## X-Bar and R Charts

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### Summary

The **X-Bar and R Charts** procedure creates control charts for a single numeric variable where the data have been collected in subgroups. It creates both an X-bar chart to monitor the subgroup means and an R chart to monitor the subgroup ranges. Out-of-control signals are highlighted, including both points beyond the control limits and any unusual runs in the data. The charts may be constructed in either **Initial Study** (Phase 1) mode, where the current data determine the control limits, or in **Control to Standard** (Phase 2) mode, where the limits come from either a known standard or from prior data.

**Sample StatFolio:** `xbarrchart.sgp`
Sample Data
The file *wafers.sgd* contains measurements made on the flow width of wafers, taken from Montgomery (2005). The data consist of $m = 45$ samples of 5 wafers each. The table below shows a partial list of the data in that file:

<table>
<thead>
<tr>
<th>Sample</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3235</td>
<td>1.4128</td>
<td>1.6744</td>
<td>1.4573</td>
<td>1.6914</td>
</tr>
<tr>
<td>2</td>
<td>1.4314</td>
<td>1.3592</td>
<td>1.6075</td>
<td>1.4666</td>
<td>1.6109</td>
</tr>
<tr>
<td>3</td>
<td>1.4284</td>
<td>1.4871</td>
<td>1.4932</td>
<td>1.4324</td>
<td>1.5674</td>
</tr>
<tr>
<td>4</td>
<td>1.5028</td>
<td>1.6352</td>
<td>1.3841</td>
<td>1.2831</td>
<td>1.5507</td>
</tr>
<tr>
<td>5</td>
<td>1.5604</td>
<td>1.2735</td>
<td>1.5265</td>
<td>1.4363</td>
<td>1.6441</td>
</tr>
<tr>
<td>6</td>
<td>1.5955</td>
<td>1.5451</td>
<td>1.3574</td>
<td>1.3281</td>
<td>1.4198</td>
</tr>
<tr>
<td>7</td>
<td>1.6274</td>
<td>1.5064</td>
<td>1.8366</td>
<td>1.4177</td>
<td>1.5144</td>
</tr>
<tr>
<td>8</td>
<td>1.419</td>
<td>1.4303</td>
<td>1.6637</td>
<td>1.6067</td>
<td>1.5519</td>
</tr>
<tr>
<td>9</td>
<td>1.3884</td>
<td>1.7277</td>
<td>1.5355</td>
<td>1.5176</td>
<td>1.3688</td>
</tr>
<tr>
<td>10</td>
<td>1.4039</td>
<td>1.6697</td>
<td>1.5089</td>
<td>1.4627</td>
<td>1.522</td>
</tr>
<tr>
<td>11</td>
<td>1.4158</td>
<td>1.7667</td>
<td>1.4278</td>
<td>1.5928</td>
<td>1.4181</td>
</tr>
<tr>
<td>12</td>
<td>1.5821</td>
<td>1.3355</td>
<td>1.5777</td>
<td>1.3908</td>
<td>1.7559</td>
</tr>
</tbody>
</table>

STATGRAPHICS refers to each row of the file as a *subgroup*. The subgroup size $n = 5$.

The first 25 rows of the file will be used in a Phase 1 study to establish control limits for the process. Rows 26-45 will then be plotted against those limits in a Phase 2 analysis.
**Data Input**

In entering data for this procedure, you may enter either:

1. the original measurements.
2. the subgroup means and ranges.

**Case #1: Entering Original Measurements**

In this case, the data to be analyzed consist of the original measurements taken on a single variable.

- **Observations**: one or more numeric columns. If more than one column is entered, each row of the file is assumed to represent a subgroup with subgroup size $n$ equal to the number of columns entered. If only one column is entered, then the *Date/Time/Labels or Size* field is used to form the groups.
- **Date/Time/Labels or Size**: If each set of \( n \) rows represents a group, enter the single value \( n \). For example, entering a 5 implies that the data in rows 1-5 form the first group, rows 6-10 form the second group, and so on. If the subgroup sizes are not equal, enter the name of an additional numeric or non-numeric column containing group identifiers. The program will scan this column and place sequential rows with identical codes into the same group.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

- **Select**: subset selection.

Note the use of the FIRST operator to select only the first \( m = 25 \) rows for the Phase 1 analysis.

**Case #2: Entering Subgroup Statistics**

In this case, the statistics for each subgroup have been computed elsewhere and entered into the datasheet, as in the table below:

<table>
<thead>
<tr>
<th>Sample</th>
<th>Means</th>
<th>Ranges</th>
<th>Sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.51188</td>
<td>0.3679</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1.49512</td>
<td>0.2517</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>1.4817</td>
<td>0.139</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>1.47118</td>
<td>0.3521</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>1.48816</td>
<td>0.3706</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>1.44918</td>
<td>0.2674</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>1.5805</td>
<td>0.4189</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>1.53432</td>
<td>0.2447</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>1.5076</td>
<td>0.3589</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>1.51344</td>
<td>0.2658</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>1.52424</td>
<td>0.3509</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>1.5284</td>
<td>0.4204</td>
<td>5</td>
</tr>
</tbody>
</table>
- **Subgroup Statistics**: the names of the column containing the subgroup means, subgroup ranges, and subgroup sizes.

- **Date/Time/Labels**: optional labels for each subgroup.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

- **Select**: subset selection.
**X-Bar Chart**

This chart plots the subgroup means \( \bar{x}_j \).

![X-bar Chart for X1-X5](image)

In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. The centerline is located at the weighted average of the subgroup means:

\[
x = \frac{\sum_{j=1}^{m} n_j \bar{x}_j}{\sum_{j=1}^{m} n_j}
\]

(1)

The control limits are placed above and below the centerline at:

\[
x \pm k \frac{\hat{\sigma}}{\sqrt{n}}
\]

(2)

where \( k \) is the sigma multiple specified on the *Control Charts* tab of the *Preferences* dialog box (\( k = 3 \) except in rare cases), \( \hat{\sigma} \) is the estimate of the process sigma, and \( n \) is the subgroup size. If the subgroup sizes are not equal, then depending on *Analysis Options*, \( n \) is replaced by either:

1. \( \bar{n} \), the average subgroup size. In this case, the control limits are the same for all subgroups.
2. \( n_j \), the individual subgroup sizes. In this case, the control limits are step functions.

The method for estimating the process sigma also depends on the settings on the *Control Charts* tab of the *Preferences* dialog box, as discussed in the *Analysis Summary* section below.

Any points beyond the control limits will be flagged using a special point symbol. Any point excluded from the analysis, usually by clicking on a point on the chart and pressing the *Exclude/Include* button, will be indicated by an X. If so indicated on the *Pane Options* dialog
box, unusual sequences of points may also be flagged. In the current chart, no unusual points or out-of-control signals are indicated.

**Pane Options**

- **Outer Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 2 sigma.

- **Inner Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 1 sigma.

- **Moving Average**: check this box to add a moving average smoother to the chart. In addition to the subgroup means, the average of the most recent \( q \) points will also be displayed, where \( q \) is the order of the moving average. The default value \( q = 9 \) since the 1-sigma inner warning limits for the original subgroup means are equivalent to the 3-sigma control limits for that order moving average.

- **Exponentially Weighted Moving Average**: check this box to add an EWMA smoother to the chart. In addition to the subgroup means, an exponentially weighted moving average of the subgroup means will also be displayed, where \( \lambda \) is the smoothing parameter of the EWMA. The default value \( \lambda = 0.2 \) since the 1-sigma inner warning limits for the original subgroup means are equivalent to the 3-sigma control limits for that EWMA.

- **Decimal Places for Limits**: the number of decimal places used to display the control limits.

- **Mark Runs Rules Violations**: flags with a special point symbol any unusual sequences or runs. The runs rules applied by default are specified on the *Runs Tests* tab of the *Preferences* dialog box.
- **Color Zones**: check this box to display green, yellow and red zones.

- **Display Specification Limits**: whether to add horizontal lines to the chart displaying the location of the specification limits (if any).

**Example: Chart with Warning Limits and EWMA Smoother**

The chart below shows inner and outer warning limits, together with an EWMA smoother with $\lambda = 0.2$:

The EWMA smoother gives a running estimate of the process mean. Since the EWMA lies completely within the inner warning limits, there is additional evidence that the process mean was probably stable throughout the sampling period.

**R Chart**

This chart the subgroup ranges $R_j$. 

© 2013 by StatPoint Technologies, Inc. X-Bar and R Charts - 8
In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. The centerline is located at:

\[
CL = d_2(\bar{n})\hat{\sigma}
\]  

(3)

If sigma is estimated from the average range, this equals \( \bar{R} \). The control limits are placed above and below the centerline at the following locations:

\[
CL \pm kd_3(n)\hat{\sigma}
\]  

(4)

where \( k \) is the sigma multiple specified on the *Control Charts* tab of the *Preferences* dialog box (\( k = 3 \) except in rare cases), \( \hat{\sigma} \) is the estimate of the process sigma, and \( n \) is the subgroup size. If the subgroup sizes are not equal, *Analysis Options* specifies whether to use the average subgroup size or the individual subgroup sizes.

The R chart for the sample data shows no unusual signals.

*Pane Options*

The same options exist as for the X-Bar chart.

### Subgroup Reports

This pane tabulates the values plotted on the control charts:

<table>
<thead>
<tr>
<th>Subgroup</th>
<th>Size</th>
<th>( \bar{X} )-bar</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1.51188</td>
<td>0.3679</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1.49512</td>
<td>0.2517</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1.4817</td>
<td>0.139</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.47118</td>
<td>0.3521</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1.48816</td>
<td>0.3706</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>1.44918</td>
<td>0.2674</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>1.5805</td>
<td>0.4189</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>1.53432</td>
<td>0.2447</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>1.5076</td>
<td>0.3589</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>1.51344</td>
<td>0.2658</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>1.52424</td>
<td>0.3509</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>1.5284</td>
<td>0.4204</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>1.3407</td>
<td>0.4992</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>1.52614</td>
<td>0.2422</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
<td>1.40832</td>
<td>0.3499</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>1.5344</td>
<td>0.6823</td>
</tr>
<tr>
<td>17</td>
<td>5</td>
<td>1.48738</td>
<td>0.3589</td>
</tr>
<tr>
<td>18</td>
<td>5</td>
<td>1.45734</td>
<td>0.3153</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>1.5777</td>
<td>0.3062</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>1.506</td>
<td>0.524</td>
</tr>
<tr>
<td>21</td>
<td>5</td>
<td>1.46914</td>
<td>0.2185</td>
</tr>
<tr>
<td>22</td>
<td>5</td>
<td>1.539</td>
<td>0.1863</td>
</tr>
<tr>
<td>23</td>
<td>5</td>
<td>1.55924</td>
<td>0.2533</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>1.5688</td>
<td>0.1156</td>
</tr>
</tbody>
</table>

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Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

**Pane Options**

- **Display**: specify the subgroups to display in the report.

**Analysis Summary**

The *Analysis Summary* summarizes the data and the control charts.

**X-bar and R Charts - X1-X5 (FIRST(25))**

Selection variable: FIRST(25)
Number of subgroups = 25
Subgroup size = 5.0
0 subgroups excluded

Distribution: Normal
Transformation: none

**X-bar Chart**

<table>
<thead>
<tr>
<th>Period</th>
<th>#1-25</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCL: +3.0 sigma</td>
<td>1.69224</td>
</tr>
<tr>
<td>Centerline</td>
<td>1.50345</td>
</tr>
<tr>
<td>LCL: -3.0 sigma</td>
<td>1.31467</td>
</tr>
</tbody>
</table>

0 beyond limits

**Range Chart**

<table>
<thead>
<tr>
<th>Period</th>
<th>#1-25</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCL: +3.0 sigma</td>
<td>0.692064</td>
</tr>
<tr>
<td>Centerline</td>
<td>0.327296</td>
</tr>
<tr>
<td>LCL: -3.0 sigma</td>
<td>0.0</td>
</tr>
</tbody>
</table>

0 beyond limits

**Estimates**

<table>
<thead>
<tr>
<th>Period</th>
<th>#1-25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process mean</td>
<td>1.50345</td>
</tr>
<tr>
<td>Process sigma</td>
<td>0.140712</td>
</tr>
<tr>
<td>Average range</td>
<td>0.327296</td>
</tr>
</tbody>
</table>
Included in the table are:

- **Subgroup Information**: the number of subgroups \( m \) and the average subgroup size

\[
\bar{n} = \frac{\sum_{j=1}^{m} n_j}{m}
\]  

(5)

If any subgroups have been excluded from the calculations, that number is also displayed.

- **Distribution**: the assumed distribution for the data. By default, the data are assumed to follow a normal distribution. However, one of 26 other distributions may be selected using *Analysis Options*.

- **Transformation**: any transformation that has been applied to the data. Using *Analysis Options*, you may elect to transform the data using either a common transformation such as a square root or optimize the transformation using the Box-Cox method.

- **X-bar Chart**: a summary of the centerline and control limits for the x-bar chart, which plots the subgroup means \( \bar{x}_j \). Using *Analysis Options*, separate limits may be calculated for different periods (sets of subgroups).

- **R Chart**: a summary of the centerline and control limits for the R chart, which plots the subgroup ranges \( R_j \).

- **Estimates**: estimates of the process mean \( \mu \) and the process standard deviation \( \sigma \). The process mean is estimated from the weighted average of the subgroup means:

\[
\hat{\mu} = \bar{x}
\]

(6)

The process sigma may be estimated in any of 3 ways, depending upon the settings on the *Control Charts* tab of the *Preferences* dialog box, accessible through the *Edit* menu. There are 3 options:

1. **From average range**: the process sigma is estimated from a weighted average of the subgroup ranges. This is the estimator used in most SPC textbooks.
2. **From pooled s with no bias correction**: the process sigma is estimated from the pooled within group variance, as in a one-way ANOVA. In this case, \( s^2 \) is an unbiased estimate of \( \sigma^2 \) but \( s \) is a biased estimator for \( \sigma \).
3. **From pooled s with bias correction**: the process sigma is estimated from the pooled within group variance and then multiplied by a factor that makes the result an unbiased estimate of \( \sigma \).

- **Average Range**: the average of the subgroup ranges:
\[ \overline{R} = \frac{\sum_{j=1}^{m} R_j}{m} \]  

(7)

Analysis Options

- **Type of Study**: determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.

- **Normalize**: if selected, all statistics will be normalized by calculating Z-scores and the Z-scores plotted on the charts. The centerline on such a chart is always located at 0, and the control limits are always located at ±k.

- **Avg. Subgroup Size**: if checked, the control limits will be horizontal lines based on the average subgroup size. If not checked, the individual subgroup sizes will be used, resulting in step function control limits if the subgroups sizes are not all equal.

- **Use Zone Format**: if checked, the statistics will be plotted using a zone chart rather than the usual format. See the example below.

- **Recalculate at**: the control limits of the chart may be recalculated at up to 4 locations along the X axis by specifying the subgroup numbers at which new estimates are to be initiated. Separate estimates of the process mean and sigma will be obtained using the data in the different sections. In such cases, the control limits will be adjusted at the start of each new period.

- **X-bar Control Limits**: specify the multiple k to use in determining the upper and lower control limits on the X-bar chart. To suppress a limit completely, enter 0.
- **Range Control Limits**: specify the multiple $k$ to use in determining the upper and lower control limits on the R chart. To suppress a limit completely, enter 0.

- **Control to Standard**: to perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter either of the following:
  - *Specify Parameters*: specify the established standard process mean and sigma (or other parameters if not assuming a normal distribution). These values will then be used to position the centerlines and control limits on the charts.
  - *Specify Control Limits*: specify the location of the centerlines and control limits exactly where you wish them to be placed.

- **Exclude button**: Use this button to exclude specific subgroups from the calculations. It will display the following dialog box:

  ![Exclude/Include Options dialog box](image)

To exclude one or more values from the calculations:

1. Select *Manual*, enter the subgroup number to exclude, and press OK. The control limits will be recalculated without that subgroup and the chart replotted.

2. Select *Automatic* and press OK. The program will then remove points one at a time from the control charts, recalculating the limits after each point is removed. It will stop removing points once all remaining points are within the current control limits. This operation is done first to the R chart and then to the X-bar chart. This option should be used with care and is designed primarily for “what if” types of analyses.

Points may also be excluded by clicking on them while viewing a control chart and then pressing the *Exclude/Include* button on the analysis toolbar.

- **Transform Button**: Use this button to specify a transformation or non-normal distribution.
If a Data Transformation option is selected, the observations will first be transformed according to the option selected before any calculations are performed. When displaying the X-bar chart, the inverse transformation will be applied to the centerline and control limits so that the chart is plotted in the original units. On such a chart, the control limits will not be equally spaced around the centerline.

If a Distribution other than the Normal is selected, then the data will be normalized (transformed to equivalent Z-scores) before the control charts are calculated. When displaying the X-bar chart, the normalization will be reversed so that the plot may be done in the original metric. As with a data transformation, the control limits will not be equally spaced around the centerline. For distribution such as the 3-parameter Weibull and 3-parameter lognormal distributions, you must also specify the value of the lower threshold parameter.

Note: The options on this dialog box are designed primarily for a Phase 2 analysis, after an extensive study of the process has been performed to determine the proper method for handling a particular variable. When used in a Phase 1 analysis, estimates of the non-normal parameters will be impacted by any out-of-control conditions and may lead to a loss of power in detecting those conditions. The same is true of the optimized Box-Cox transformation, which may be very sensitive to outlying observations.

For examples of the use of transformations and non-normal distributions, see the documentation for the Individuals Control Charts.
Example: Zone Chart

If the Zone Format option is selected, then the X-bar chart will be displayed in the following manner:

Above each observation is a running count based on the latest run of observations either above the centerline or below the centerline. Beginning at 0, a cumulative score is attached to each point on the chart in the following manner:

- If the point is within the inner warning limits, the score is incremented by 1.
- If the point is beyond the inner warning limits but within the outer warning limits, the score is incremented by 2.
- If the point is beyond the outer warning limits but within the control limits, the score is incremented by 4.
- If the point is beyond the control limits, the score is incremented by 8.

Whenever the process crosses the centerline, the score is reset to 0. Values exactly equal to the centerline do not change the score. When the score reaches a critical value (8 is the default), an out-of-control signal is generated.

The above rule is the default but may be overridden on the Runs Tests tab of the Preferences dialog box, accessible through the Edit menu.
In addition, the score may be reset to 0 immediately after each out-of-control signal.

In the chart above, the maximum score reached only 7, which was not enough to generate an out-of-control signal.

**Runs Tests**

The *Runs Tests* pane displays the results of standard tests applied to the X-bar and R charts to look for unusual sequence of points.

<table>
<thead>
<tr>
<th>Runs Tests Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) runs above or below centerline of length 8 or greater.</td>
</tr>
<tr>
<td>(B) runs up or down of length 8 or greater.</td>
</tr>
<tr>
<td>(C) sets of 5 subgroups with at least 4 beyond 1.0 sigma.</td>
</tr>
<tr>
<td>(D) sets of 3 subgroups with at least 2 beyond 2.0 sigma.</td>
</tr>
<tr>
<td>(E) sets of 15 subgroups at or within 1.0 sigma.</td>
</tr>
<tr>
<td>(F) sets of 8 subgroups beyond 2.0 sigma.</td>
</tr>
<tr>
<td>(G) sets of 8 observations alternating up and down</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Violations</th>
<th>Subgroup</th>
<th>X-bar Chart</th>
<th>Range Chart</th>
</tr>
</thead>
</table>

Depending on the default settings on the *Runs Tests* tab of the *Preferences* dialog box, STATGRAPHICS will look for up to 7 different types of patterns:

A. A group of 8 or more points, all above or all below the centerline.

B. A group of 8 or more points, all increasing or all decreasing.

C. A group of 5 points in which at least 4 are more than 1-sigma away from the centerline, on the same side of the centerline.

D. A group of 3 points in which at least 2 are more than 2-sigma away from the centerline, on the same side of the centerline.

E. A group of 15 or more points, all within 1-sigma.
F. A group of 8 or more points, all beyond 2-sigma, but not necessarily on the same side of the centerline.

G. A group of 8 or more points, all following an alternating up and down pattern.

Any such runs will be indicated in the above table and also on the control charts (unless suppressed).

Runs tests are designed to make standard Shewhart charts more sensitive to small shifts in the process.

Pane Options

Select the runs tests to be applied and the parameters that define those tests. For example, some practitioners prefer to test for runs of length 7 rather than 8.

Tolerance Chart

Control charts are designed to determine whether or not a process is in a state of statistical control, not how well it meets a specification. However, it is sometimes useful to plot the same data with its specification limits. In the current situation, the specification for the wafer width was 1.50 ± 0.50 microns. The Tolerance Chart plots the data with these limits:
Each of the individual measurements is plotted. A horizontal line is also plotted at the overall sample mean.

**Pane Options**

- **Specifications**: the upper specification limit, nominal or target value, and lower specification limit. Any of these entries may be left blank if not relevant.

- **Decimal Places for Limits**: the number of decimal places to display.

- **Plot**: select *Points* to plot point symbols. Otherwise, only the ranges will be displayed. Select *Nominal* to include a horizontal line at the nominal or target value.
Capability Indices

The *Capability Indices* pane displays the values of selected indices that measure how well the data conform to the specification limits.

### Capability Indices for X1-X5

**Specifications**
- USL = 2.0
- Nom = 1.5
- LSL = 1.0

<table>
<thead>
<tr>
<th>Indices</th>
<th>Short-Term</th>
<th>Long-Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sigma</td>
<td>0.140712</td>
<td>0.123636</td>
</tr>
<tr>
<td>Cp/Pp</td>
<td>1.18445</td>
<td>1.34804</td>
</tr>
<tr>
<td>CR/PR</td>
<td>84.4272</td>
<td>74.1818</td>
</tr>
<tr>
<td>Cpk/Ppk</td>
<td>1.17628</td>
<td>1.33874</td>
</tr>
<tr>
<td>K</td>
<td></td>
<td>0.0069008</td>
</tr>
</tbody>
</table>

Based on 6 sigma limits. Short-term sigma estimated from average range.

The indices displayed by default depend on the settings of the *Capability* tab on the *Preferences* dialog box. A detailed discussion of these indices may be found in the documentation for *Process Capability (Variables)*.

### Pane Options

- **Indices**: select the indices to be displayed.

- **Specifications**: the upper specification limit, nominal or target value, and lower specification limit. Any of these entries may be left blank if not relevant.
Phase 2 Analyses

The discussion above deals with a study in which 25 samples of 5 wafers each was used to study data from a manufacturing process. Based on that study, it appears that the process is in a state of statistical control. The estimated process parameters, displayed on the *Analysis Summary*, are:

- Process mean: \( \hat{\mu} = 1.50345 \)
- Process standard deviation: \( \hat{\sigma} = 0.140712 \)

In a Phase 2 study, additional data is collected and plotted with control limits based on that standard. Rows 26-45 of the wafers data file represent such a case.

To perform a Phase 2 study on the wafers, the data input dialog box will first be modified to remove the earlier *Subset selection*: 
Next, the Type of Study will be switched to Control to Standard in the Analysis Options dialog box and the standard mean and sigma will be entered:
The resulting chart has the same centerline and control limits as before:

Examining the subgroups beyond the initial 25, it appears that the process continued to remain in control until approximately subgroup 38, when the mean width jumped. The runs tests generated the first out-of-control signal at subgroup 40, due to 2 out of the last 3 points being beyond the 2-sigma outer warning limits. Eventually, at time periods 43 and 45, the subgroup means exceeded the upper control limit.
**OC Curve**

The *OC (Operating Characteristic) Curve* is designed to illustrate the properties of a Phase 2 control chart.

The chart displays the probability that a subgroup mean will be within the control limits on the X-bar chart, as a function of the true process mean. For example, if the process mean were to shift to 1.75 microns, the chart would generate a point beyond the control limits about 80% of the time. On the other hand, a shift of half that magnitude would cause a signal only about 20% of the time.

**ARL Curve**

The *ARL Curve* is another way to view the performance of a Phase 2 X-bar chart.
The ARL curve plots the average run length (average number of subgroups plotted up to and including the first point beyond the control limits) as a function of the true process mean. Assuming that the process mean suddenly shifts to a new value, the chart shows how long it takes on average until an out-of-control signal is generated (not counting any signals from run rules violations). For very small shifts, it can take in excess of 350 subgroups on average to detect the shift. Expanding the chart in the vicinity of 1.75 shows:

![ARL Curve for X-bar](image)

At a shift to $\mu = 1.64$, the ARL is approximately 5 subgroups.

The responsiveness of the X-bar chart depends on the subgroup size. To help determine a good subgroup size, see the documentation for the Control Chart Design procedure.

**Save Results**

The following results can be saved to the datasheet:

1. **Means** – the subgroup means.
2. **Ranges** – the subgroup ranges.
3. **Sizes** – the subgroup sizes.
4. **Labels** – the subgroup labels.
5. **Process Mean** – the estimated process mean.
6. **Process Sigma** – the estimated process standard deviation.
Calculations

Estimate of Process Sigma

(1) From average range:

\[ \hat{\sigma} = \frac{\sum_{j=1}^{k} \left( f_j R_j \right)}{\sum_{j=1}^{k} f_j} \]  

(8)

where

\[ f_j = \frac{d_j^2(n_j)}{d_j^2(n_j)} \]  

(9)

(2) From pooled s with no bias correction:

\[ \hat{\sigma} = \sqrt{\frac{\sum_{j=1}^{k} (n_j-1) s_j^2}{\sum_{j=1}^{k} (n_j-1)}} \]  

(10)

(3) From pooled s with bias correction:

\[ \hat{\sigma} = \frac{1}{c_4(d)} \sqrt{\frac{\sum_{j=1}^{k} (n_j-1) s_j^2}{\sum_{j=1}^{k} (n_j-1)}} \]  

(11)

where

\[ d = 1 + \sum_{j=1}^{k} (n_j - 1) \]  

(12)

Average Run Length

\[ ARL(\mu) = \frac{1}{1 - \beta(\mu)} \]  

(13)

where

\[ \beta(\mu) = \Phi \left( \frac{UCL - \mu}{\sigma / \sqrt{n}} \right) - \Phi \left( \frac{LCL - \mu}{\sigma / \sqrt{n}} \right) \]  

(14)