Acceptance Sampling (Variables)

Summary

When lots containing a relatively large number of items require inspection, acceptance sampling plans can provide reasonable protection against shipping or receiving an unacceptable fraction of non-conforming items without inspecting 100% of the lot. The Acceptance Sampling (Variables) procedure generates acceptance sampling plans for situations when items can be measured and the measurements compared against established specification limits. In such plans, a sample of size \( n \) is drawn from a lot of \( N \) items and the lot is accepted if the sample mean lies no closer than \( k \) standard deviations to the nearer specification limit.

STATGRAPHICS generates three types of acceptance sampling plans:

- **OC Plans** - plans that control the alpha and beta risks, i.e., the probability of accepting a bad lot and the probability of rejecting a good lot. For such a plan, “good” and “bad” must be well-defined.

- **AOQL Plans** - plans that minimize the average outgoing quality limit, i.e., the maximum fraction of non-conforming items accepted on average. Such a plan requires 100% inspection and rectification of all rejected lots.

- **LTPD Plans** - plans that minimize total inspection while controlling the risk of rejecting a bad lot, where “bad” must again be well-defined. Such a plan also requires 100% inspection and rectification of all rejected lots.

Sample StatFolio: acceptvariables.sgp

Sample data:
None.
Acceptance Sampling Plans for Variables

In a variables plan, a sample of \( n \) items is taken from a lot of \( N \), and each item is measured. The sample mean \( \bar{x} \) and the sample standard deviation \( s \) are calculated. The lot is then accepted or rejected according to the following rules:

- If the sample mean \( \bar{x} \) lies no closer than a critical distance \( k\sigma \) from the nearer specification limit, the lot is accepted.

- Otherwise, one of two actions is taken:
  1. If the lot is rectifiable, then all remaining items in the lot are measured. Any non-conforming items are replaced by conforming ones to yield \( N \) conforming items.
  2. If the lot is not rectifiable, the lot is rejected without further inspection and returned to the producer.

In the discussion that follows, several terms are important:

1. AQL = acceptable quality level, defined as the poorest level of quality that the consumer finds acceptable on average.

2. LTPD = lot tolerance percent defective, defined as the poorest level of quality that the consumer is willing to tolerate in any given lot.

3. OC(\( \theta \)) = operating characteristic, defined as the probability that a sampling plan will accept a lot when the lot contains a fraction \( \theta \) of non-conforming items.

4. AOQL = average outgoing quality limit, defined as the maximum percent of defective items accepted by a given sampling plan assuming that all rejected lots are subjected to 100% inspection and all non-conforming items in such lots are replaced with conforming items.
Data Input

The data input dialog box defines the desired features of the sampling plan.

Acceptance Sampling - Variables

- **Action** - defines the type of action desired:
  1. **Create OC Plan** - creates a plan that controls the probability of accepting a lot at both the AQL and the LTPD. Rejected lots are sent back to the producer without being rectified.
  2. **Create AOQL Plan** - creates a plan that minimizes the total number of inspected units at a selected fraction of non-conforming items while insuring that the maximum percentage of non-conforming items accepted does not exceed a specified value. Rejected lots are subjected to 100% inspection and rectified.
  3. **Create LTPD Plan** - creates a plan that minimizes the total number of inspected units at a selected fraction of non-conforming items while controlling the probability of accepting a lot at the LTPD. Rejected lots are subjected to 100% inspection and rectified.
  4. **Analyze Existing Plan** - computes the operating characteristic curve for a sampling plan specified by the user.

- **Quality Levels** - defines the percentage defective in “good” and “bad” lots:
1. **AQL (acceptable quality level)** - the poorest level of quality that the consumer finds acceptable on average.

2. **LTPD (lot tolerance percent defective)** - the poorest level of quality that the consumer is willing to tolerate in any given lot.

- **Lot Size** - the number of items $N$ in the lot.

- **Process Sigma** – whether to assume that the value of $\sigma$ is known or will be replaced by the sample estimate $s$.

- **Desired Features** - characteristics desired of the sampling plan, depending upon the type of plan selected:

<table>
<thead>
<tr>
<th>Type of Plan</th>
<th>Feature #1</th>
<th>Feature #2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OC Plan</strong></td>
<td>Producer’s risk $\alpha$ - the probability of rejecting a lot with a percent defective equal to the AQL.</td>
<td>Consumer’s risk $\beta$ - the probability of accepting a lot with a percent defective equal to the LTPD.</td>
</tr>
<tr>
<td><strong>AOQL Plan</strong></td>
<td>Average percent defective at which inspection will be minimized.</td>
<td>The AOQL or maximum % of non-conforming items accepted after rectification</td>
</tr>
<tr>
<td><strong>LTPD Plan</strong></td>
<td>Average percent defective at which inspection will be minimized.</td>
<td>Consumer’s risk $\beta$ - the probability of accepting a lot with a percent defective equal to the LTPD.</td>
</tr>
</tbody>
</table>

- **Current Plan** - If *Analyze Existing Plan* is selected, the sample size $n$ and critical distance $k$ of the plan to be analyzed.
Analysis Summary

The Analysis Summary displays the generated plan:

<table>
<thead>
<tr>
<th>Acceptance Sampling for Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot size: 10000</td>
</tr>
</tbody>
</table>

Desired features

- Producer's risk (alpha): 5.0%
- Consumer's risk (beta): 10.0%

Generated plan

- Sample size (n) = 138
- Critical distance (k) = 2.43545

Plan attributes

- Acceptable quality level (AQL): 0.5%
- Producer's risk (alpha) = 4.95538%
- Lot tolerance percent defective (LTPD): 1.0%
- Consumer's risk (beta) = 9.99992%

- Average Outgoing Quality Limit (AOQL) = 0.48748% at 0.567986% defective
- Average Total Inspection (ATI) =
  - 626.7 units per lot at the AQL
  - 1417.4 units per lot at the AOQL
  - 9013.81 units per lot at the LTPD

There are several important sections of the output:

- **Desired Features** - summarizes the user-specified features upon which the plan is based. In the above example, the plan was constructed so as to have a producer’s risk of no more than 5% and a consumer’s risk of no more than 10%.

- **Generated plan** - shows the smallest sampling plan that has the desired features. In the example, n = 138 items are to be sampled from the lot of N = 10,000 and the lot accepted if the sample mean is at least 2.43545 standard deviations inside of all specification limits.

- **Plan Attributes** - exact results for the generated plan. This includes:

  - *Producer’s risk at the AQL* - probability of rejecting a “good” lot.
  - *Consumer’s risk at the LTPD* - probability of accepting a “bad” lot.
  - *Average Outgoing Quality Limit* - Assuming that rejected lots are 100% inspected and that any non-conforming items are replaced by good items, this is the maximum fraction of non-conforming items that are accepted.
  - *Average Total Inspection* - assuming that rejected lots are 100% inspected and that any non-conforming items are replaced by good items, this is the average percentage of items in a lot that will be inspected.

The plan generated in the output above would be implemented as follows:

1. A sample of n = 138 items will be taken from each lot of N = 10,000 items.
2. The sample mean $\bar{x}$ and sample standard deviation $s$ will be computed.

3. Z scores will then calculated from:

$$Z_L = \frac{\bar{x} - LSL}{\sigma}$$  
$$Z_U = \frac{USL - \bar{x}}{\sigma}$$  \hspace{1cm} (1)

if the process standard deviation $\sigma$ is known, or from

$$Z_L = \frac{\bar{x} - LSL}{s}$$  
$$Z_U = \frac{USL - \bar{x}}{s}$$  \hspace{1cm} (2)

if the process sigma is to be estimated from the data.

4. If both $Z$-scores are greater than $k$, the lot is accepted. Otherwise, it is either sent back to the producer or rectified by measuring every item and replacing any non-conforming items with conforming ones.

A typical example is a bottle manufacturer who produces lots of 10,000 bottles, each of which has a lower specification on breaking strength of 200 psi. From each lot of $N = 10,000$ bottles, a sample of $n = 138$ bottles would be taken and the strengths of each calculated. Assuming that $\sigma$ is known, the average of the sample must be such that

$$Z_L = \frac{\bar{x} - LSL}{\sigma} \geq 2.435$$  \hspace{1cm} (3)

or

$$\bar{x} \geq 200 + 2.435\sigma$$  \hspace{1cm} (4)

Note: If $\sigma$ is estimated from the data rather than assumed to be known, the required sample size is a much larger $n = 546$, which may be verified by setting the Process sigma field to Use sample estimate on the data input dialog box.
**OC Curve**

The *OC Curve* shows the probability that a lot with a percentage of non-conforming items equal to 1000% will be rejected by the current sampling plan:

For an *OC Plan* such as was generated for the current example, the OC Curve passes through \((1-\alpha)\) at the AQL and \(\beta\) at the LTPD.

**AOQ Curve**

The AOQ curve shows the average outgoing quality of lots with a percentage of non-conforming items equal to 1000% when subjected to the current sampling plan:

The curve peaks at the AOQL.
**ATI Curve**
The ATI curve shows the average number of items inspected for lots with a percentage of non-conforming items equal to 100% when subjected to the current sampling plan:

The ATI curve assumes that rejected lots are rectified.

**Calculations**

**Acceptance Probability**
The probability of accepting a lot containing a fraction non-conforming items equal to \( \theta \) is computed from the standard normal distribution according to

\[
P(\text{accept}|\theta) = \Phi \left( Z_{\theta} \sqrt{n} \right)
\]

(5)

if \( \sigma \) is known and from

\[
P(\text{accept}|\theta) = \Phi \left( \frac{Z_{\theta}}{\sqrt{\frac{1}{n} + \frac{k^2}{2n}}} \right)
\]

(6)

if \( \sigma \) is estimated from the data, where \( Z_{\theta} \) is the value of the standard normal distribution leaving an area of \( \theta \) in the upper tail of the curve.

**Producer’s Risk**

\[
\alpha = P(\text{reject}|AQL) = 1 - P(\text{accept}|AQL)
\]

(7)
Consumer’s Risk

$$\beta = P(\text{accept}|LTPD)$$  \hspace{1cm} (8)

Average Outgoing Quality

$$AOQ(\theta) = \theta P(\text{accept} | \theta) \left( \frac{N-n}{N} \right)$$  \hspace{1cm} (9)

Average Outgoing Quality Limit

$$AOQL = \max_{\theta} \left[ \theta P(\text{accept} | \theta) \left( \frac{N-n}{N} \right) \right]$$  \hspace{1cm} (10)

Average Total Inspection

$$ATI(\theta) = n + (1 - P(\text{accept}|\theta))(N-n)$$  \hspace{1cm} (11)

Reference