Sample Size Determination (Capability Indices)

Summary

This procedure determines a suitable sample size for estimating three capability indices:

- **Cp** – This two-sided capability index compares the distance between the specification limits to *6*-sigma:

$$C_P = \frac{USL - LSL}{6\sigma} \tag{1}$$

- **Cpk** – The smaller of two one-sided indices:

$$C_{PK} = \min\left(\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right) \tag{2}$$

- **Cpm** – A modified version of *Cp* measuring variation around the target or nominal value *T* rather than the estimated process mean:

$$C_{PM} = \frac{C_P}{\sqrt{1 + \frac{(\mu - T)^2}{\sigma^2}}} \tag{3}$$

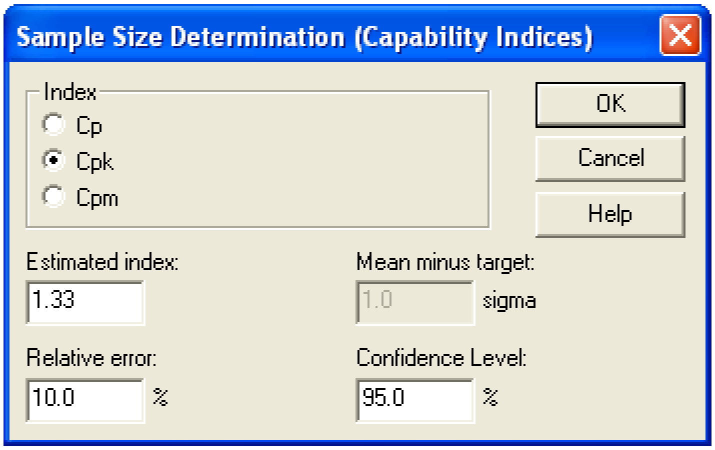
where LSL and USL are the lower and upper specification limits, respectively, μ is the process mean, and σ is the process sigma. The estimates of the indices, which will be labeled $\hat{C}_P$, $\hat{C}_{PK}$, and $\hat{C}_{PM}$ are calculated by replacing the mean and sigma with estimated values.

Sample StatFolio: *capsize.sgp*

Sample Data:
None.

Data Input

The first dialog box displayed by this procedure is used to specify the problem of interest to the analyst.



- **Index:** the capability index to be estimated. It is assumed that a random sample of size *n* will be used to estimate the indicated index. The procedure will determine a suitable value for *n*.

- **Relative error**: the maximum desired difference between the estimate of the capability index and its lower confidence bound.

- **Estimated index**: the anticipated value of the capability index (required for Cpk only). This is the value at which the *relative error* will be fixed. For Cp and Cpm, the relative error is the same for all values of the index.

- **Confidence level**: the level of confidence for the lower confidence bound.

- **Mean minus target**: the anticipated difference between the estimated mean and the target value (required for Cpm only). This is the value at which the *relative error* will be fixed.

For example, the above dialog box indicates that the lower 95% confidence bound for Cpk should be no more than 10% below the estimated value when $\hat{C}_{PK}$ equals 1.33.

Analysis Summary

The *Analysis Summary* displays the required sample size:

Sample Size Determination (Capability Indices)
Capability index: Cpk
Estimate: 1.33
Relative error: 10.0%
Confidence level: 95.0%

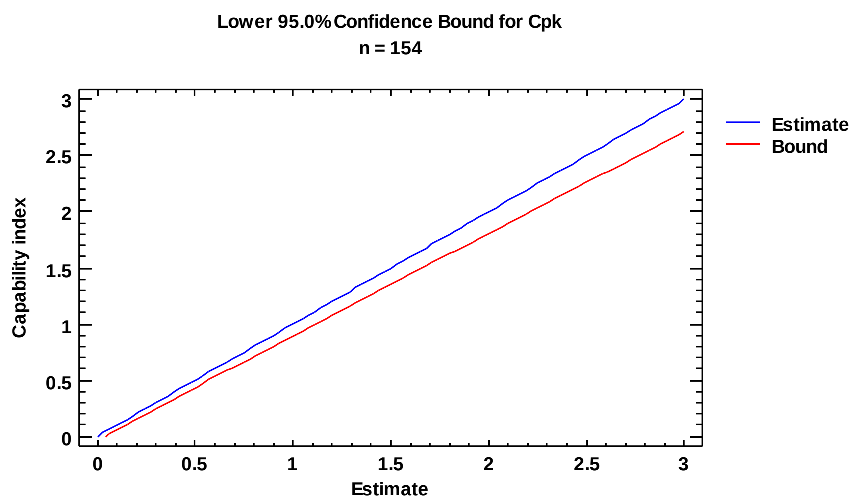
The required sample size is 154.

The StatAdvisor
To be 95.0% confident that the true value of Cpk is no less than 10.0% below the estimated value, the required sample size is 154 if the estimate equals 1.33.

In the current example, a sample of n = 154 observations is required to achieve the desired lower bound.

Confidence Bounds

The *Confidence Bounds* plot shows the lower confidence bound as a function of the estimated capability index:



For a sample of n = 154, the bound is 10% below the estimate when $\hat{C}_{PK}$ equals 1.33. For Cpk, the relative error increases as the estimate increases.

Calculations

The program finds the smallest n such that the lower confidence bound for the ratio of the true capability index to its estimated value satisfies the condition specified in the *relative error* field of the dialog box.

Let α equal 1 minus the desired confidence level. Then the equations for the lower confidence bound of that ratio are shown below:

- **Cp**

$$\sqrt{\frac{\mathrm{X}^2_{\alpha,n-1}}{n-1}} \tag{4}$$

where $\mathrm{X}^2_{\alpha,n-1}$ is the value at which the cumulative chi-square distribution with n-1 degrees of freedom equals α.

- **Cpk**

$$1 - z_a \sqrt{\frac{1}{9n} + \frac{\hat{C}^2_{PK}}{2(n-1)}} / \hat{C}_{PK} \tag{5}$$

where z_α is the value at which the cumulative standard normal distribution equals 1-α and $\hat{C}_{PK}$ is the value entered in the *estimated index* field on the dialog box.

- **Cpm**

$$\sqrt{\frac{n\mathrm{X}^2_{\alpha,f}}{(n-1)f}} \tag{6}$$

where

$$f = \frac{(n+\lambda)^2}{n+2\lambda} \tag{7}$$

and

$$\lambda = n\delta^2 \tag{8}$$

where δ is the value entered in the *mean minus target* field on the dialog box.

Reference: "Sample Size Determination for the Estimate of Process Capability Indices" by Chin-Chuan Wu and Hsin-Lin Kuo, Information and Management Sciences, Vol. 15, No. 1, pp. 1-12.