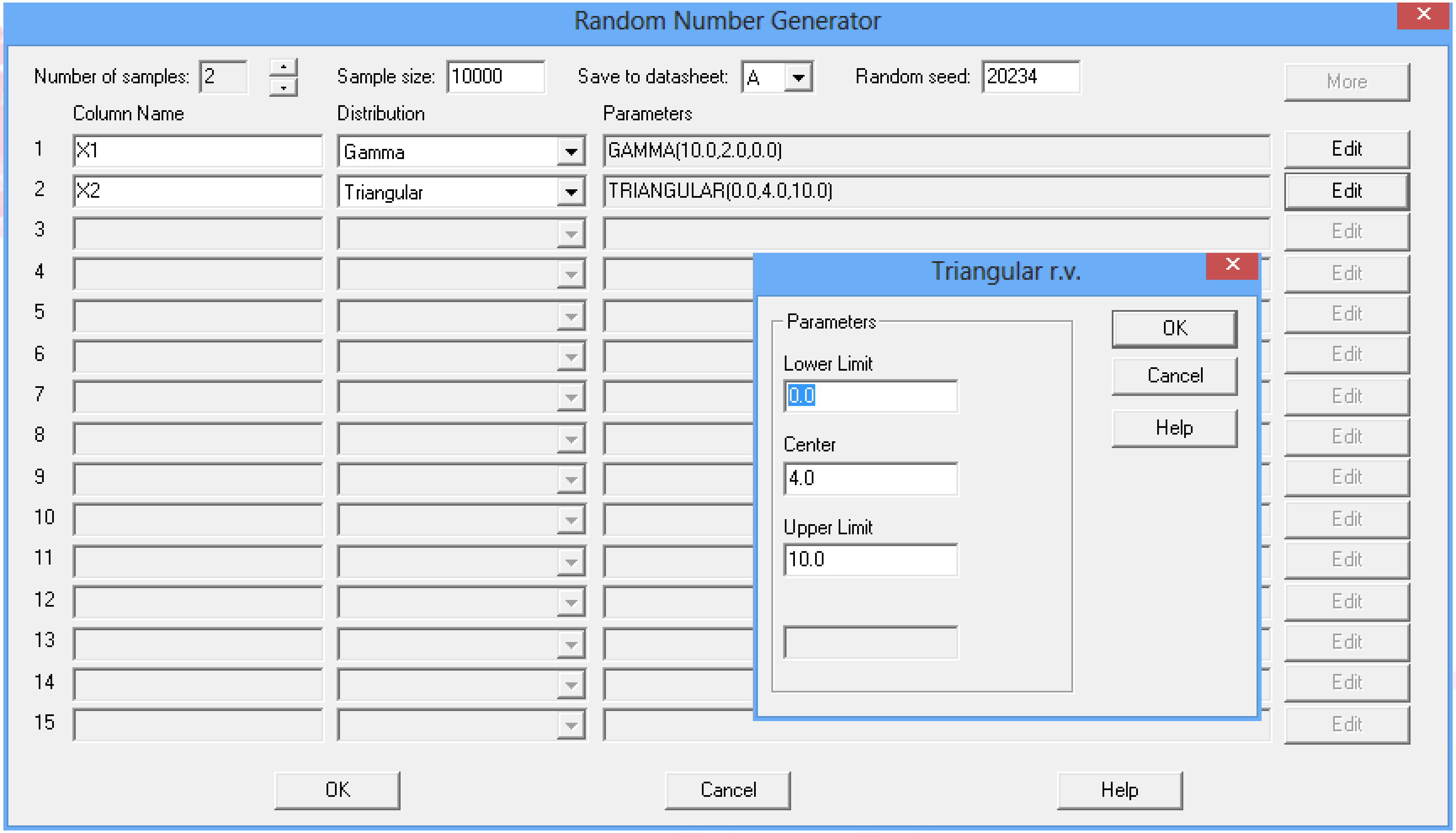
Example – Triangular Distribution



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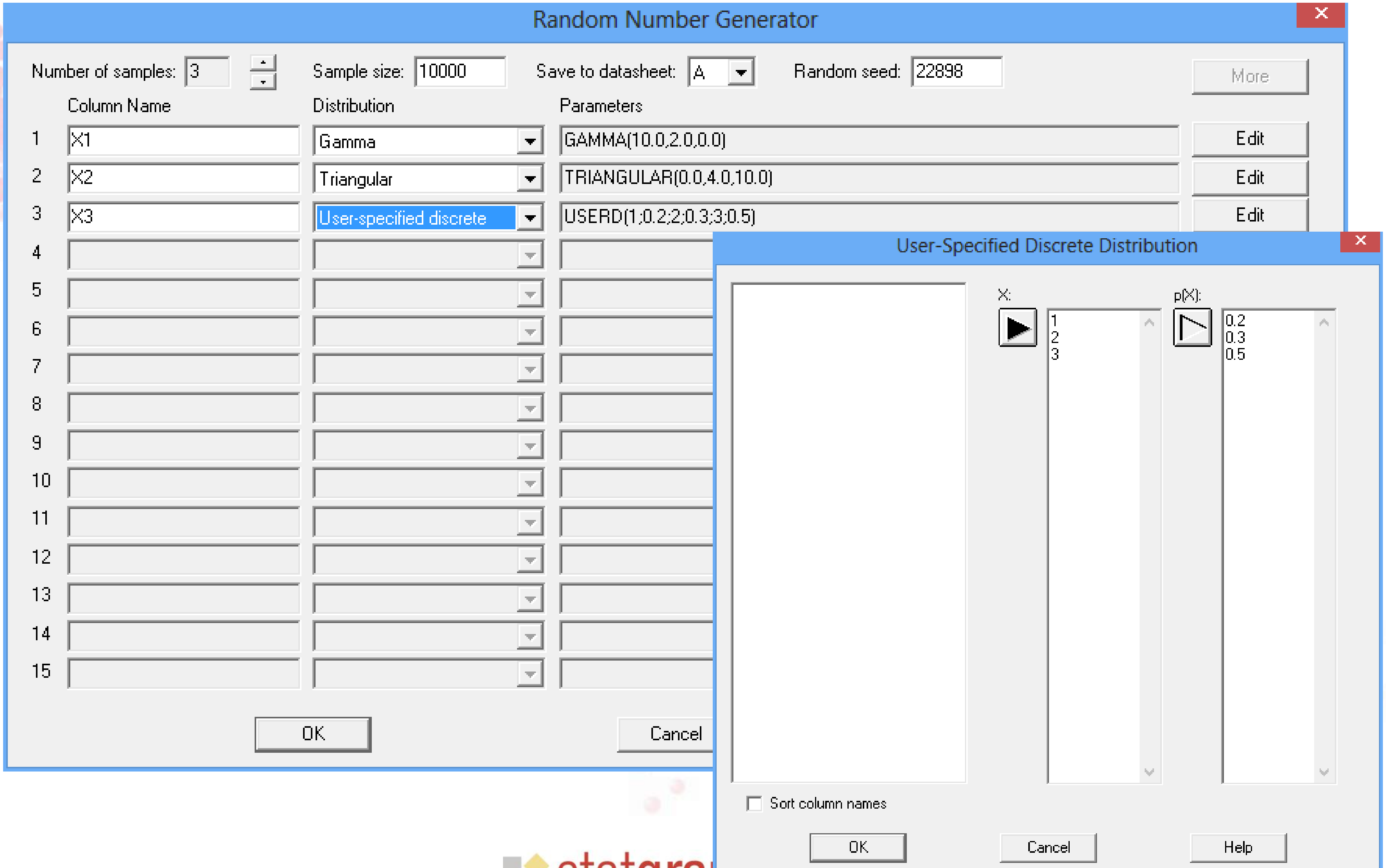
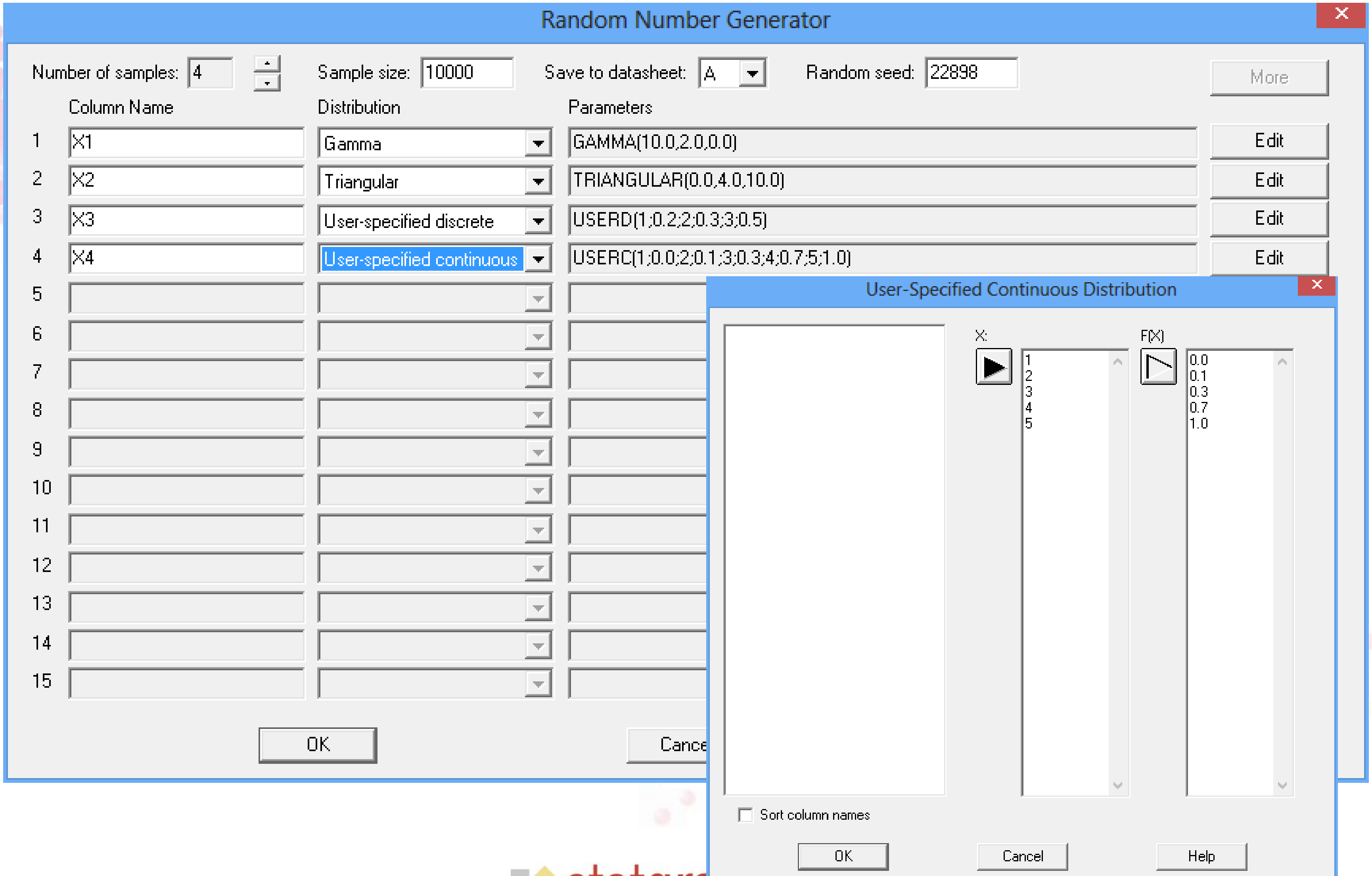
Example – User Specified Discrete



Example – User Specified Continuous



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Datasheet

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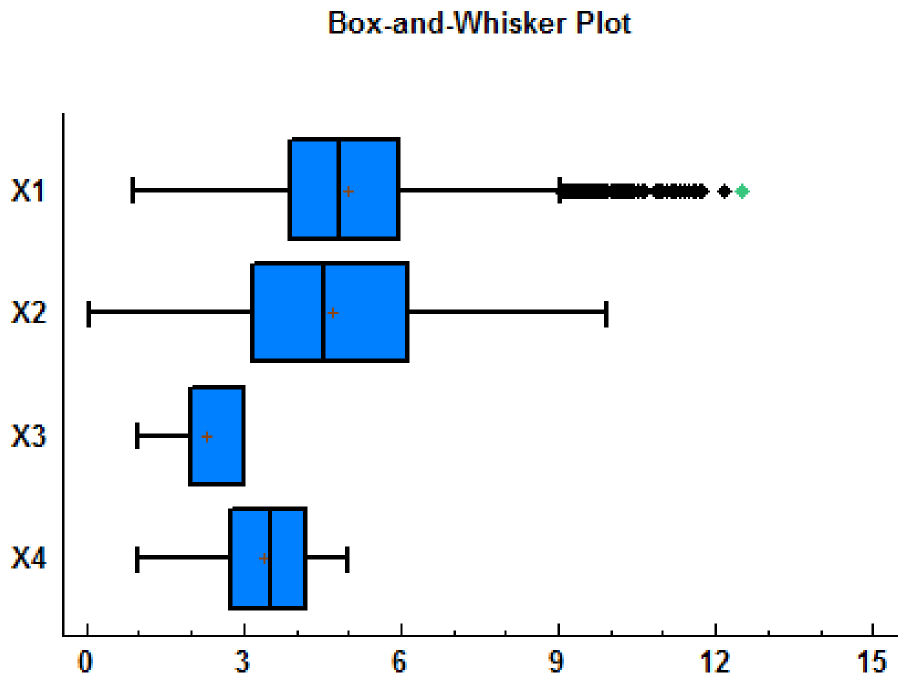
	X1	X2	X3	X4
1	4.4263	8.13002	3.0	4.43163
2	5.34978	3.42481	2.0	3.93414
3	5.09337	2.23698	3.0	3.38972
4	5.16672	4.29812	3.0	2.75472
5	3.42699	1.80771	2.0	3.99378
6	5.53609	0.932991	3.0	3.75126
7	2.89852	4.8701	2.0	3.24061
8	4.39979	4.49062	3.0	1.61507
9	5.33839	1.01202	3.0	2.94342
10	5.71663	7.69211	2.0	4.96178
11	6.15975	5.00934	3.0	3.13898
12	3.82894	8.36813	1.0	1.04288
13	4.61559	3.20668	3.0	4.79058
14	4.84695	4.00717	3.0	2.28208
15	4.51131	5.62881	2.0	3.14273
16	8.05416	3.10584	2.0	3.20243
17	5.78243	4.03803	3.0	4.97599
18	7.45231	2.58465	3.0	2.34332

A B C

Box-and-Whisker Plot



General Simulation Models

- Begin with a mathematical function:

$$Y = f(X_1, X_2, X_3, \ldots, X_p)$$

- Specify probability distributions for X_1 through X_p.

- Generate *n* sets of random numbers and plug them into the function.

- Calculate statistics from the *n* values of *Y*.

Example: Oil Reserves

Murtha presents the following volumetric model for oil in place:

$$N = 7{,}758Ah\phi(1 - S_w) / B_o \quad \text{stock-tank barrels}$$

where

- A = area
- h = net pay
- ϕ = porosity
- S_w = water saturation
- B_0 = formation volume factor

Question: Given a selected site, how small or large might N reasonably be?

Source: Murtha, J.A. 1994. "Incorporating Historical Data Into Monte Carlo Simulation". *SPE Comp App* **6** (2): 11-17. SPE-26245-PA.

How do get parameter distributions?

- If we have historical data, we can fit a probability distribution using the *Distribution Fitting (Uncensored Data)* procedure in Statgraphics.
  - Murtha used data from 83 similar reservoirs

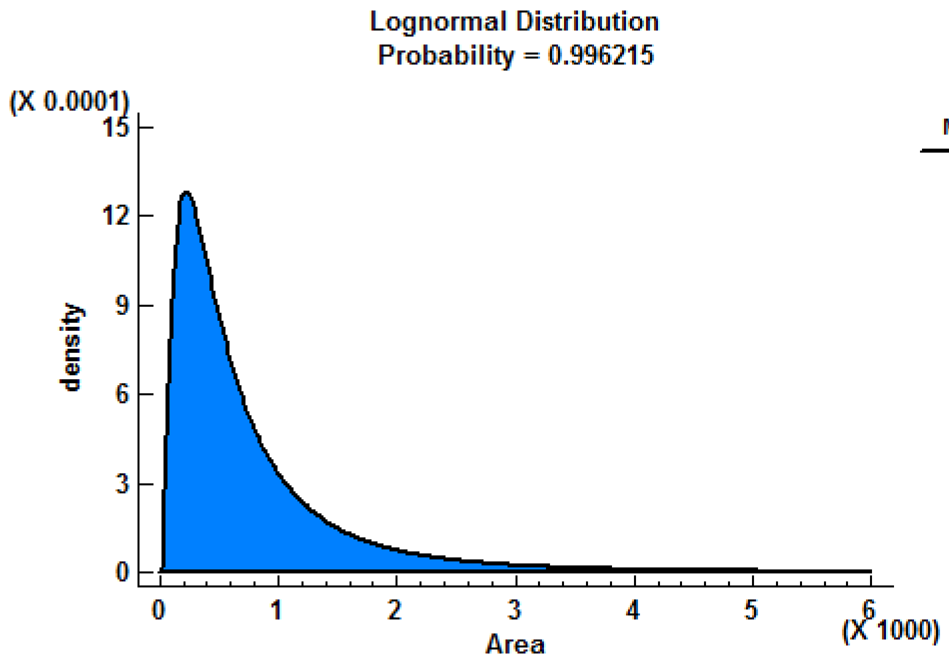
- We could specify the minimum value, most likely value, and maximum value and use a triangular distribution.

- We could specify the mean and standard deviation and use a normal distribution.

- We could specify 2 or more percentiles and find a matching non-normal distribution.

Area

- Characterized by a lognormal distribution with mean = 788 and standard deviation = 906



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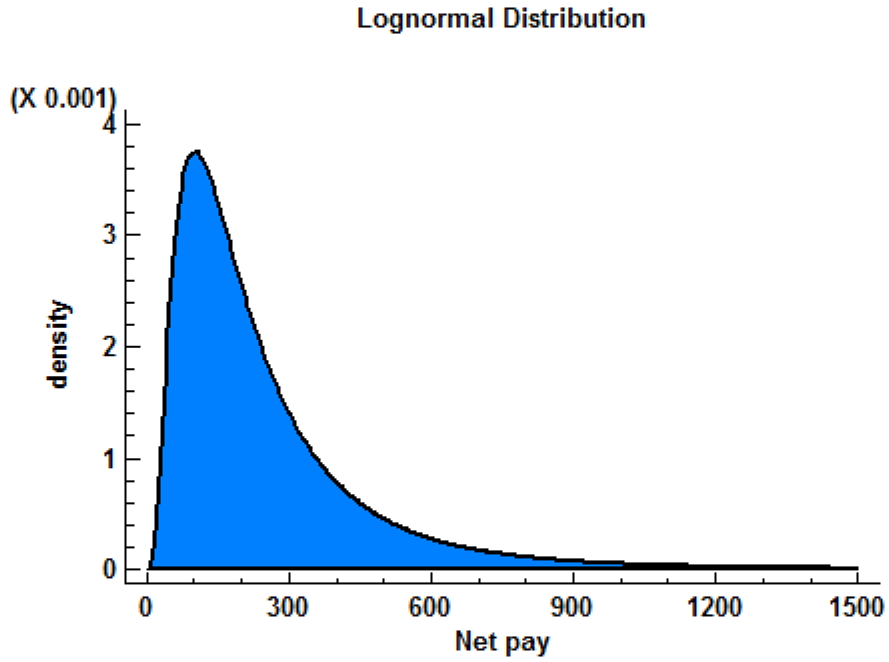
Net pay

- Characterized by a lognormal distribution with mean = 250 and standard deviation = 227



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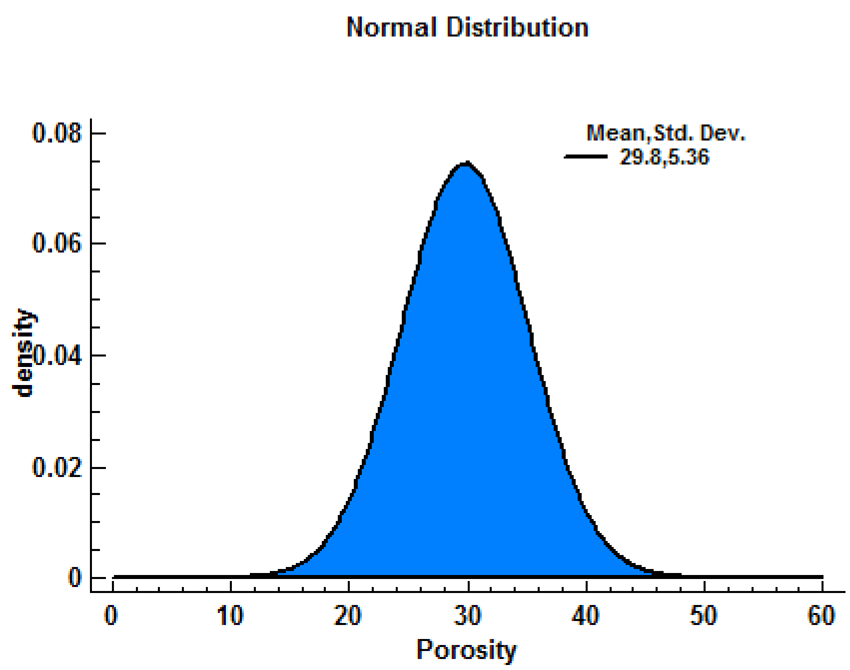
Porosity as %

- Characterized by a normal distribution with mean = 29.8 and standard deviation = 5.36



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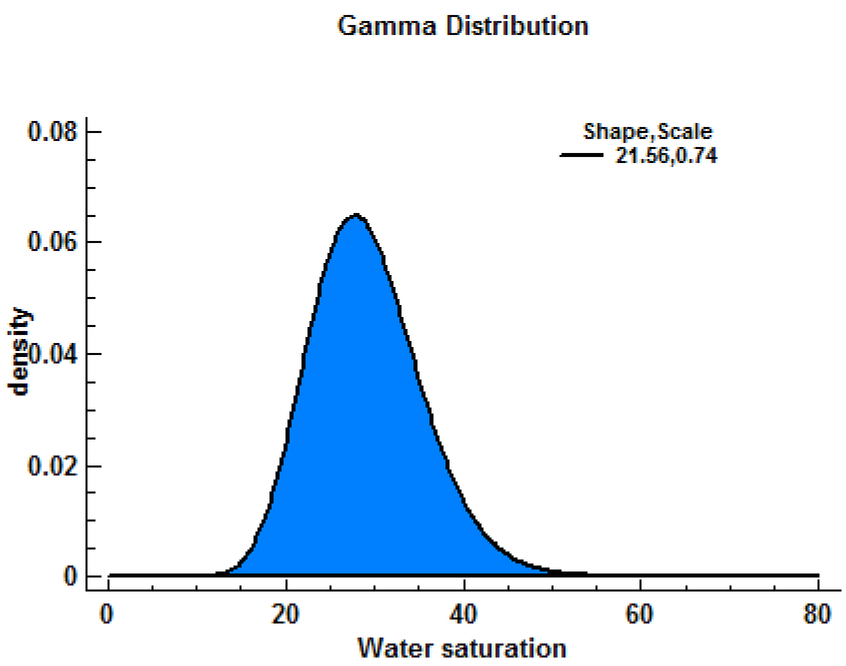
Water saturation as %

- Characterized by a gamma distribution with shape = 21.56 and scale = 0.74



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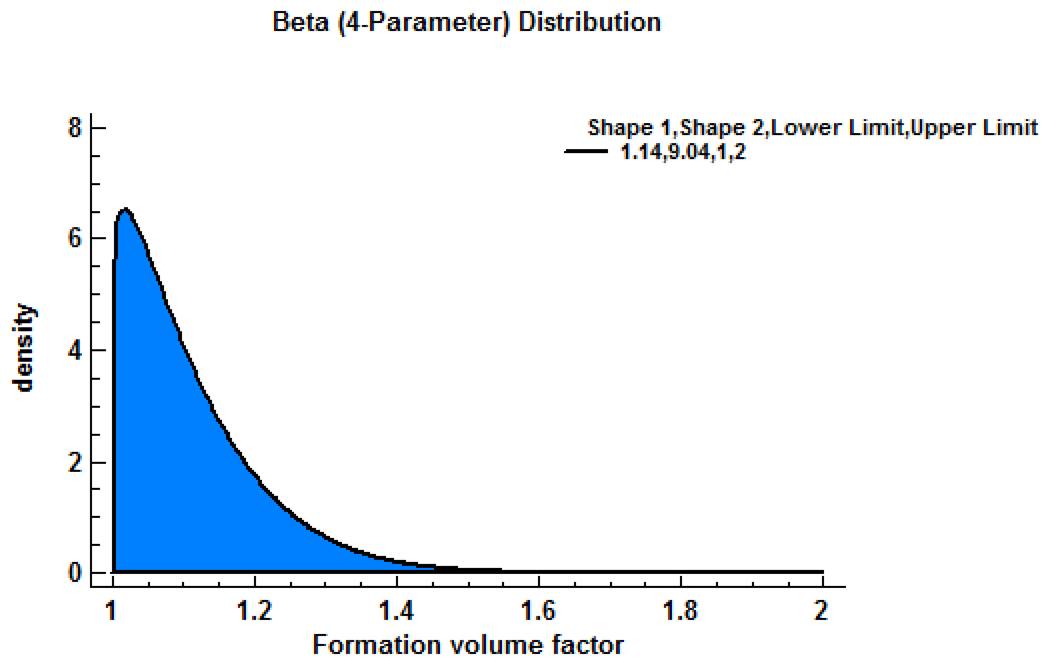
Formation volume factor

- Characterized by a beta distribution with shape1 = 1.14 and shape2 = 9.04, in the interval [1,2]



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Matching Percentiles

- *Water saturation*: suppose we are 90% certain that it is between 20 and 40 and want to use a gamma distribution.

- I have written an R script that I've saved in *matchpercentiles.sgp* that will find the parameters of various distributions for which the cdf satisfies:
  - F(20) = 0.05
  - F(40) = 0.95

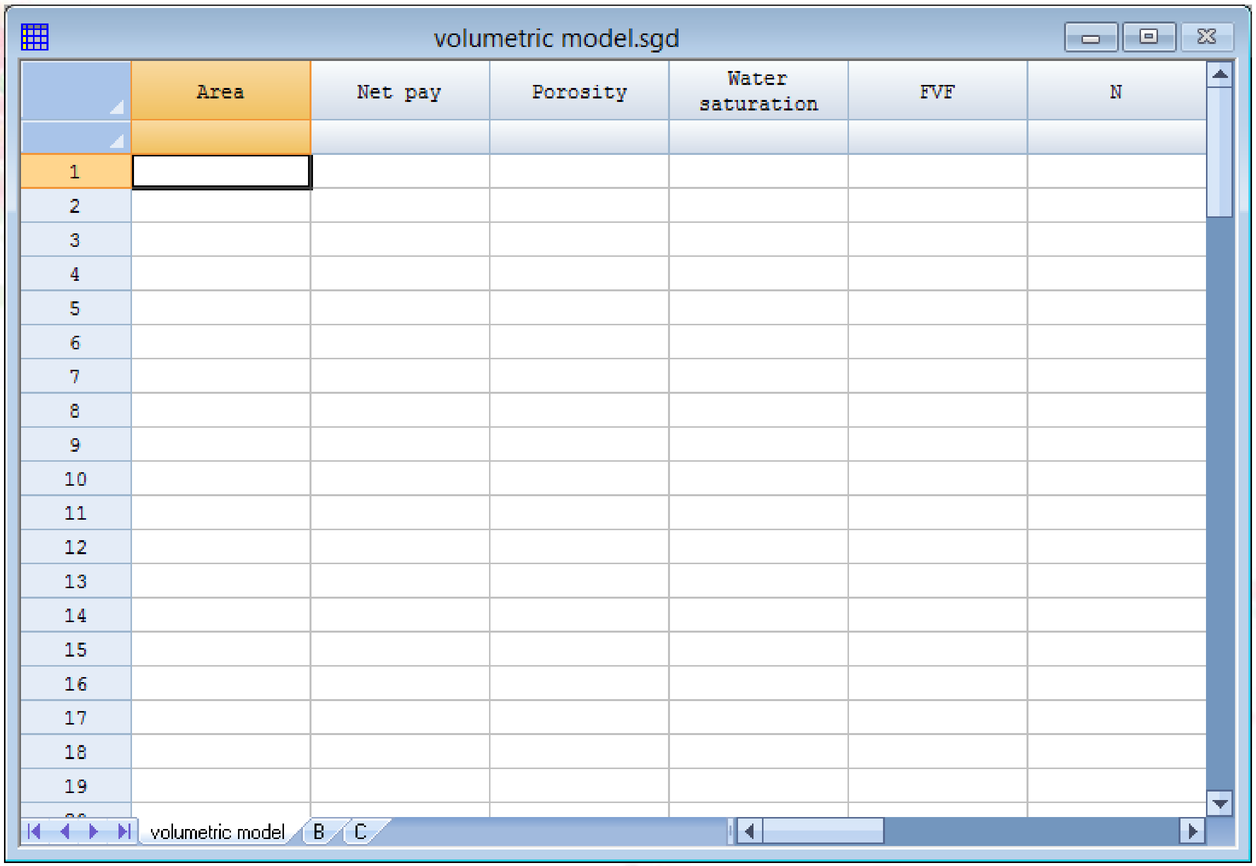
R Script

Interface to R - Execute Script

```
#LOAD REQUIRED LIBRARY
library(nleqslv)

## Warning: package 'nleqslv' was built under R version 3.2.5

#SET VALUES FOR X WITH PROBABILITIES
X<-c(20,40)
p<-c(0.05,0.95)

#DEFINE FUNCTION TO RETURN DIFFERENCES FROM TARGET VALUES
fn <- function(parms,X,p,dist) {
result<-c(999,999)
if(parms[1]>0.0&&parms[2]>0.0) {
     if(dist==1) result <- p - pgamma(X,parms[1],parms[2])
     if(dist==2) result <- p - plnorm(X,parms[1],parms[2])
     if(dist==3) result <- p - pnorm(X,parms[1],parms[2])  }
return (result) }

#SOLVE FOR GAMMA PARAMETER VALUES
sol<-nleqslv(c(1.0,0.01),fn,p=p,X=X,dist=1,control=list(allowSingular=TRUE))
sol$x

## [1] 22.9498931  0.7839015

#SOLVE FOR LOGNORMAL PARAMETER VALUES
sol<-nleqslv(c(10,1),fn,p=p,X=X,dist=2,control=list(allowSingular=TRUE))
c(exp(sol$x[1]+0.5*sol$x[2]^2),sqrt(exp(2*sol$x[1]+sol$x[2]^2)*(exp(sol$x[2]^2)-1)))

## [1] 28.919135  6.161571

#SOLVE FOR NORMAL PARAMETER VALUES
sol<-nleqslv(c(2000,500),fn,p=p,X=X,dist=3,control=list(allowSingular=TRUE))
sol$x

## [1] 30.000000  6.079568
```

General Simulation Models

- Step 1: create datasheet with variables



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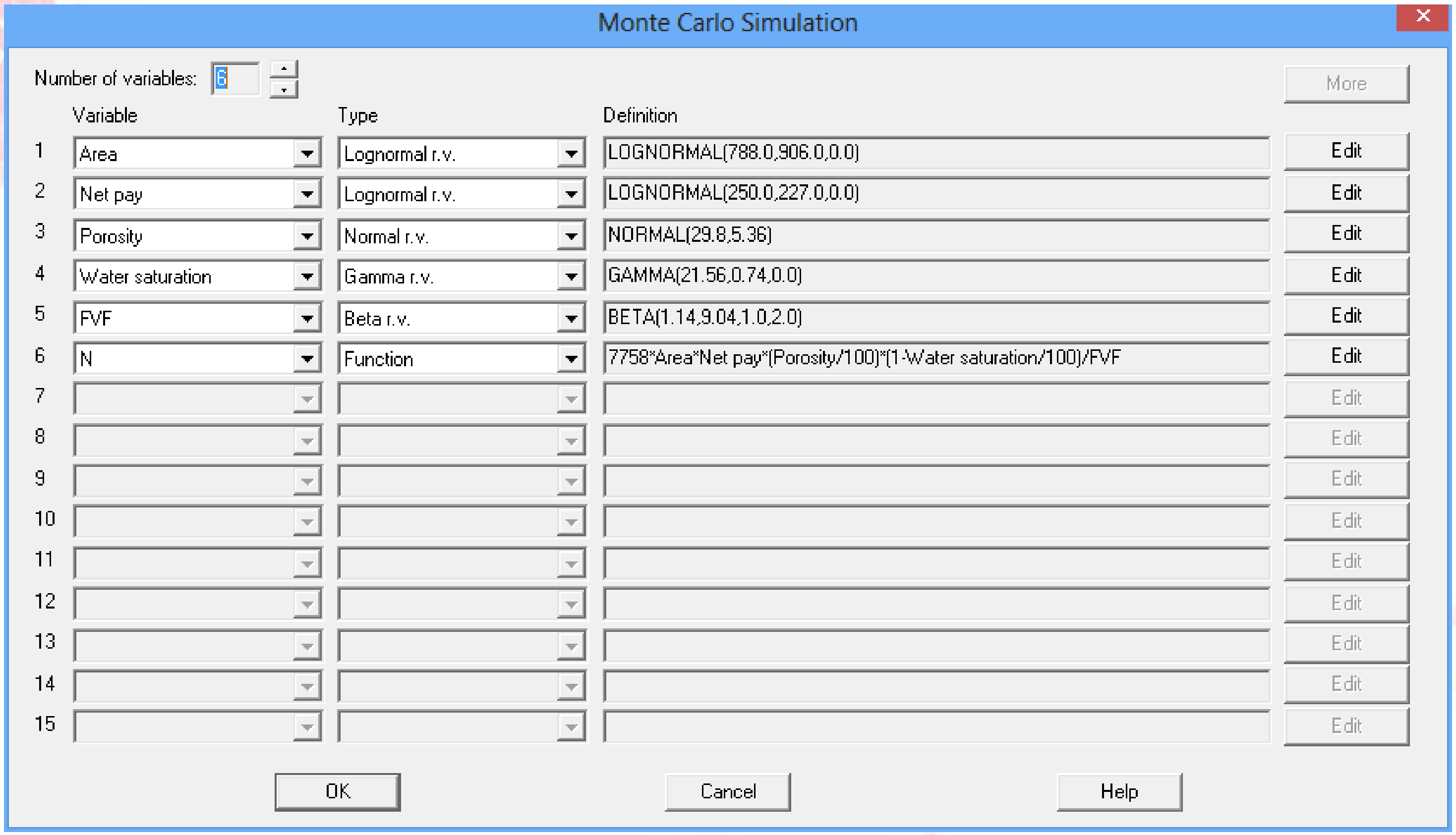
General Simulation Models

- Step 2: define the simulation



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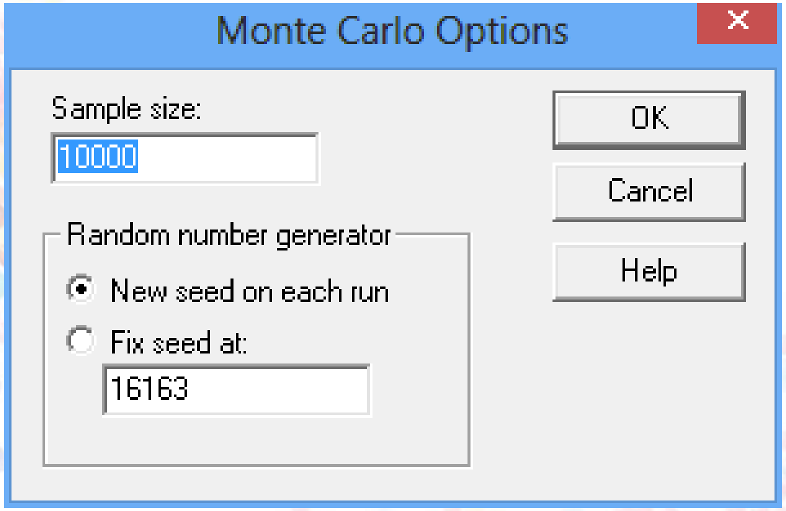
General Simulation Models

- Step 3: run the simulation



General Simulation Models

- Step 4: calculate summary statistics

Summary Statistics

	Area	*Net pay*	*Porosity*	*Water saturation*	*FVF*	*N*
Count	10000	10000	10000	10000	10000	10000
Average	798.257	254.146	29.7738	29.1382	1.11129	3.02279E8
Median	520.151	187.103	29.7858	28.6549	1.08525	1.41263E8
Standard deviation	921.451	235.396	5.34707	6.2976	0.094791	5.81547E8
Coeff. of variation	115.433%	92.6223%	17.959%	21.6128%	8.5298%	192.388%
Minimum	13.2084	11.7636	3.71162	10.7716	1.00003	2.31833E6
Maximum	18030.1	3364.35	50.0127	58.1647	1.62905	1.8165E10
Range	18016.9	3352.59	46.3011	47.3931	0.62902	1.81627E10
Lower quartile	284.913	110.926	26.1484	24.6363	1.03933	6.15614E7
Upper quartile	963.019	318.933	33.3238	33.0429	1.15893	3.22183E8
Stnd. skewness	189.92	139.082	-0.101744	19.5701	55.7774	372.656
Stnd. kurtosis	882.858	425.331	0.724102	7.0223	41.8117	3189.81

General Simulation Models

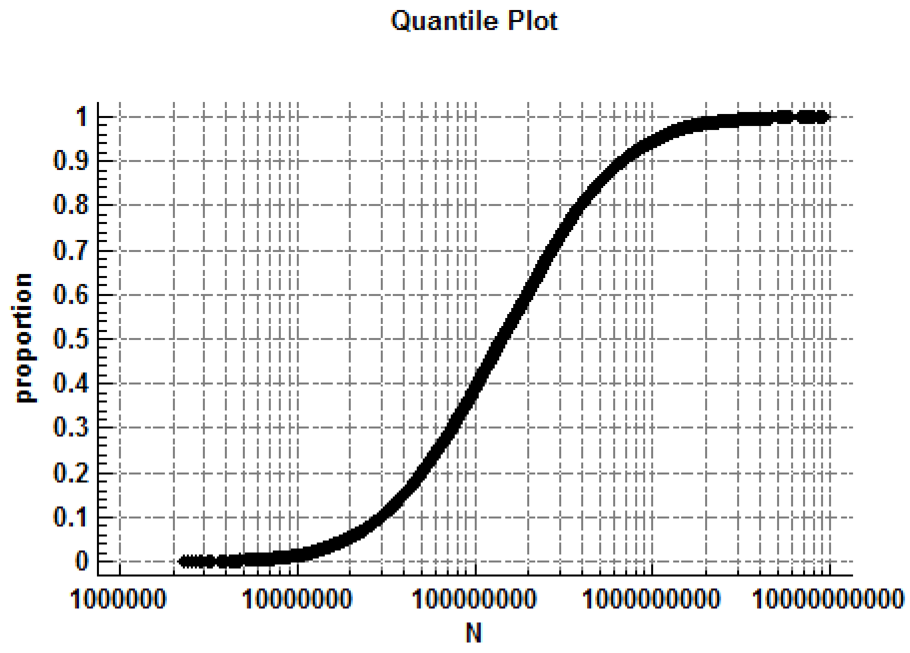
- Step 5: calculate percentiles

Percentiles

Percentage	*Area*	*Net pay*	*Porosity*	*Water saturation*	*FVF*	*N*
1.0%	63.8958	30.9285	17.3382	16.6558	1.00185	8.72609E6
5.0%	117.535	50.8366	21.0517	19.7407	1.00836	1.90795E7
10.0%	162.918	67.8868	22.8418	21.4664	1.01613	2.95456E7
25.0%	284.913	110.926	26.1484	24.6363	1.03933	6.15614E7
50.0%	520.151	187.103	29.7858	28.6549	1.08525	1.41263E8
75.0%	963.019	318.933	33.3238	33.0429	1.15893	3.22183E8
90.0%	1733.26	512.351	36.5647	37.4472	1.24328	6.72699E8
95.0%	2352.44	676.942	38.6333	40.3498	1.30265	1.07355E9
99.0%	4462.55	1157.71	42.4143	46.0589	1.41934	2.5423E9

General Simulation Models

- Step 6: create a quantile plot for the response



Example #2: Consumer Price Index

All urban consumers, all items, 1982-2016

C:\Data\ConsumerPriceIndex.sgd

	Year	Month	CPI	Date	CPI Adjusted
1	1982	Jan	94.3	1/1982	94.4
2	1982	Feb	94.6	2/1982	94.7
3	1982	Mar	94.5	3/1982	94.7
4	1982	Apr	94.9	4/1982	95.0
5	1982	May	95.8	5/1982	95.9
6	1982	Jun	97.0	6/1982	97.0
7	1982	Jul	97.5	7/1982	97.5
8	1982	Aug	97.7	8/1982	97.7
9	1982	Sep	97.9	9/1982	97.7
10	1982	Oct	98.2	10/1982	98.1
11	1982	Nov	98.0	11/1982	98.0
12	1982	Dec	97.6	12/1982	97.7
13	1983	Jan	97.8	1/1983	97.9
14	1983	Feb	97.9	2/1983	98.0
15	1983	Mar	97.9	3/1983	98.1
16	1983	Apr	98.6	4/1983	98.8
17	1983	May	99.2	5/1983	99.2
18	1983	Jun	99.5	6/1983	99.4
19	1983	Jul	99.9	7/1983	99.8
20	1983	Aug	100.2	8/1983	100.1

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Source: https://www.bls.gov/cpi/data.htm

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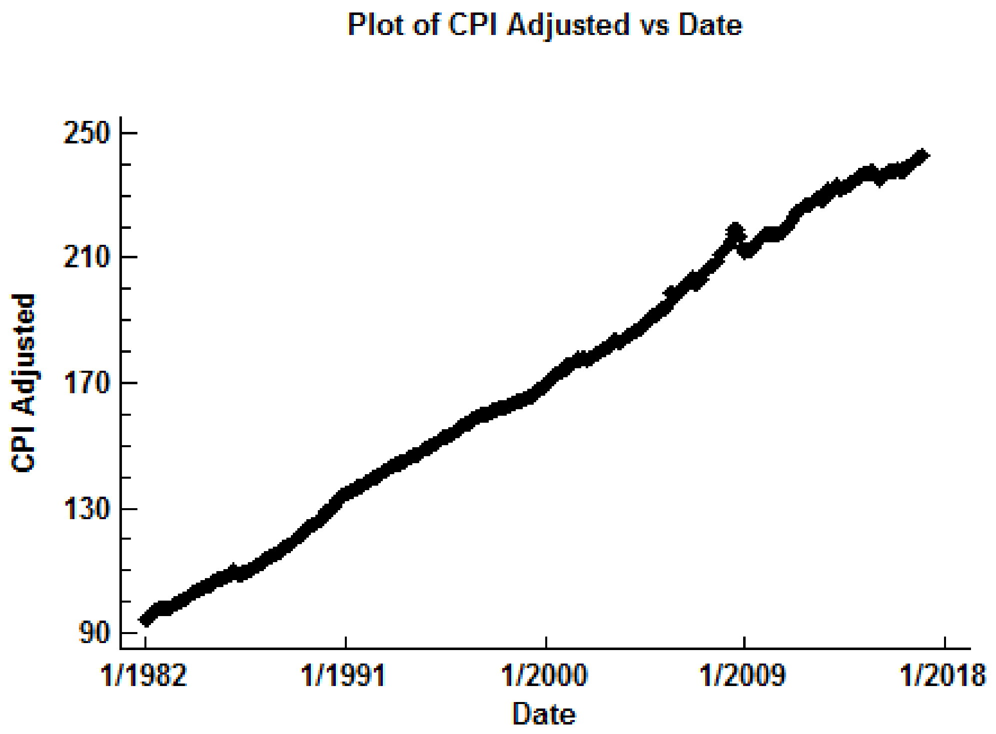
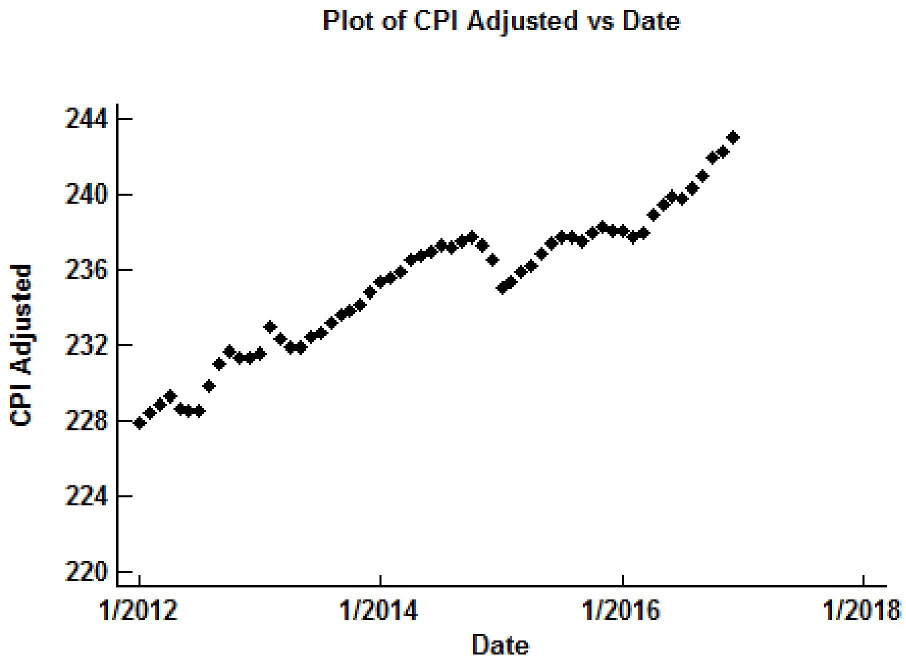
CPI during last 35 years



Average yearly change: 4.3

CPI during last 5 years



Average yearly change: 3.2

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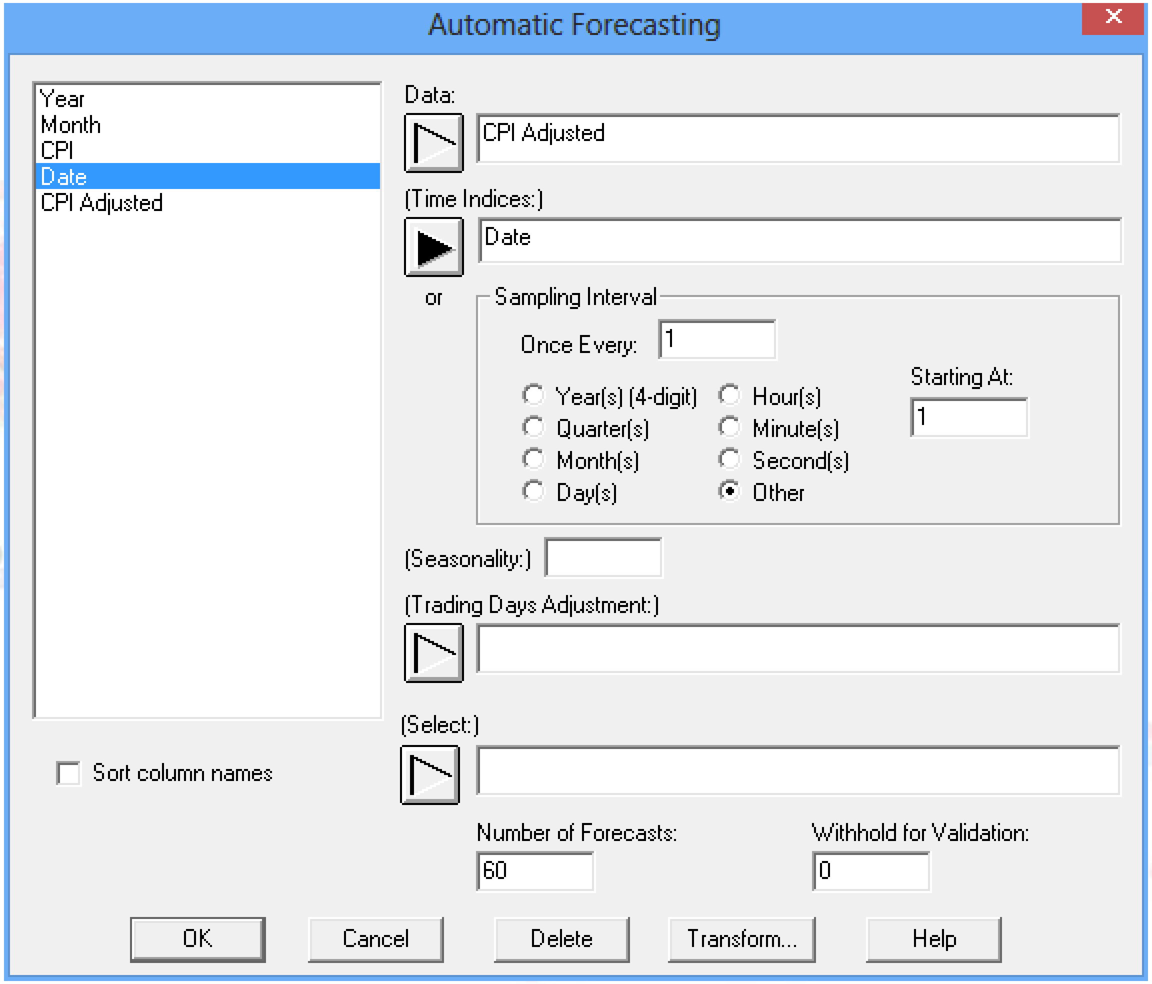
Simulating ARIMA Time Series Models

- Steps
  - Fit an ARIMA model to the CPI data
  - Generate several possible realizations
  - Plot the realizations on a single graph
- Useful as input to financial models

Automatic Forecasting



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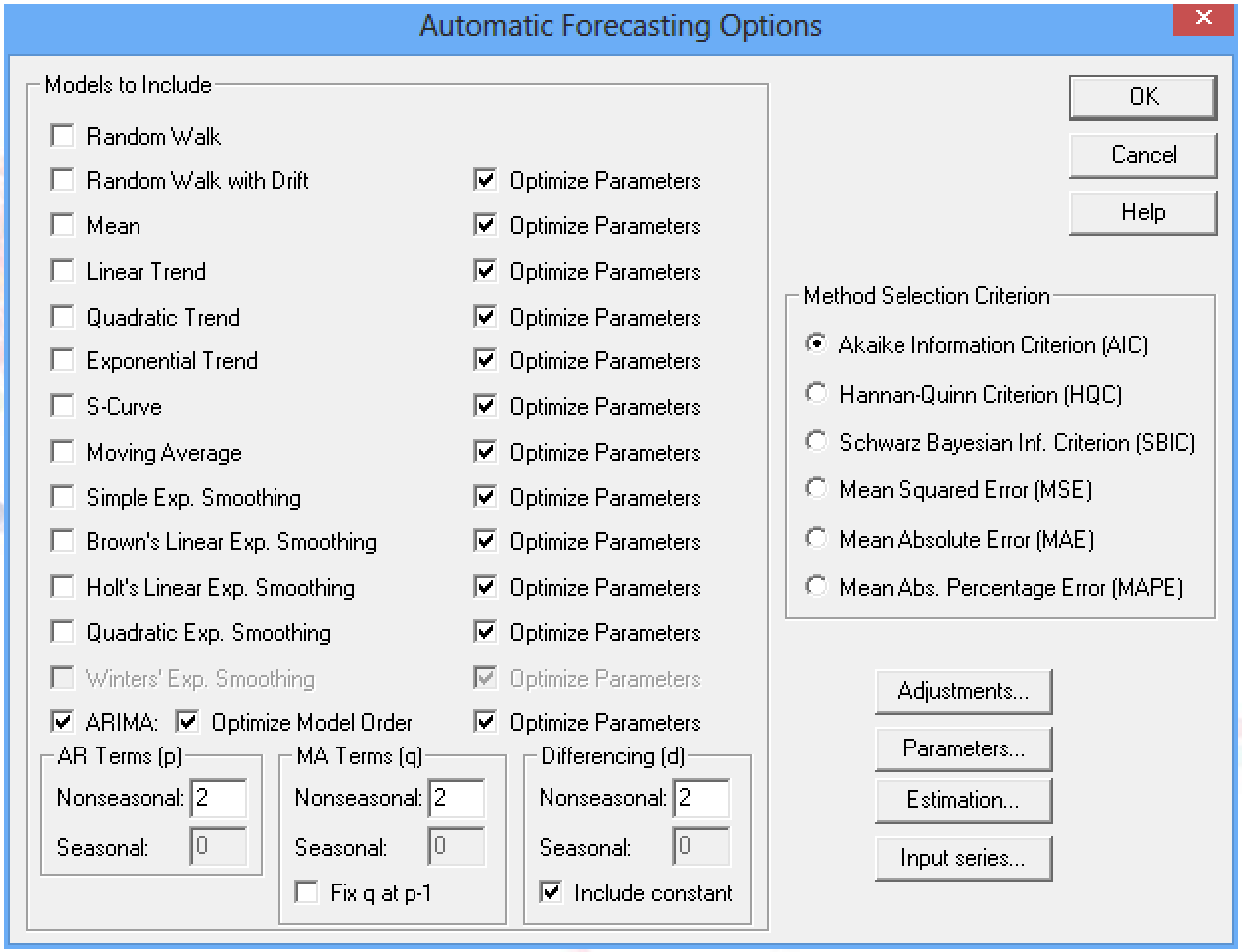
Analysis Options



Estimated Model

Forecast Summary
Forecast model selected: ARIMA(1,1,2) with constant
Number of forecasts generated: 60
Number of periods withheld for validation: 0

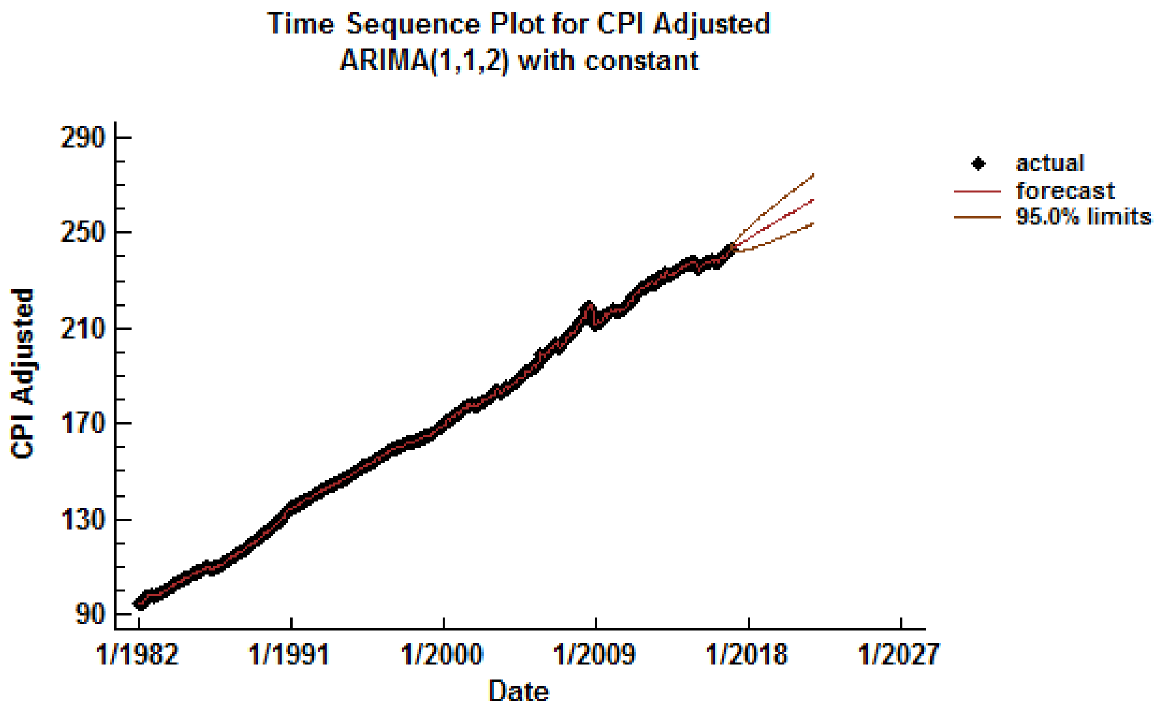
	Estimation	*Validation*
Statistic	*Period*	*Period*
RMSE	0.42377	
MAE	0.268858	
MAPE	0.155123	
ME	-0.0000700932	
MPE	-0.000434589	

ARIMA Model Summary

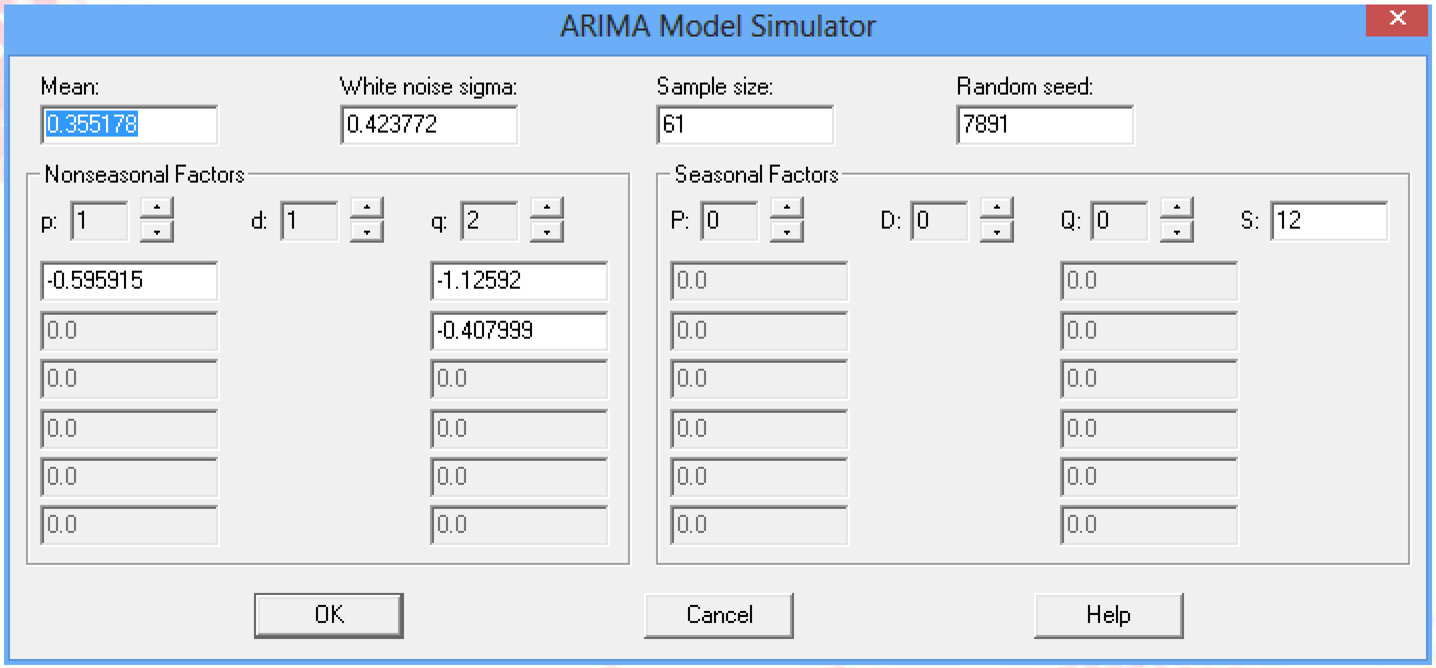
Parameter	*Estimate*	*Stnd. Error*	*t*	*P-value*
AR(1)	-0.595915	0.219866	-2.71036	0.007000
MA(1)	-1.12592	0.209141	-5.38357	0.000000
MA(2)	-0.407999	0.0889357	-4.58757	0.000006
Mean	0.355178	0.0328146	10.8238	0.000000
Constant	0.566833			

Backforecasting: yes
Estimated white noise variance = 0.179583 with 415 degrees of freedom
Estimated white noise standard deviation = 0.423772
Number of iterations: 8

Forecast Plot



ARIMA Model Simulation



Simulated Time Series

ARIMA Model Simulation

ARIMA Model Simulation

Sample size: 61
Seed for random number generator: 7872

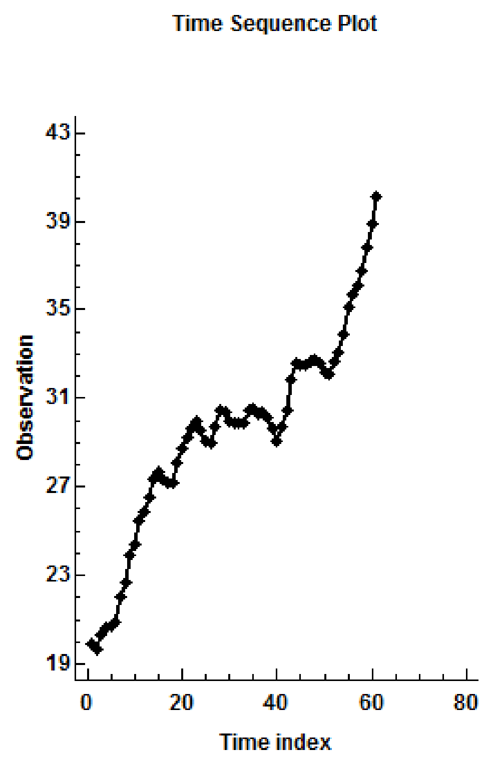
Mean: 0.355178
Sigma: 0.423772

Nonseasonal Factors

	Order	*Parameters*
AutoRegressive	p=1	-0.595915
Differencing	d=1	
Moving Average	q=2	-1.12592,-0.407999

The StatAdvisor

This procedure generates random samples from ARIMA time serie
datasheet.



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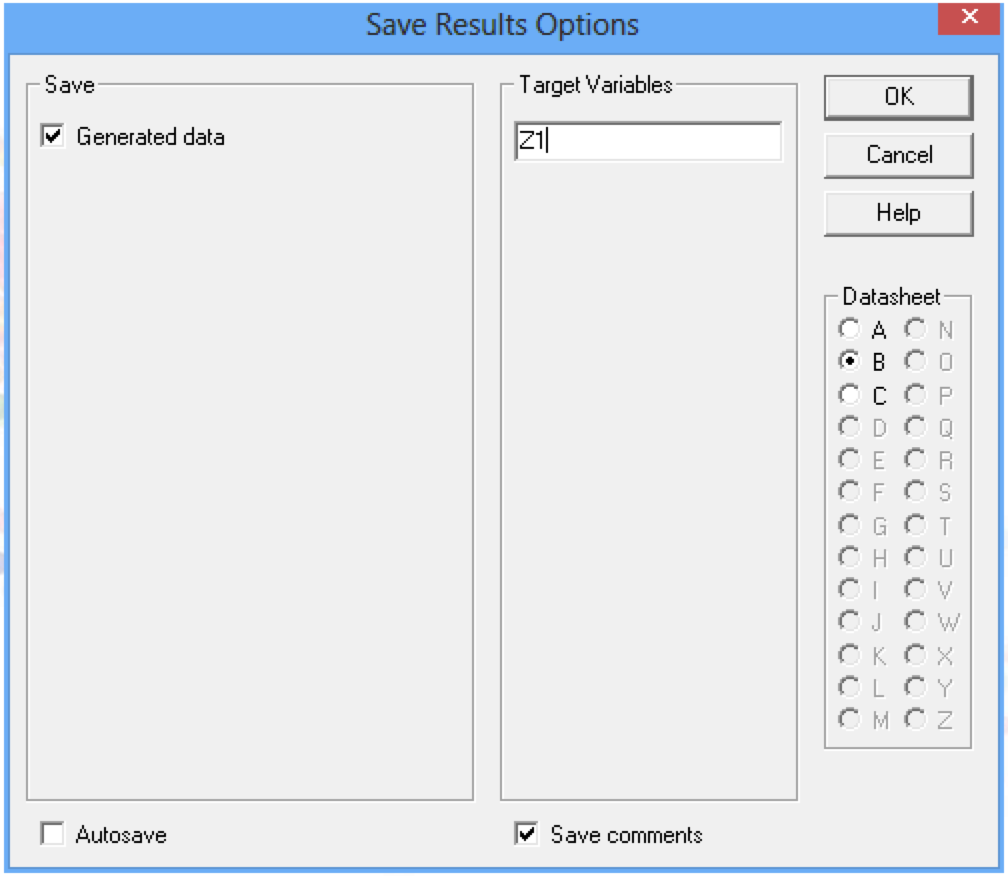
Save Data



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Multiple Realizations

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	Z1	Z2	Z3	Z4	Z5
	Generated data	Generated data	Generated data	Generated data	Generated data
1	19.4499	23.7789	9.86124	22.7656	25.7688
2	19.6846	23.7468	9.89014	22.9462	26.0434
3	20.4016	24.9258	10.174	22.7485	26.573
4	20.9872	25.4891	10.2415	22.7572	27.1589
5	20.7981	25.9981	10.3698	22.8612	27.5835
6	21.4019	25.495	10.3129	23.0409	27.9823
7	21.7082	26.138	10.3202	22.8159	27.578
8	21.809	26.666	10.4096	22.724	27.5002
9	21.9993	26.5306	10.1109	23.6937	27.9449
10	23.1193	26.264	9.43815	24.2098	28.2149
11	23.8161	26.2418	9.85697	24.1366	29.0662
12	24.9937	25.7055	10.7021	24.2449	29.8427
13	25.5259	25.939	10.8019	24.959	30.2293
14	26.5991	26.0924	11.5299	24.7239	29.8991
15	26.9825	26.1303	12.0654	24.841	30.1412
16	27.6518	26.8526	11.7374	24.9927	30.2666
17	28.4898	27.6915	11.6341	25.2819	31.0328
18	28.7695	28.2499	12.6396	25.9487	31.6175
19	28.4309	28.7619	13.3927	25.6802	32.0427
20	28.2571	28.5678	14.3091	25.8167	32.633

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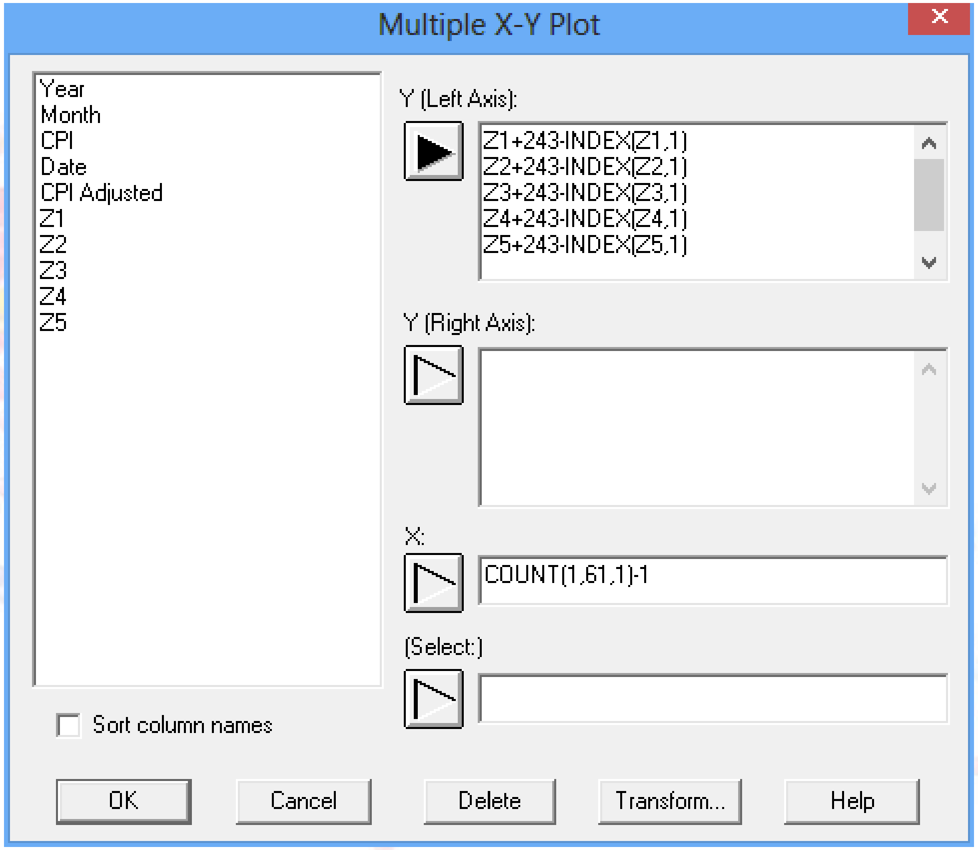
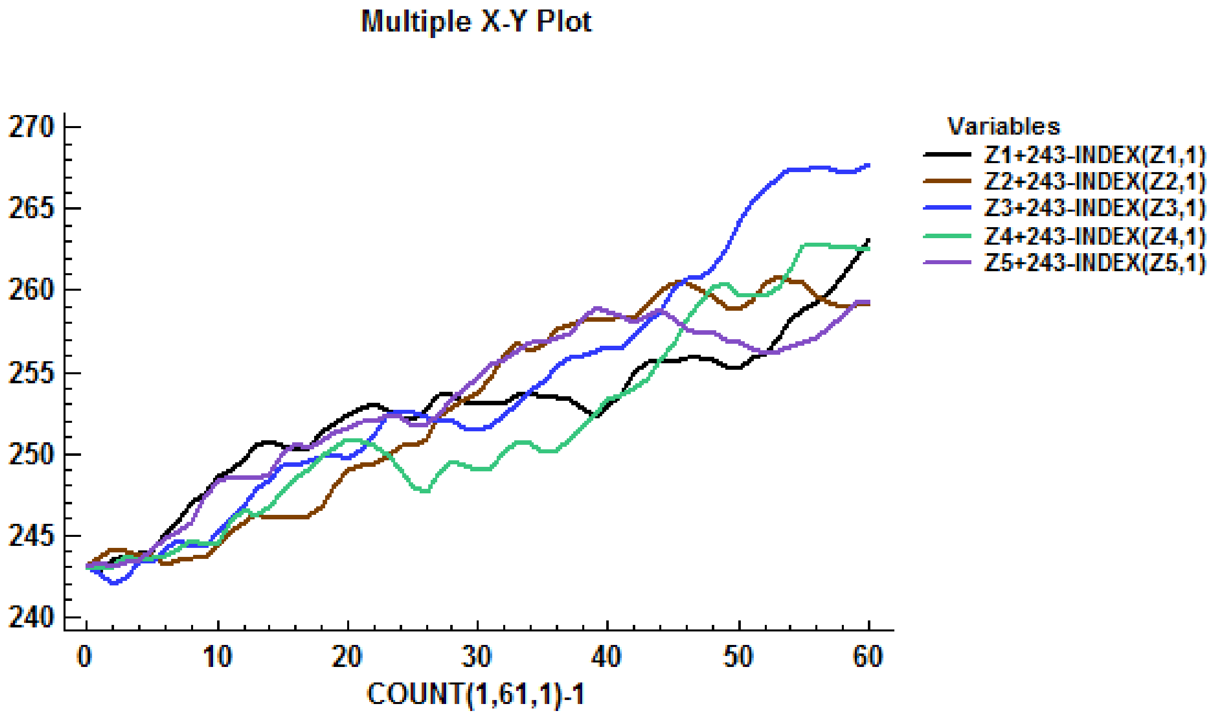
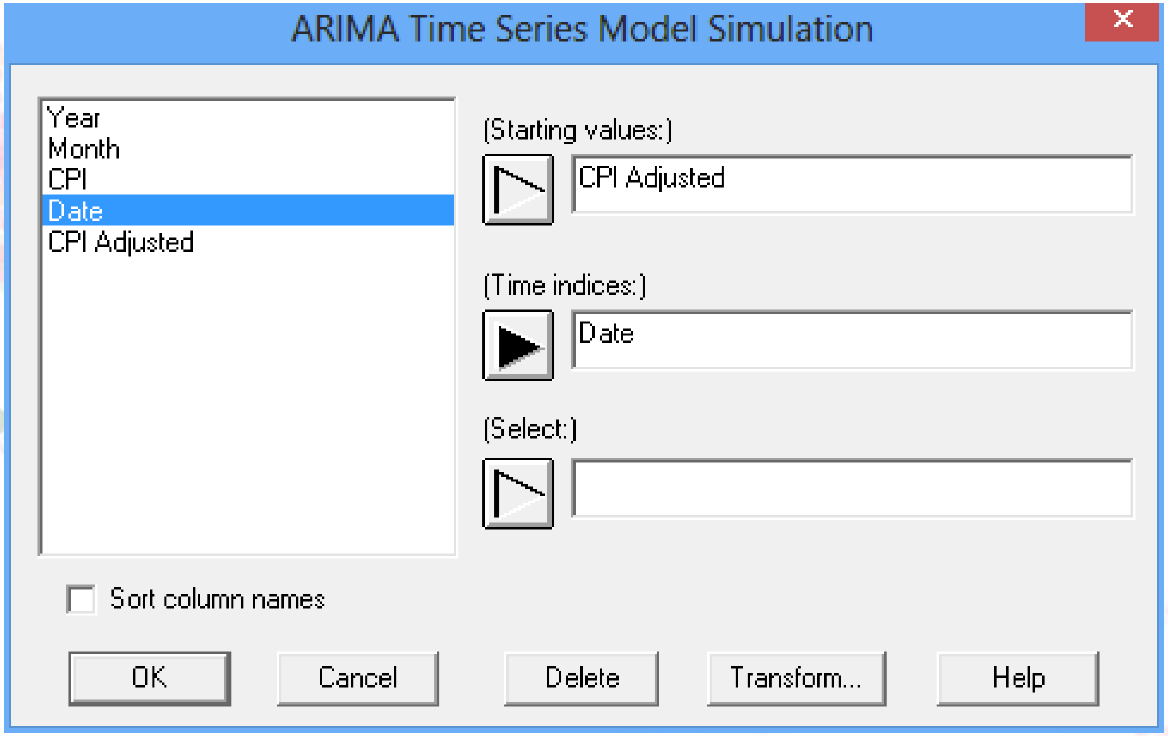
Plot Multiple Realizations



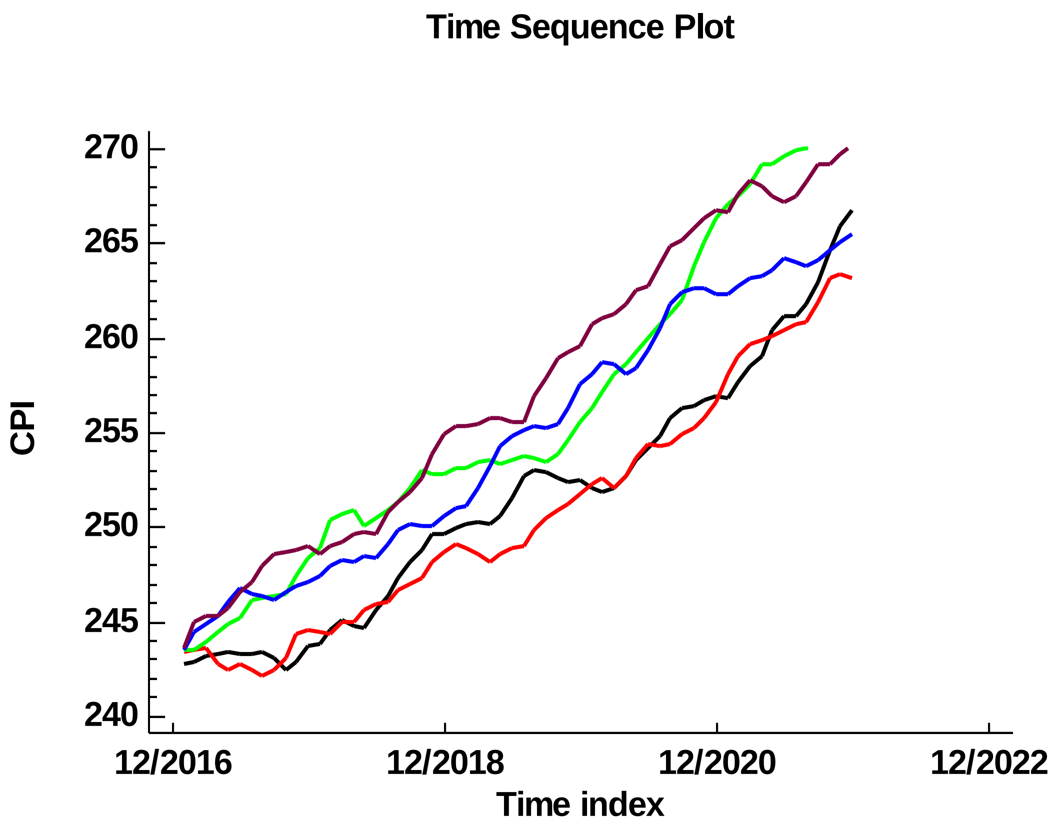
Plot Multiple Realizations



Upcoming Enhancement



Using Historical Data as Starting Values



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Resources

- StatFolios and data files are at: www.statgraphics.com/webinars

- Recorded webinar will be posted soon.