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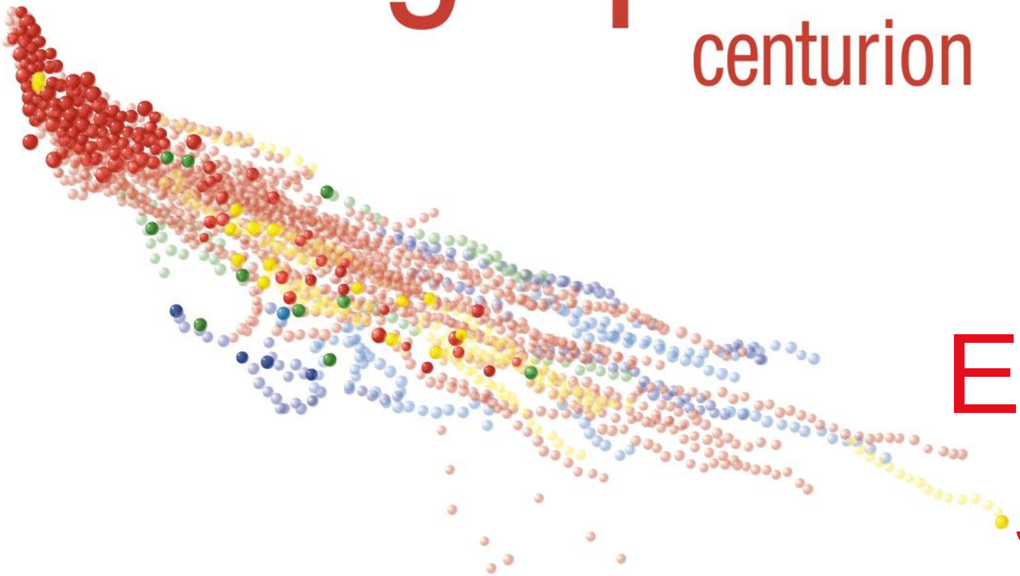


Forecasting Economic Time Series Using Statgraphics Centurion

Presented by Dr. Neil W. Polhemus

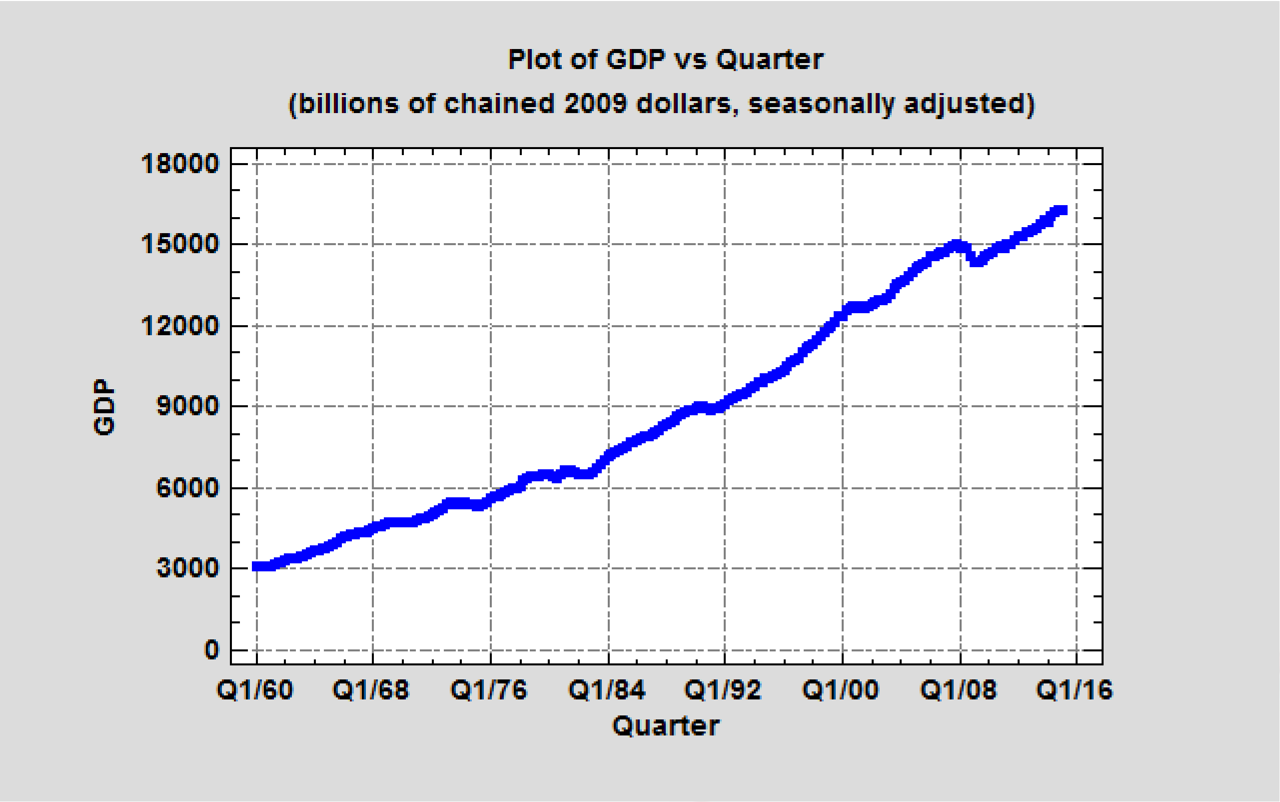
Time Series

- "A sequence of numerical data points in successive order, using occurring in uniform intervals." – www.investopida.com

- Examples
  - Daily closing stock prices
  - Monthly unemployment rates
  - Quarterly GDP

- Notation: $\{Y_t\}$, $t = 1, 2, \ldots, n$

Example – U.S. Quarterly GDP



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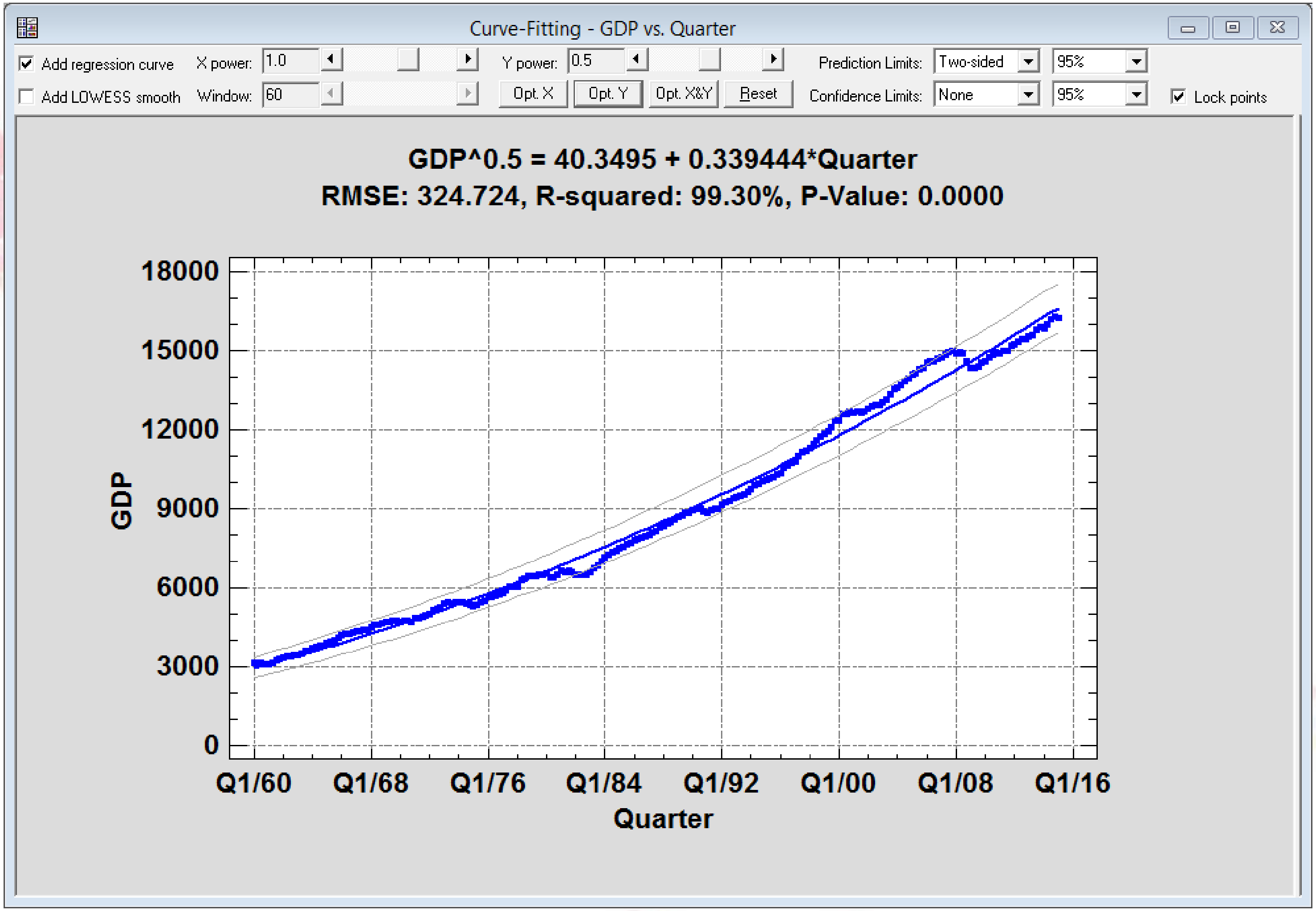
Time Series Components

- Trend
- Cycle
- Seasonality
- Random or irregular component

Trend Analysis



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Differencing Operators

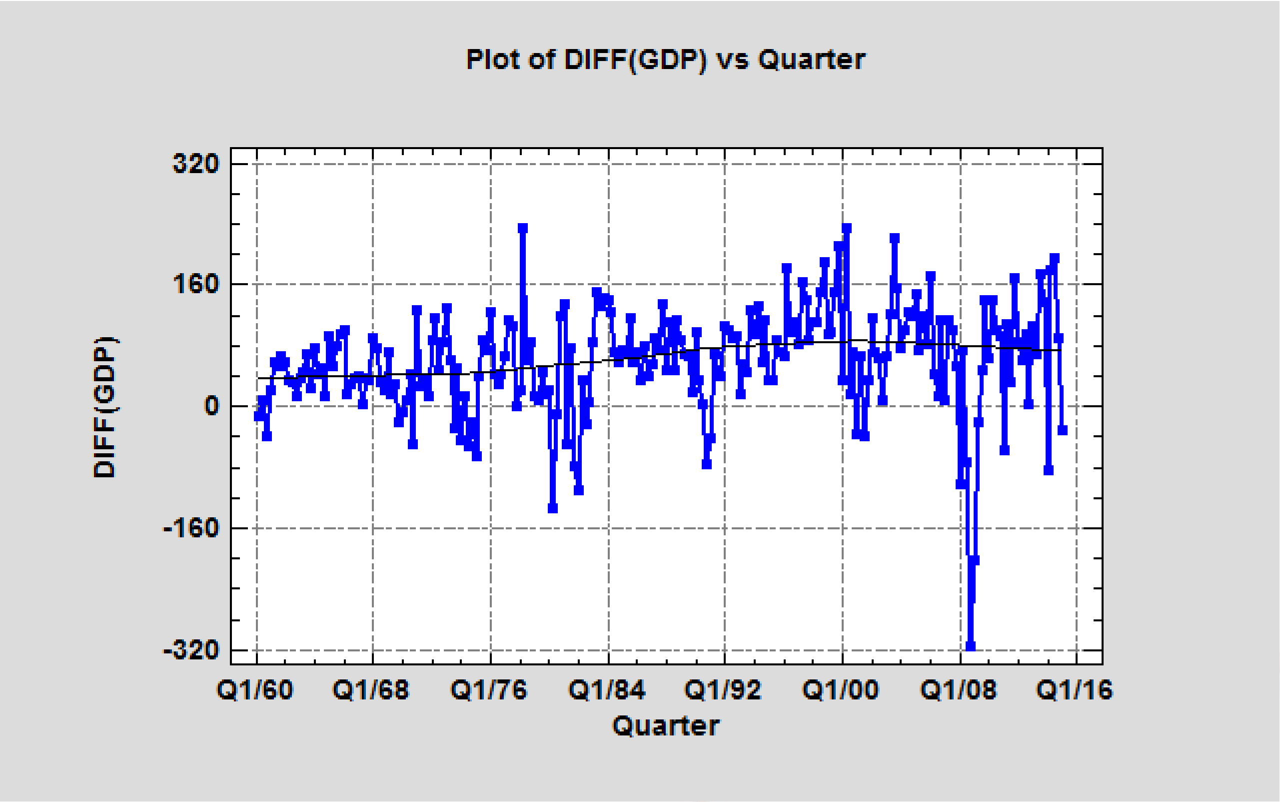
- First Differences

$$\nabla Y_t = Y_t - Y_{t-1}$$

- Second Differences

$$\nabla^2 Y_t = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2})$$

First Differences: $\nabla Y_t = Y_t - Y_{t-1}$



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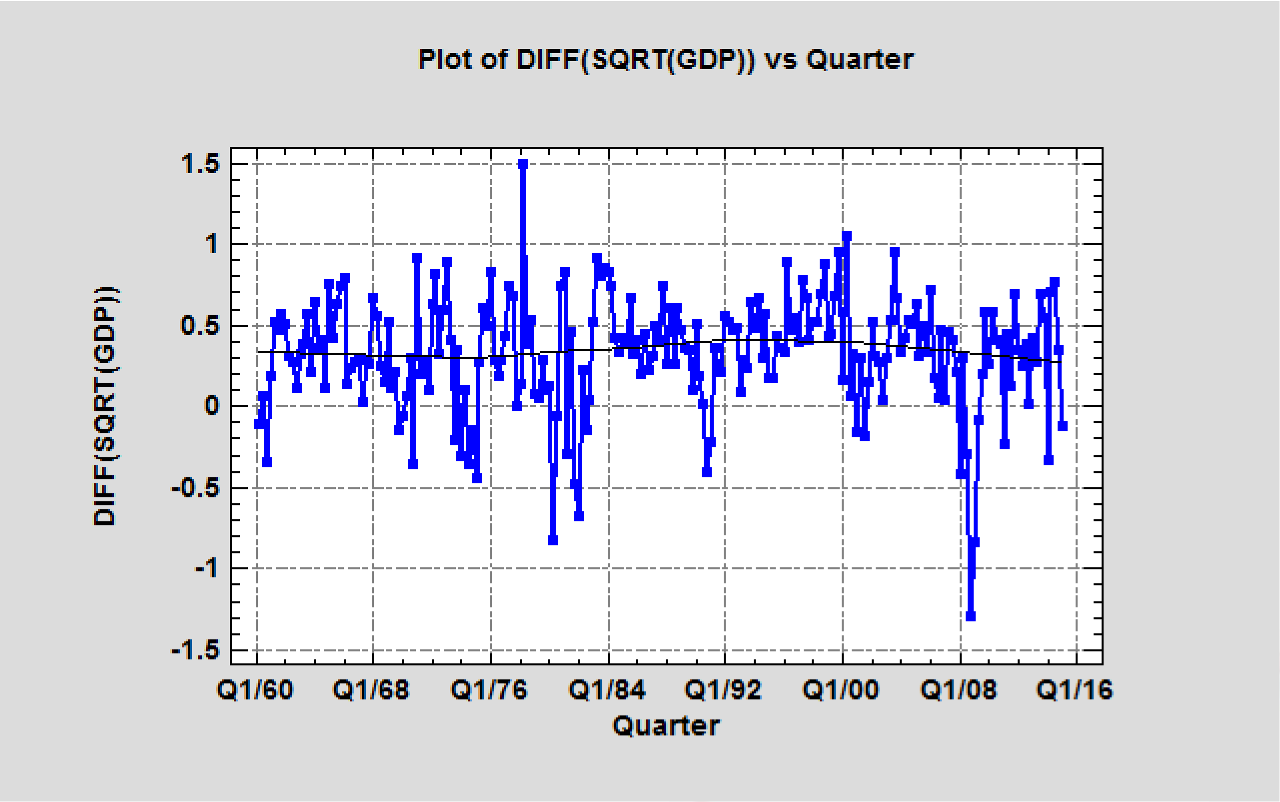
First Differences after Square Root

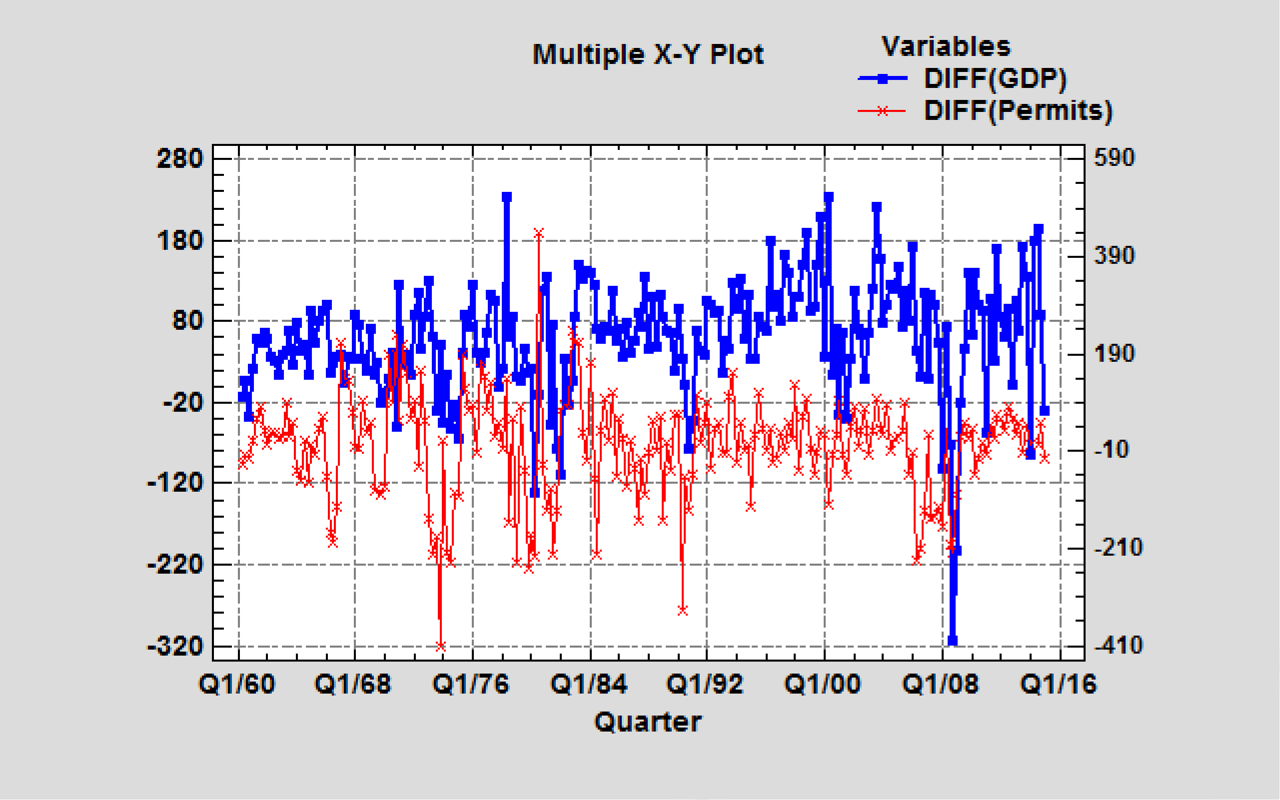


Types of Forecasting Models

- **Autoprojective models** – models that involve only the time series to be forecast. These models capture the dynamics of past time series movements and project them into the future.

- **Models with leading indicators** – models that include past values of other time series variables.

GDP and New Construction Permits



Notation

- Time series to be forecast:

  $\{Y_t\}$, $t = 1, 2, 3, \ldots, n$

- Forecasts:

  $F_t(k)$ = forecast of Y_{t+k} using information available at time t

- One-ahead forecast errors:

  $$\hat{\varepsilon}_t = Y_t - F_{t-1}(1)$$

Types of Autoprojective Models

1. **Random walk** - current value has all relevant information.

without constant: $F_t(k) = Y_t$ for all $k \geq 1$

with constant: $F_t(k) = Y_t + k\hat{\Delta}$

where $\hat{\Delta}$ is mean difference between consecutive periods

Types of Autoprojective Models

2. **Trend models** – time series follows a deterministic trend with random fluctuations around the trend.

$$F_t(k) = \hat{a} + \hat{b}(t + k)$$

$$F_t(k) = \exp\left(\hat{a} + \hat{b}(t + k)\right)$$

$$F_t(k) = \exp\left(\hat{a} + \hat{b}/(t + k)\right)$$

Types of Autoprojective Models

3. **Moving averages** – averages recent history to forecast future behavior.

$$F_t(k) = \frac{\sum_{i=0}^{c-1} Y_{t-i}}{c}$$

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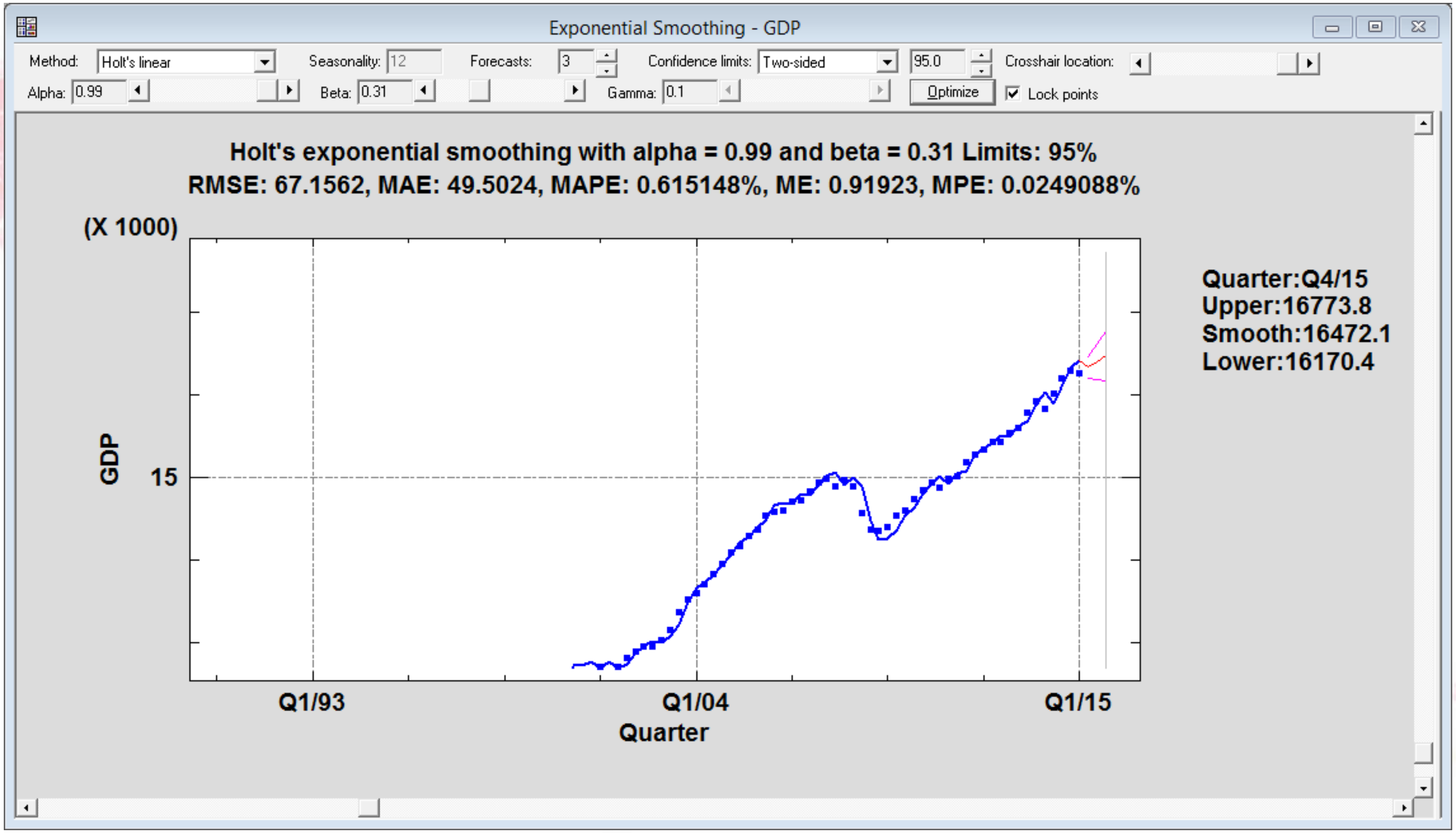
Types of Autoprojective Models

4. **Exponential smoothing** – combines new information with previous forecasts to generate new forecasts.

$$F_t(k) = \alpha Y_t + (1 - \alpha)F_{t-1}(1)$$

Statgraphics has simple, linear, quadratic and seasonal smoothers.

Holt’s Linear Exp. Smoothing



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Types of Autoprojective Models

5. **ARIMA Models** – parametric models which describe system dynamics.

ARIMA(p,d,q) model has:

autoregressive term of order p

moving average term of order q

applied to the differences of order d

Autoregressive Models

- AR(1)

$$Y_t = \mu + \phi_1(Y_{t-1} - \mu) + \varepsilon_t$$

- AR(2)

$$Y_t = \mu + \phi_1(Y_{t-1} - \mu) + \phi_2(Y_{t-2} - \mu) + \varepsilon_t$$

Moving Average Models

- MA(1)

$$Y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

- MA(2)

$$Y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

ARMA Models

- ARMA(1,1)

$$Y_t = \mu + \phi_1 (Y_{t-1} - \mu) + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

ARIMA Models

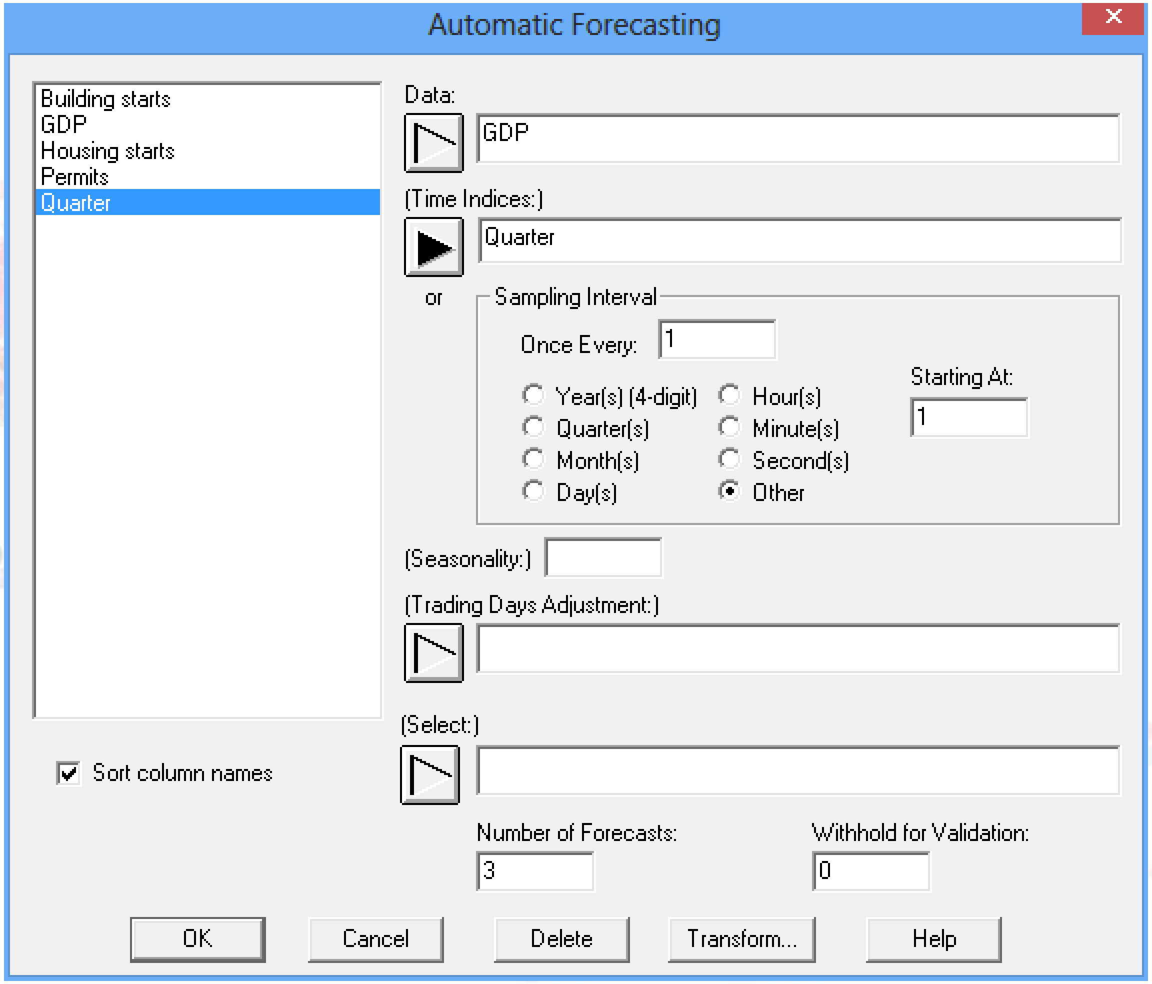
- ARIMA(1,1,1)

$$\nabla Y_t = Y_t - Y_{t-1}$$

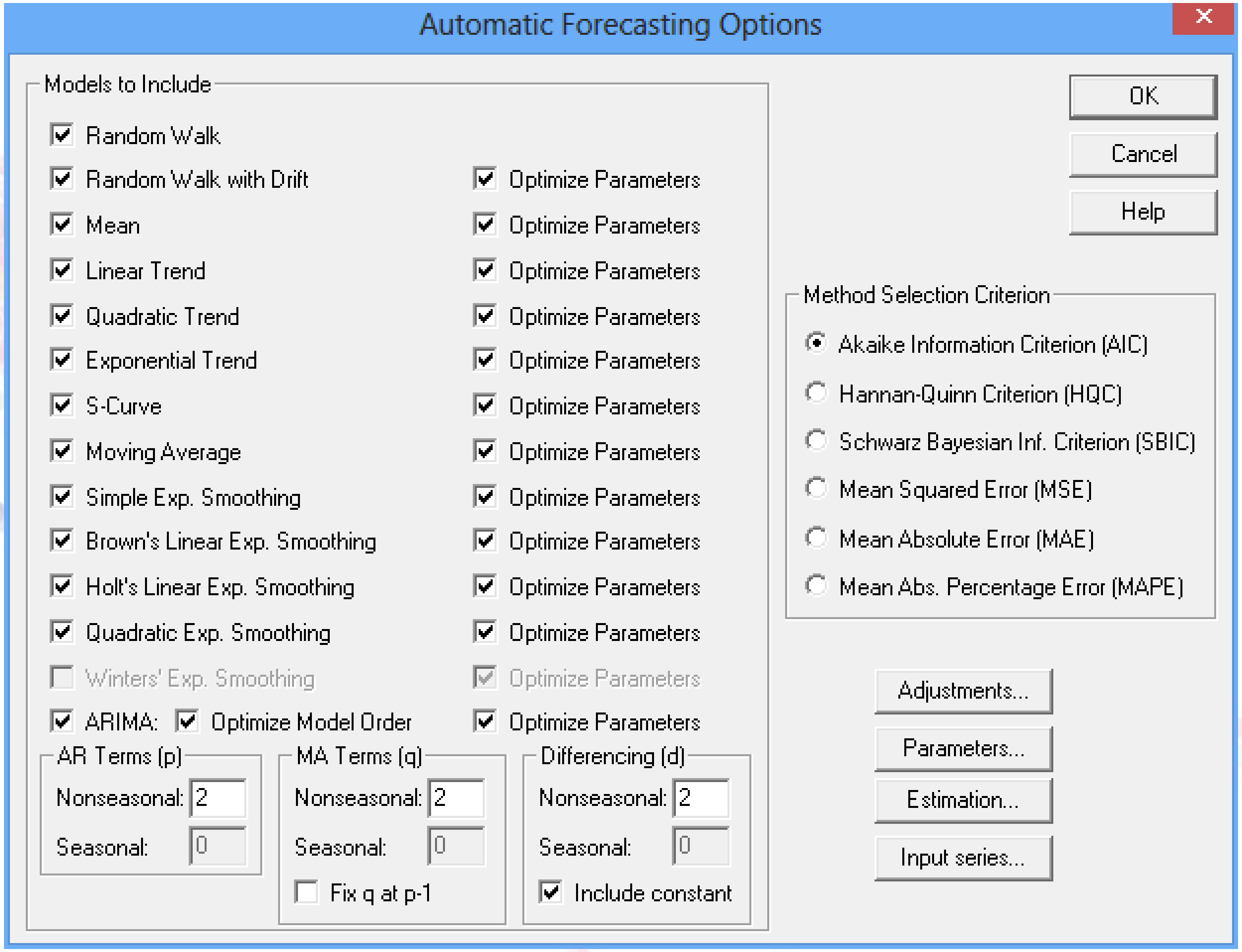
$$\nabla Y_t = \mu + \phi_1 (\nabla Y_{t-1} - \mu) + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

Note: μ is sometimes omitted.

Automatic Forecasting



Analysis Options



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Method Selection Criterion

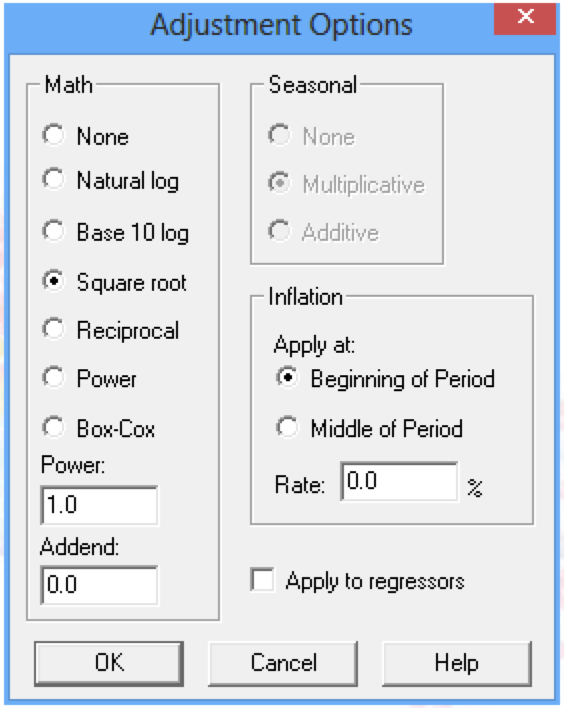
Akaike Information Criterion

$$AIC = 2\ln\left(RMSE\right) + \frac{2c}{n}$$

c = number of coefficients in fitted model

RMSE = root mean squared error calculated from the one-period ahead forecast errors

Adjustments



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Adjustments

1. **Trading days adjustment** – used to normalize monthly data by dividing each data value by number of trading days in the month.
2. **Math adjustment** – transforms each data value before fitting models.
3. **Seasonal adjustment** – removes seasonal effects using seasonal decomposition prior to fitting models.
4. **Inflation adjustment** – corrects each data value for a constant rate of inflation.

Analysis Summary

Automatic Forecasting - GDP

Data variable: GDP (billions of chained 2009 dollars, seasonally adjusted)

Number of observations = 221
Time indices: Quarter (from BEA)

Forecast Summary
Math adjustment: Square root
Forecast model selected: ARIMA(2,1,0) with constant
Number of forecasts generated: 3
Number of periods withheld for validation: 0

Statistic	*Estimation Period*	*Validation Period*
RMSE	64.215	
MAE	46.0611	
MAPE	0.572054	
ME	-0.00425189	
MPE	-0.00142829	

ARIMA Model Summary

Parameter	*Estimate*	*Stnd. Error*	*t*	*P-value*
AR(1)	0.28558	0.0672908	4.24396	0.000033
AR(2)	0.145362	0.0672999	2.15991	0.031876
Mean	0.32359	0.0391041	8.27509	0.000000
Constant	0.184141			

Backforecasting: yes
Estimated white noise variance = 0.111779 with 217 degrees of freedom
Estimated white noise standard deviation = 0.334334
Number of iterations: 1

Model Comparisons

Model Comparison
Data variable: GDP
Number of observations = 221

Models
(A) Random walk
(B) Random walk with drift = 0.32566
(C) Constant mean = 91.6055
(D) Linear trend = 53.9272 + 0.339444 t
(E) Quadratic trend = 55.3126 + 0.302169 t + 0.000167904 t^2
(F) Exponential trend = exp(4.06538 + 0.00380989 t)
(G) S-curve trend = exp(4.52201 + -1.24712 /t)
(H) Simple moving average of 2 terms
(I) Simple exponential smoothing with alpha = 0.9999
(J) Brown's linear exp. smoothing with alpha = 0.6898
(K) Holt's linear exp. smoothing with alpha = 0.9999 and beta = 0.0046
(L) Brown's quadratic exp. smoothing with alpha = 0.4852
(M) ARIMA(2,1,0) with constant
(N) ARIMA(1,1,1) with constant
(O) ARIMA(1,1,0) with constant
(P) ARIMA(0,1,2) with constant
(Q) ARIMA(2,1,1) with constant

Model Statistics

Estimation Period

Model	*RMSE*	*MAE*	*MAPE*	*ME*	*MPE*	*AIC*	*HQC*	*SBIC*
(A)	91.8364	76.2823	0.93793	59.7314	0.743744	9.04002	9.04002	9.04002
(B)	68.7816	48.0714	0.601831	0.0671557	-0.00743105	8.47092	8.47713	8.4863
(C)	4076.34	3501.1	48.6727	472.255	-19.5762	16.635	16.6412	16.6503
(D)	367.469	296.302	3.37438	3.30028	-0.0266323	11.8314	11.8438	11.8621
(E)	369.041	281.256	2.93507	2.92671	-0.0809698	11.849	11.8676	11.8951
(F)	589.219	391.496	4.05372	-20.4197	-0.127704	12.7757	12.7881	12.8064
(G)	3825.45	3207.88	40.476	846.503	-9.99178	16.517	16.5294	16.5477
(H)	126.82	106.906	1.30998	90.1888	1.11967	9.69458	9.70079	9.70996
(I)	91.8422	75.943	0.933758	59.4671	0.740453	9.04919	9.0554	9.06457
(J)	68.679	50.2451	0.624134	-0.290729	0.0072373	8.46794	8.47415	8.48331
(K)	71.1219	51.7957	0.651243	17.2005	0.246029	8.54689	8.55931	8.57764
(L)	76.6172	56.2215	0.702866	-1.16001	-0.0153938	8.68669	8.6929	8.70207
(M)	64.215	46.0611	0.572054	-0.00425189	-0.00142829	8.35162	8.37025	8.39775
(N)	64.2656	46.193	0.574924	0.125512	0.000328955	8.3532	8.37182	8.39933
(O)	64.6991	46.524	0.577422	-0.0032121	-0.00330691	8.3576	8.37001	8.38835
(P)	64.587	46.1862	0.572778	0.0258493	-0.00296255	8.36318	8.3818	8.40931
(Q)	64.3437	46.0917	0.571796	0.0936972	-0.000419893	8.36468	8.38951	8.42618

Model Residual Analysis

Model	*RMSE*	*RUNS*	*RUNM*	*AUTO*	*MEAN*	*VAR*
(A)	91.8364	OK	**	***	OK	OK
(B)	68.7816	OK	**	***	OK	OK
(C)	4076.34	***	***	***	***	OK
(D)	367.469	***	***	***	OK	OK
(E)	369.041	***	***	***	*	***
(F)	589.219	***	***	***	OK	***
(G)	3825.45	***	***	***	***	**
(H)	126.82	***	***	***	OK	OK
(I)	91.8422	OK	**	***	OK	OK
(J)	68.679	OK	OK	*	OK	OK
(K)	71.1219	OK	***	***	OK	OK
(L)	76.6172	OK	OK	***	OK	OK
(M)	64.215	OK	OK	OK	OK	OK
(N)	64.2656	OK	OK	OK	OK	OK
(O)	64.6991	*	OK	OK	OK	OK
(P)	64.587	*	OK	OK	OK	OK
(Q)	64.3437	OK	OK	OK	OK	OK

Key:
RMSE = Root Mean Squared Error
RUNS = Test for excessive runs up and down
RUNM = Test for excessive runs above and below median
AUTO = Box-Pierce test for excessive autocorrelation
MEAN = Test for difference in mean 1st half to 2nd half
VAR = Test for difference in variance 1st half to 2nd half
OK = not significant (p >= 0.05)
* = marginally significant (0.01 < p <= 0.05)
** = significant (0.001 < p <= 0.01)
*** = highly significant (p <= 0.001)

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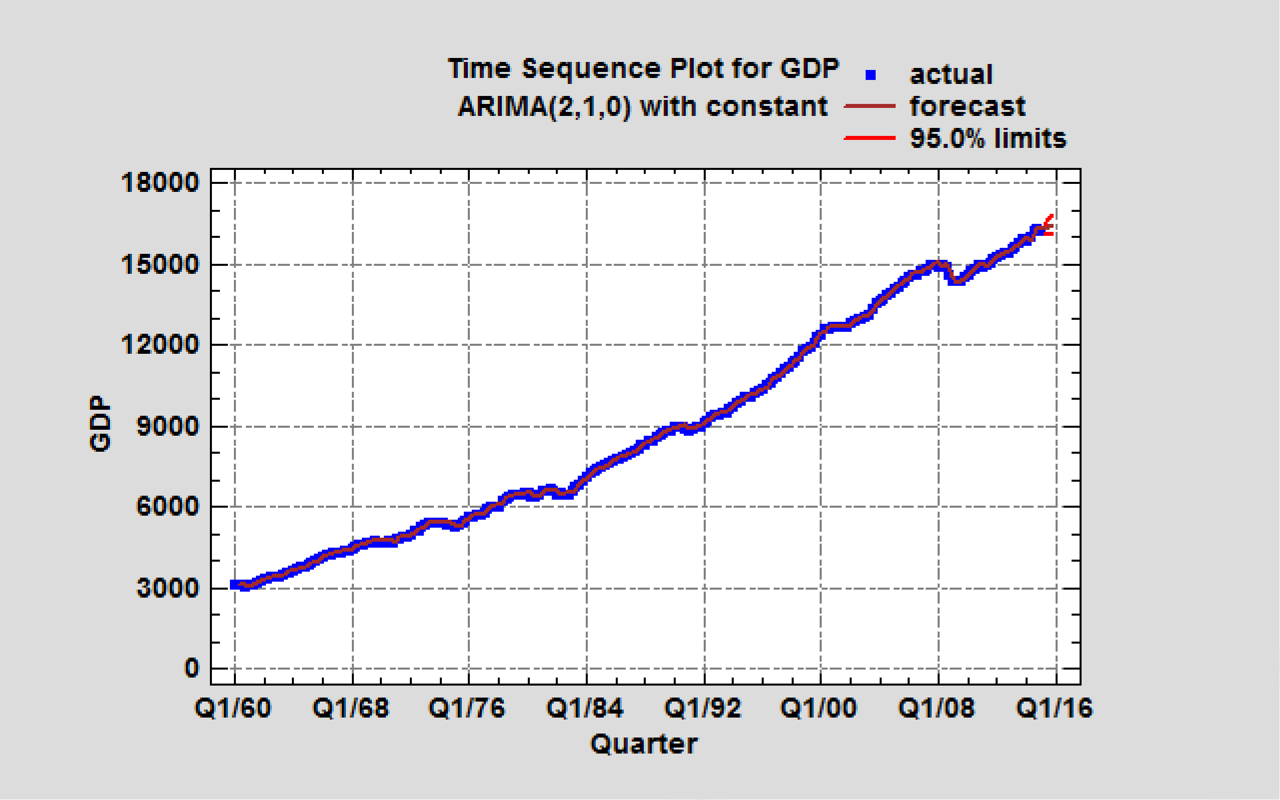
Time Sequence Plot



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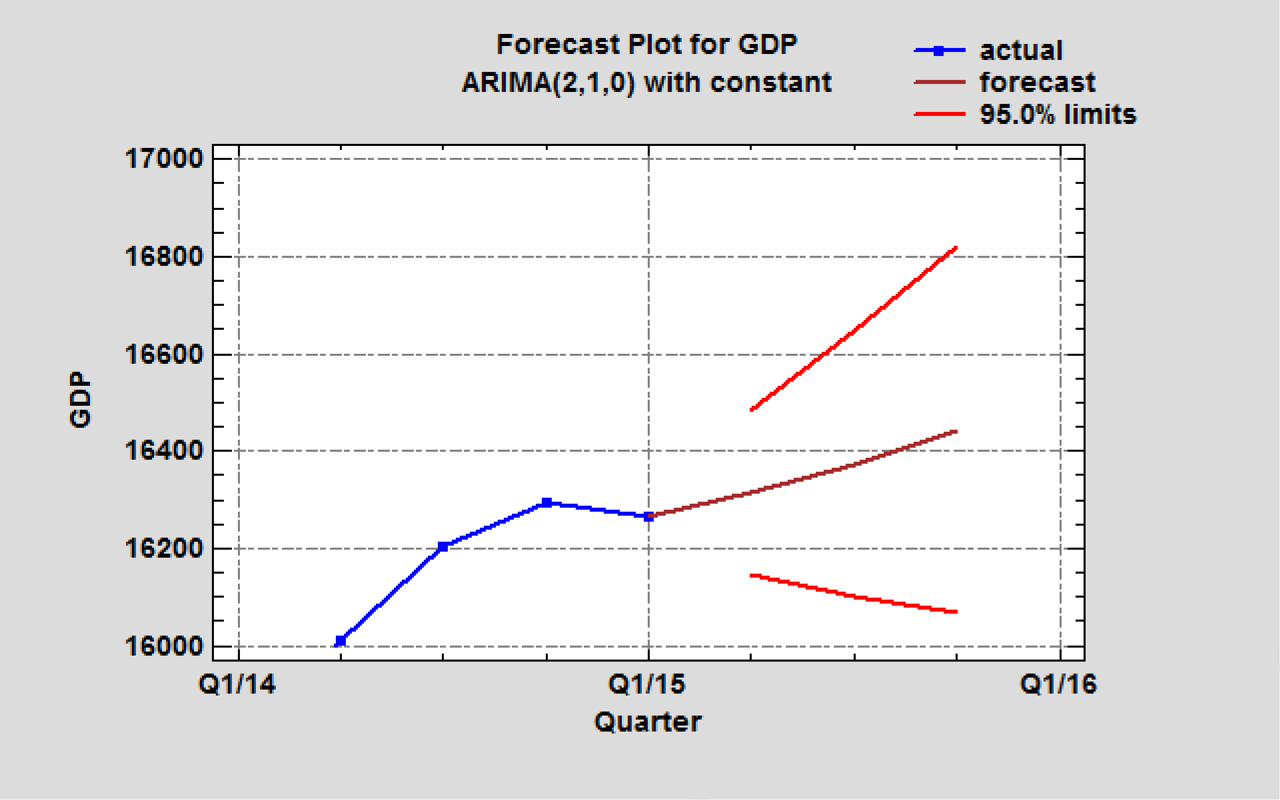
Forecast Plot



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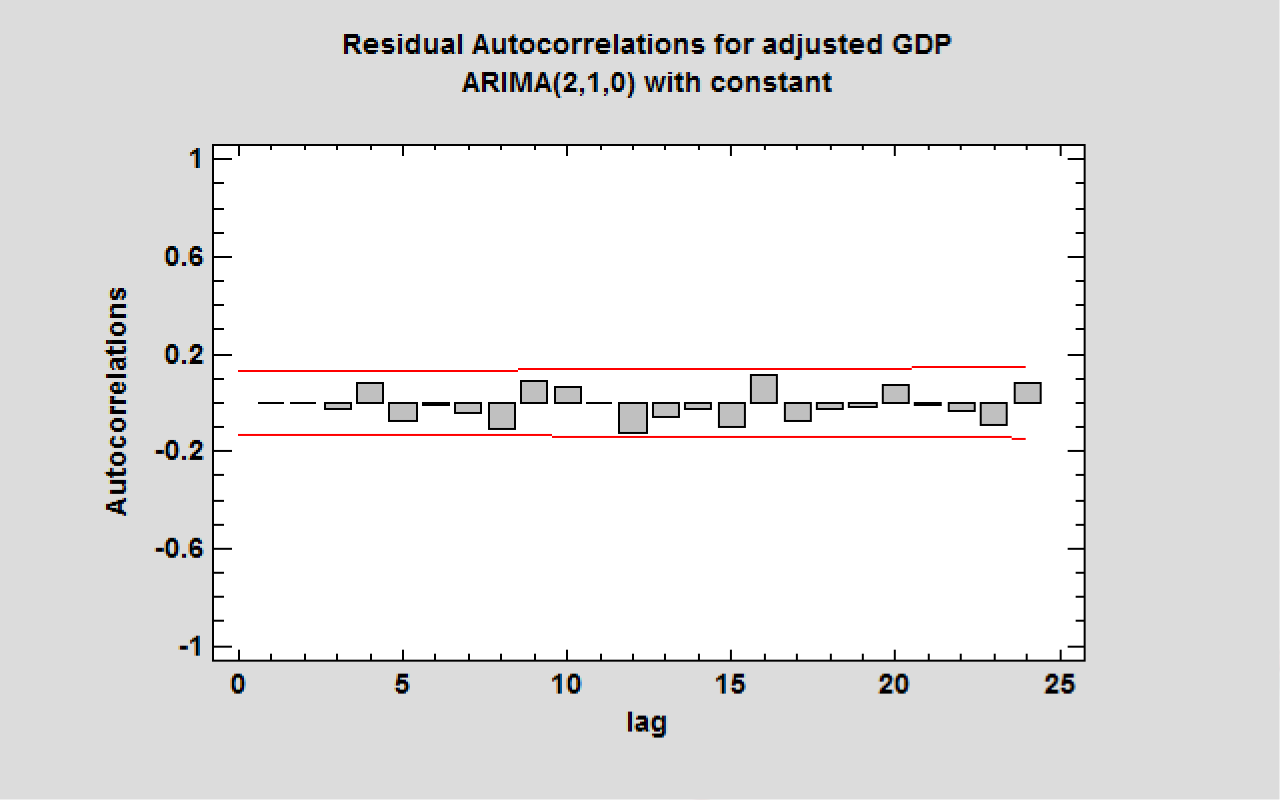
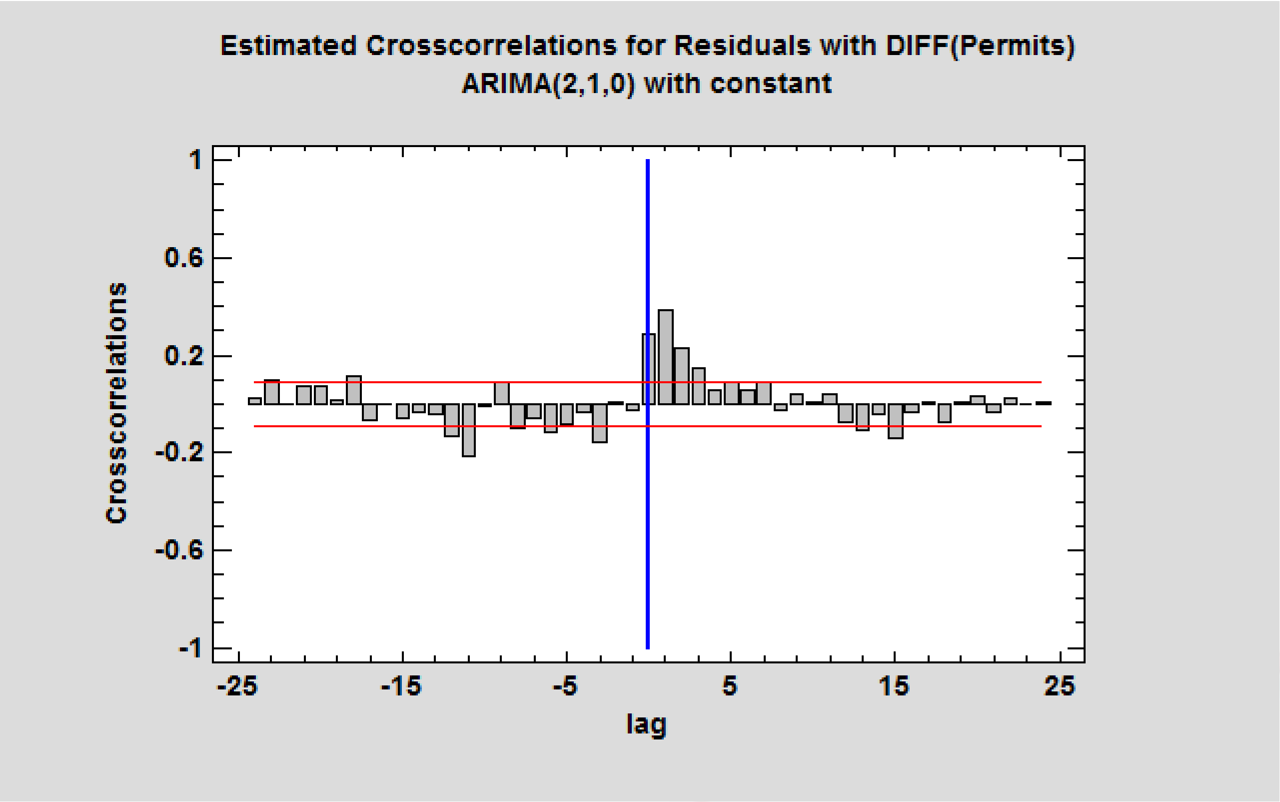
Forecast Table

		Lower 95.0%	*Upper 95.0%*
Period	*Forecast*	*Limit*	*Limit*
Q2/15	16315.3	16147.4	16484.1
Q3/15	16372.6	16099.1	16648.4
Q4/15	16443.6	16070.2	16821.4

Residual ACF



Residual Crosscorrelations



Models with Leading Indicators

- The ARIMA model is modified by adding additional terms involving one or more regressors $\{X_t\}$.
- The same differencing and AR operators are applied to $\{X_t\}$ as are applied to $\{Y_t\}$.
- We are essentially fitting an ARIMA model to the errors of the regression of Y on X.
- Helpful discussion of this by Prof. Robert Nau at *people.duke.edu/~rnau/arimreg.htm*

One Complication

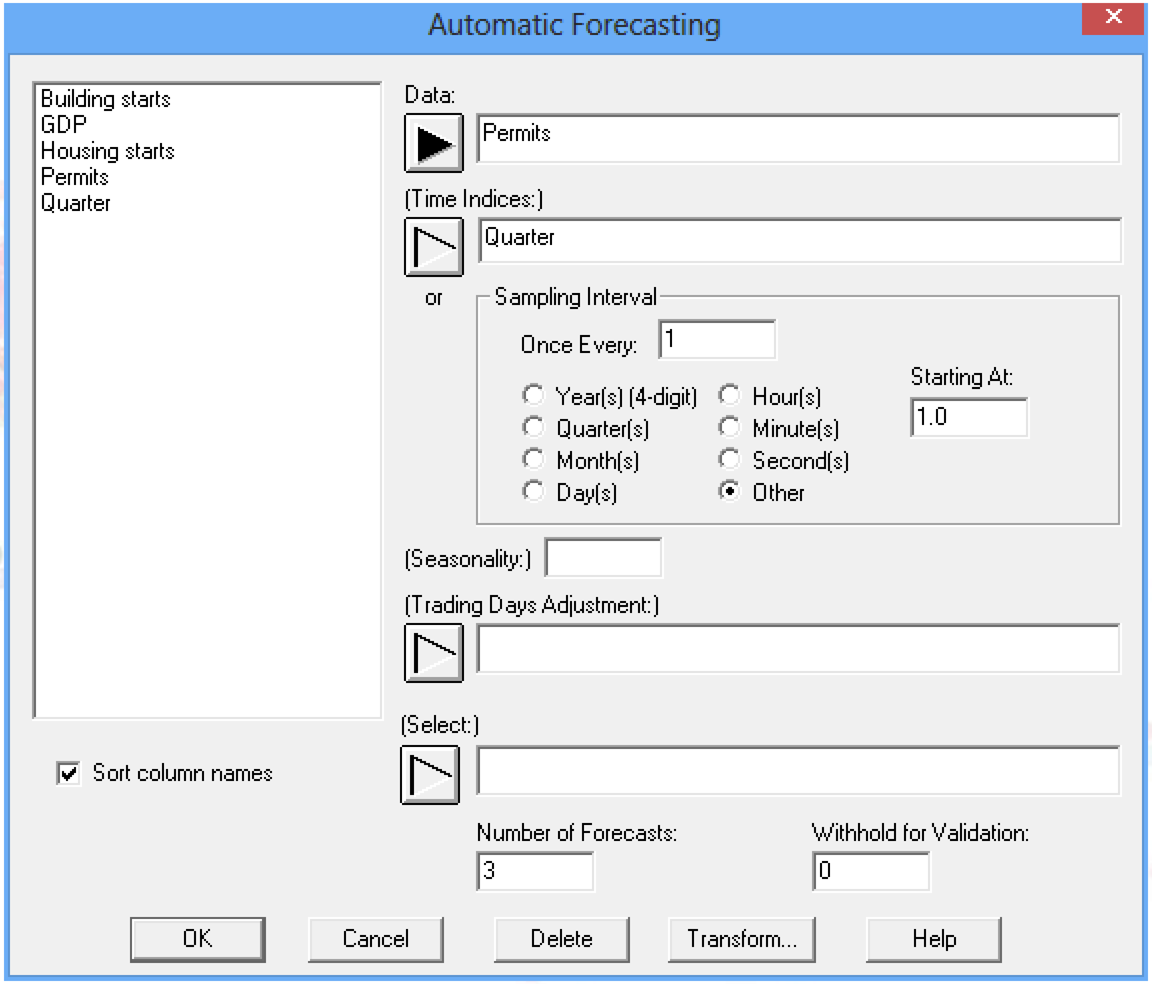
To use a time series such as *Permits* in our forecast model:

– Generate a forecasting model for the regressor variable(s).

– Add the forecasts to the bottom of the datasheet.

– Add the regressors to our model using Analysis Options.

Forecasting Construction Permits



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Adding Forecasts to Data Table

C:\Data\webinar\GDPdata.sgd

	Quarter	GDP	Building starts	Housing starts	Permits
	from BEA	billions of chained 2009 dollars, seasonally adjusted	from OECD, seasonally adjusted	thousands of units, from FRED	thousands of permits, from FRED
214	Q2/13	15606.6	72055.6	866	994
215	Q3/13	15779.9	73500.0	883	983
216	Q4/13	15916.2	85416.7	1012	1029
217	Q1/14	15831.7	77083.3	934	1031
218	Q2/14	16010.4	82111.1	984	1041
219	Q3/14	16205.6	85805.6	1029	1045
220	Q4/14	16294.7	88555.6	1055	1092
221	Q1/15	16264.1	80722.2	975	1065
222	Q2/15				1088.90
223	Q3/15				1117.32
224	Q4/15				1148.04
225					
226					
227					

GDPdata | B | C

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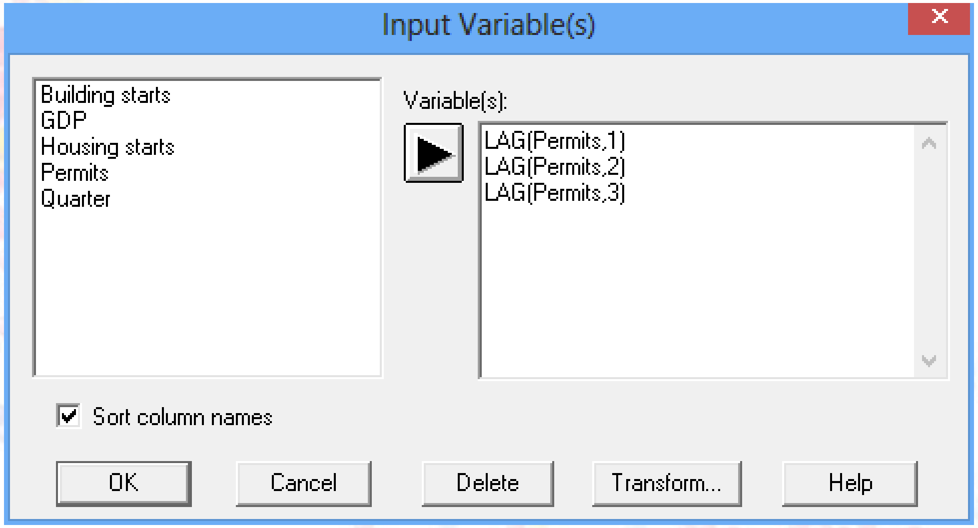
Add Regressor Variables



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Revised Model

Automatic Forecasting - GDP
Data variable: GDP (billions of chained 2009 dollars, seasonally adjusted)

Number of observations = 221
Time indices: Quarter (from BEA)

Forecast Summary
Math adjustment: Square root
Forecast model selected: ARIMA(1,1,0) with constant + 3 regressors
Number of forecasts generated: 3
Number of periods withheld for validation: 0

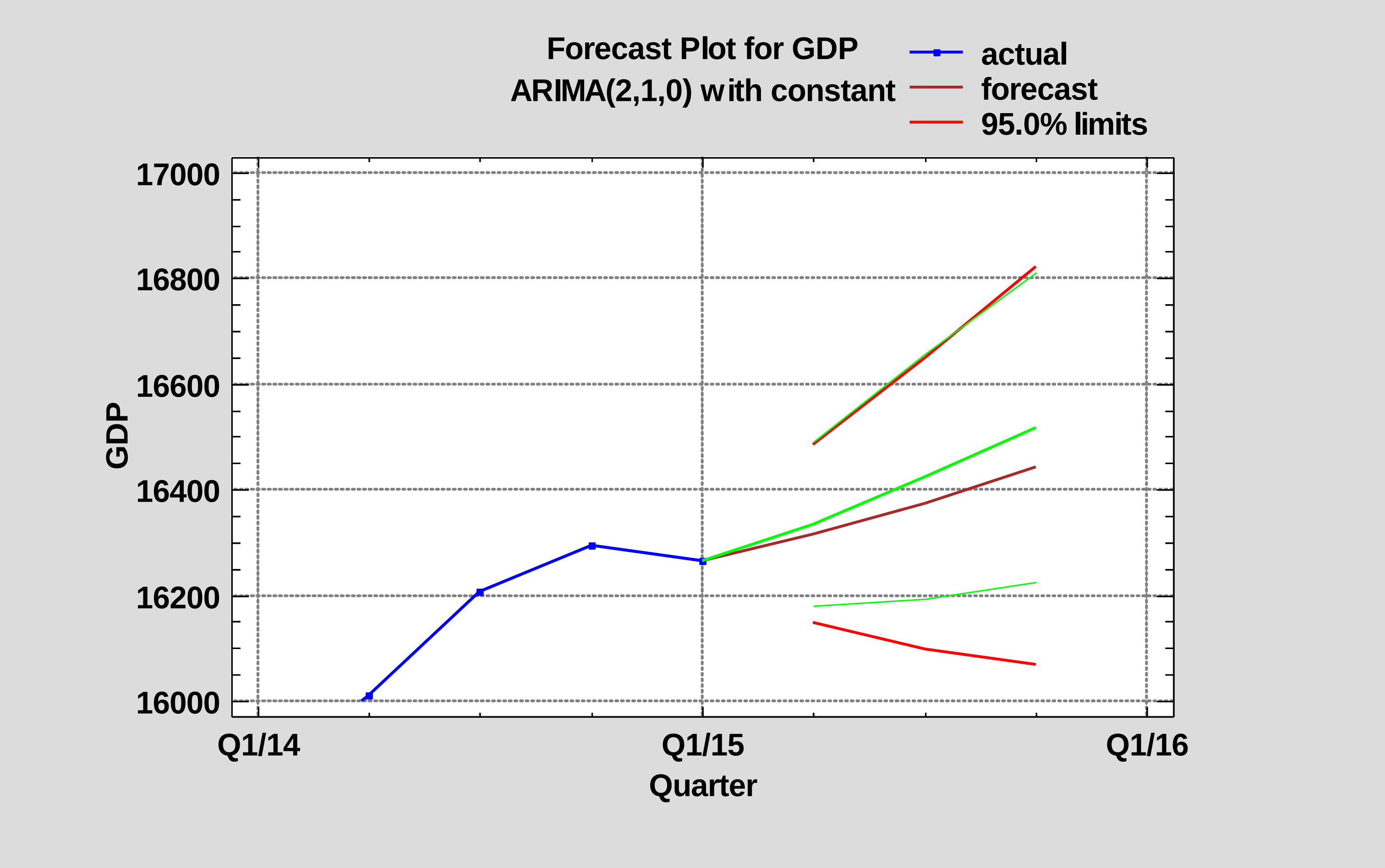
	Estimation	*Validation*
Statistic	*Period*	*Period*
RMSE	59.9928	
MAE	42.9407	
MAPE	0.520399	
ME	0.147357	
MPE	-0.00510692	

ARIMA Model Summary

Parameter	*Estimate*	*Stnd. Error*	*t*	*P-value*
AR(1)	0.126197	0.0688097	1.834	0.068056
LAG(Permits,1)	0.000963898	0.000202409	4.76213	0.000004
LAG(Permits,2)	0.000713768	0.000200952	3.55193	0.000471
LAG(Permits,3)	0.000495638	0.00020176	2.45658	0.014830
Mean	0.330946	0.0238401	13.8819	0.000000
Constant	0.289182			

Backforecasting: yes
Estimated white noise variance = 0.0936222 with 212 degrees of freedom
Estimated white noise standard deviation = 0.305977
Number of iterations: 7

Revised Forecasts



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Note

- At 8:30AM this morning (June 24) the BEA announced a "third" estimate of the Q1/2015 GDP. It raised the estimate from 16,264.1 to 16,287.7.

- That's a revision from -0.7% to -0.2% in the annual rate compared to the previous quarter.

- That changes our Q2/2015 forecast from 16,334.8 to 16,361.5. That's an increase from about 2.0% to 2.2% growth year-over-year.

Data Sources

- New Private Housing Units Authorized by Building Permits – Federal Reserve Bank of St. Louis

- GDP – Bureau of Economic Analysis, U.S. Department of Commerce

References

- George Box and Gwilym Jenkins (2008) - Time Series Analysis: Forecasting and Control (fourth edition).

- Burton Malkiel (2015) - A Random Walk Down Wall Street: The Time-Tested Strategy for Successful Investing (eleventh edition)

Recorded Webinar

- You may find the recorded webinar, PowerPoint slides and sample data at:

  www.statgraphics.com

- Look for “Instructional Videos”.