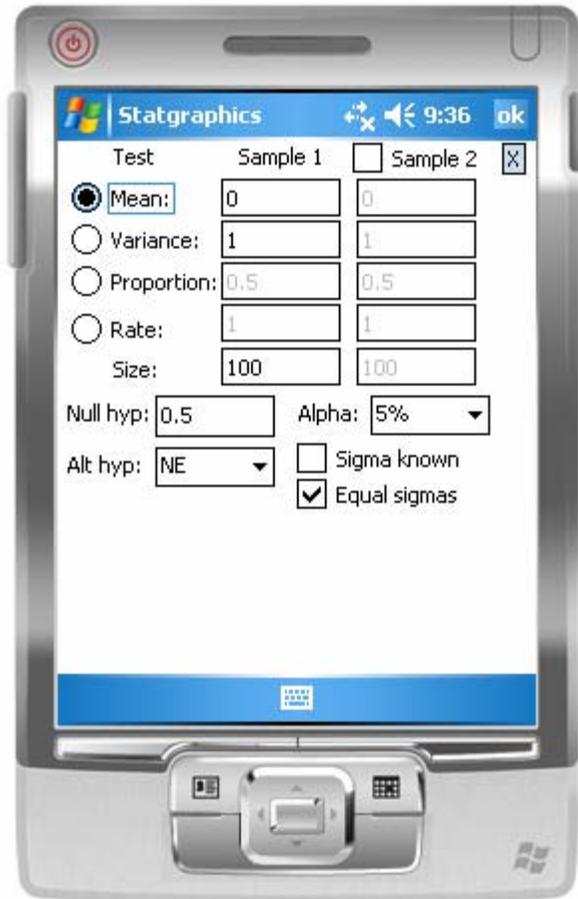


Tools – Hypothesis Tests

The *Tools* menu provides access to a *Hypothesis Tests* procedure that calculates confidence intervals and performs hypothesis tests for means, variances, rates and proportions. It is controlled by the dialog box shown below:



The input fields are:

- **Test:** select the radio button for the test you wish to perform. Tests are available for the mean of a normal distribution, the standard deviation of a normal distribution, a binomial proportion, and a Poisson rate. Alongside the radio button, enter the values of the relevant sample statistics.
- **Sample 2:** check this box if you wish to compare two samples.
- **Size:** indicate the number of observations in each sample.
- **Null hyp:** enter the value of the population parameter to be tested.
- **Alt. hyp:** select a two-sided test (NE for “not equal”) or one-sided test (LT for “less than” or GT for “greater than”).

- **Alpha:** the alpha risk of the test (probability of rejecting a true null hypothesis). This value is used to generate confidence intervals.
- **Sigma known:** When testing normal means, check this box to perform a z-test rather than a t-test.
- **Equal sigmas:** When comparing two normal means, check this box to assume that the standard deviations of both populations are equal (this is the usual assumption).

Press the *OK* button to perform the test, or the *X* button to end the procedure.

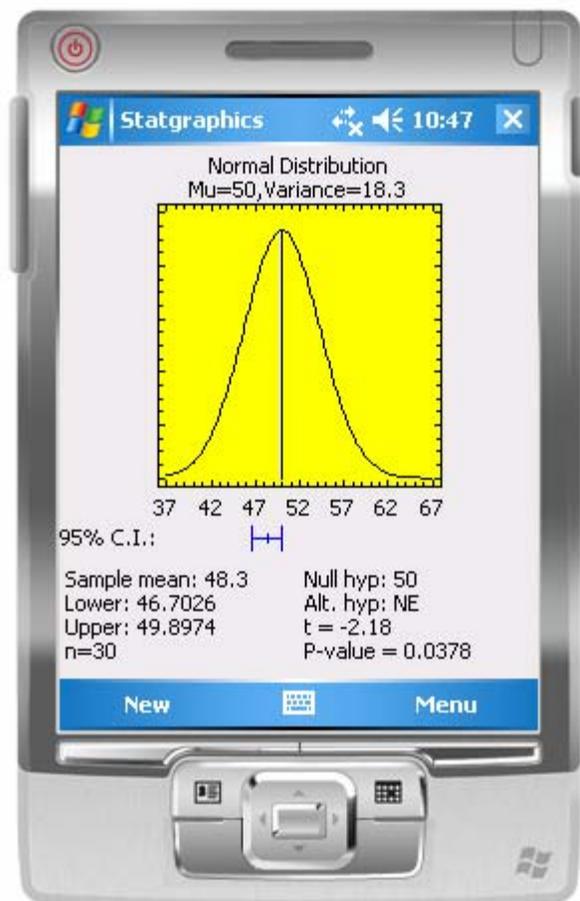
If you wish to return to the dialog box to make changes once statistics have been calculated, select *Menu – Recalc.*

Test of a Single Mean

To test the value of a single population mean, enter:

1. *Sample 1 mean*: the sample mean \bar{x} .
2. *Sample 1 variance*: If *Sigma known* is not checked, enter the sample variance s^2 . Otherwise, enter the known value of the population variance σ^2 .
3. *Size*: the sample size n .
4. *Null hyp*: the hypothesized value of the population mean μ .

The example below tests the hypothesis $\mu = 50$ based on $\bar{x} = 48.3$, $s^2 = 18.3$, and $n = 30$.



The output includes:

1. A plot of the normal distribution with the mean specified by the null hypothesis and the indicated variance.

2. A $100(1-\alpha)\%$ two-sided confidence interval or one-sided confidence bound for μ . The limits are shown graphically and also displayed alongside *Lower* and *Upper* in the tabular output. The two-sided confidence interval is calculated by

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad (1)$$

if σ is known and by

$$\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \quad (2)$$

if σ is not known.

3. Value of the test statistic for the null hypothesis. The test statistic is

$$z = \frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}} \quad (3)$$

if σ is known and by

$$t = \frac{\bar{x} - \mu_o}{s / \sqrt{n}} \quad (4)$$

if σ is not known.

4. A P-value for the hypothesis test. If $P < \alpha$, then the null hypothesis is rejected in favor of the alternative hypothesis. P is calculated using a standard normal distribution if σ is known and from Student's t-distribution with $n - 1$ degrees of freedom if σ is not known.

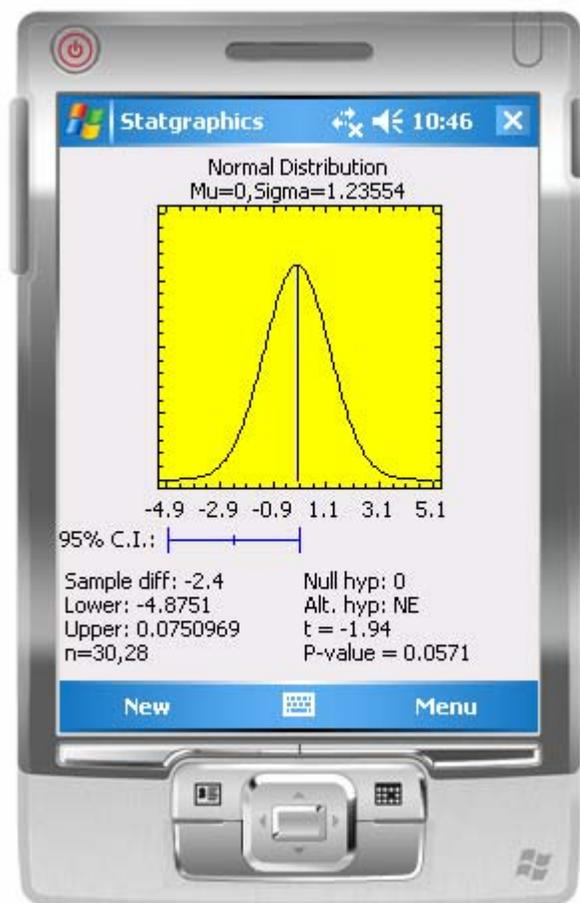
For the sample data, the 95% confidence interval for the mean runs from 46.7 to 49.9. The hypothesis that the mean equals 50 is rejected at the 5% significance level since $P < 0.05$.

Test to Compare Two Means

To compare the values of two population means, enter:

1. *Sample 1 and sample 2 means*: the sample means \bar{x}_1 and \bar{x}_2 .
2. *Sample 1 and sample 2 variances*: If *Sigma known* is not checked, enter the sample variances s_1^2 and s_2^2 . Otherwise, enter the known values of the population variances σ_1^2 and σ_2^2 .
3. *Size*: the sample sizes n_1 and n_2 .
4. *Null hyp*: the hypothesized value of the difference between the population means $\Delta = \mu_1 - \mu_2$.

The sample below tests the hypothesis $\Delta = 0$ based on $\bar{x}_1 = 48.3$, $\bar{x}_2 = 50.7$, $s_1^2 = 18.3$, $s_2^2 = 26.2$, $n_1 = 30$, and $n_2 = 28$, assuming equal sigmas.



The output includes:

1. A plot of a normal distribution with the mean specified by the null hypothesis and the known or estimated standard error for the difference between the means.
2. A $100(1-\alpha)\%$ two-sided confidence interval or one-sided confidence bound for Δ . If σ_1 and σ_2 are assumed to be known, then the two-sided interval is

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \quad (5)$$

If σ_1 and σ_2 are estimated from the data and assumed to be equal, then the two-sided interval is

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2, \nu} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad (6)$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \quad (7)$$

and

$$\nu = n_1 + n_2 - 2 \quad (8)$$

If σ_1 and σ_2 are estimated from the data and not assumed to be equal, then the two-sided interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2, m} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad (9)$$

where

$$\frac{1}{m} = \frac{c^2}{n_1 - 1} + \frac{(1 - c)^2}{n_2 - 1} \quad (10)$$

and

$$c = \frac{s_1^2 / n_1}{s_1^2 / n_1 + s_2^2 / n_2} \quad (11)$$

3. Value of the test statistic for the null hypothesis. If σ_1 and σ_2 are assumed to be known, the test statistic is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \quad (12)$$

If σ_1 and σ_2 are estimated from the data and assumed to be equal, the test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_v \quad (13)$$

If σ_1 and σ_2 are estimated from the data and not assumed to be equal, the test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t_m \quad (14)$$

4. A P-value for the hypothesis test. If $P < \alpha$, then the null hypothesis is rejected in favor of the alternative hypothesis. P is calculated using a standard normal distribution if the variances are known and from Student's t distribution if the variances are not known.

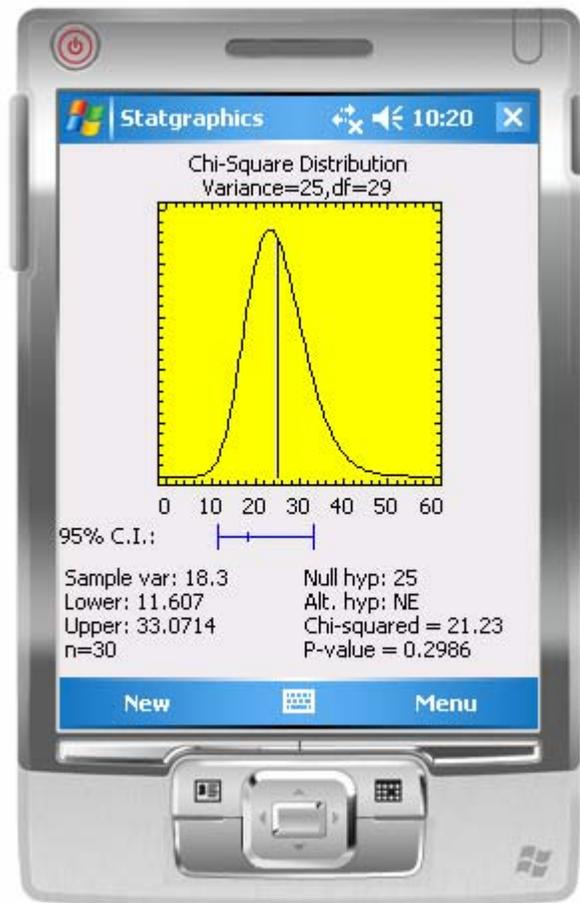
For the sample data, the 95% confidence interval for the difference between the means runs from -4.875 to 0.075. The hypothesis that the difference between the means equals 0 is not rejected at the 5% significance level since $P \geq 0.05$.

Test of a Single Variance

To test the value of a single population variance, enter:

1. *Sample 1 variance*: the sample variance s^2 .
2. *Size*: the sample size n .
3. *Null hyp*: the hypothesized value of the population variance σ^2 .

The sample below tests the hypothesis $\sigma^2 = 25$ based on $s^2 = 18.3$, and $n = 30$.



The output includes:

1. A plot of a scaled chi-square distribution with a vertical line drawn at the value of the variance specified by the null hypothesis.
2. A $100(1-\alpha)\%$ two-sided confidence interval or one-sided confidence bound for σ^2 . The two-sided interval is

$$\left[\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}} \right] \quad (15)$$

3. Value of the test statistic for the null hypothesis. The test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \quad (16)$$

4. A P-value for the hypothesis test. If $P < \alpha$, then the null hypothesis is rejected in favor of the alternative hypothesis. P is calculated using a chi-square distribution with $n - 1$ degrees of freedom.

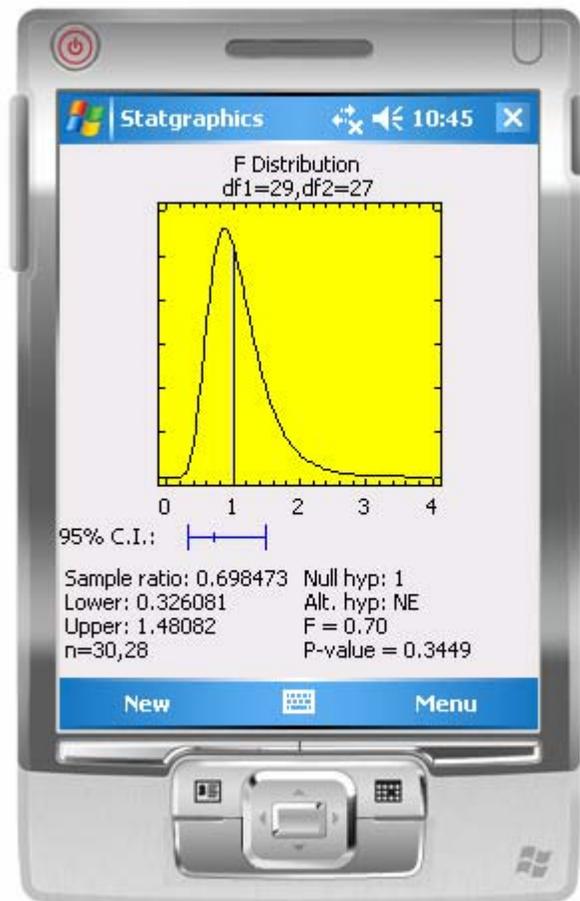
For the sample data, the 95% confidence interval for the variance runs from 11.6 to 33.1. The hypothesis that the variance equals 25 is not rejected at the 5% significance level since $P \geq 0.05$.

Test to Compare Two Variances

To compare the values of two population variances, enter:

1. *Sample 1 and sample 2 variances*: the sample variances s_1^2 and s_2^2 .
2. *Size*: the sample sizes n_1 and n_2 .
3. *Null hyp*: the hypothesized value of the ratio of the population variances $\rho = \sigma_1^2 / \sigma_2^2$.

The sample below tests the hypothesis that $\rho = 1$ based on $s_1^2 = 18.3$, $s_2^2 = 26.2$, $n_1 = 30$, and $n_2 = 28$.



The output includes:

1. A plot of a scaled F distribution with a vertical line drawn at the value of the variance ratio specified by the null hypothesis.
2. A $100(1-\alpha)\%$ two-sided confidence interval or one-sided confidence bound for ρ . The two-sided interval is

$$\left[\frac{s_1^2}{s_2^2} \frac{1}{F_{\alpha/2, n_1-1, n_2-1}}, \frac{s_1^2}{s_2^2} F_{\alpha/2, n_2-1, n_1-1} \right] \quad (17)$$

3. Value of the test statistic for the null hypothesis. The test statistic is

$$F = \frac{s_1^2 / s_2^2}{\rho_0} \quad (18)$$

4. A P-value for the hypothesis test. If $P < \alpha$, then the null hypothesis is rejected in favor of the alternative hypothesis. P is calculated using an F distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom.

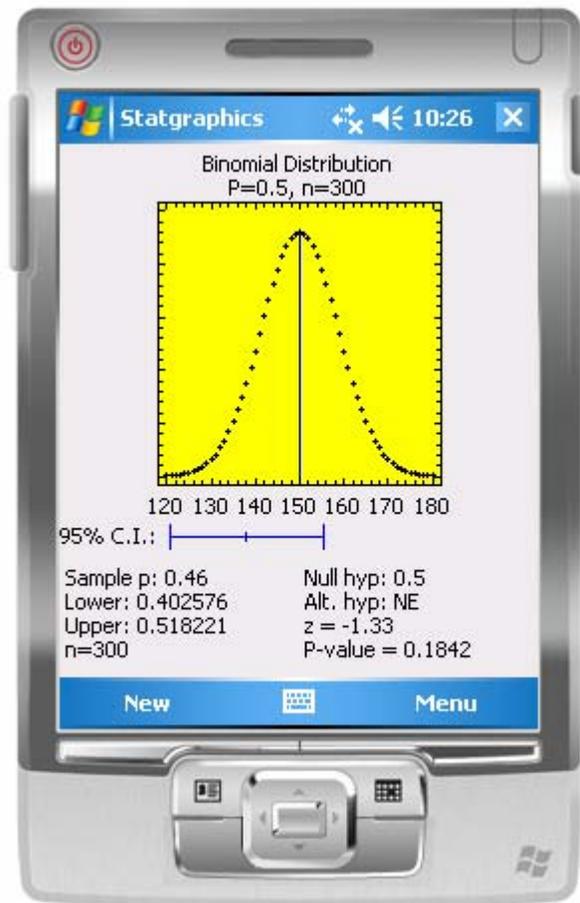
For the sample data, the 95% confidence interval for the ratio of the variances runs from 0.33 to 1.48. The hypothesis that the ratio of the variances equals 1 is not rejected at the 5% significance level since $P \geq 0.05$.

Test of a Single Proportion

To test the value of a single proportion, enter:

1. *Sample 1 proportion*: the sample proportion p .
2. *Size*: the sample size n .
3. *Null hyp*: the hypothesized value of the population proportion θ .

The sample below tests the hypothesis $\theta = 0.5$ based on $p = 0.46$ and $n = 300$.



The output includes:

1. A plot of a binomial distribution with a vertical line drawn at the value of the mean specified by the null hypothesis.
2. A $100(1-\alpha)\%$ two-sided confidence interval or one-sided confidence bound for θ . The two-sided interval is

$$\left[\frac{v_1 F_{1-\alpha/2, v_1, v_2}}{v_2 + v_1 F_{1-\alpha/2, v_1, v_2}}, \frac{v_3 F_{\alpha/2, v_3, v_4}}{v_4 + v_3 F_{\alpha/2, v_3, v_4}} \right] \quad (19)$$

where

$$v_1 = 2np \quad (20)$$

$$v_2 = 2(n - np + 1) \quad (21)$$

$$v_3 = 2(np + 1) \quad (22)$$

$$v_4 = 2(n - np) \quad (23)$$

3. For large samples, the value of the test statistic for the null hypothesis. The test statistic is

$$z = \frac{p - \theta_0}{\sqrt{\frac{\theta_0(1 - \theta_0)}{n}}} \quad (24)$$

4. A P-value for the hypothesis test. If $P < \alpha$, then the null hypothesis is rejected in favor of the alternative hypothesis. For large samples, P is calculated using a standard normal distribution. For small samples, P is calculated directly from the cumulative binomial distribution. For a two-sided test

$$P = \min\{2F_B(np | n, \theta_0), 2(1 - F_B(np - 1 | n, \theta_0))\} \quad (25)$$

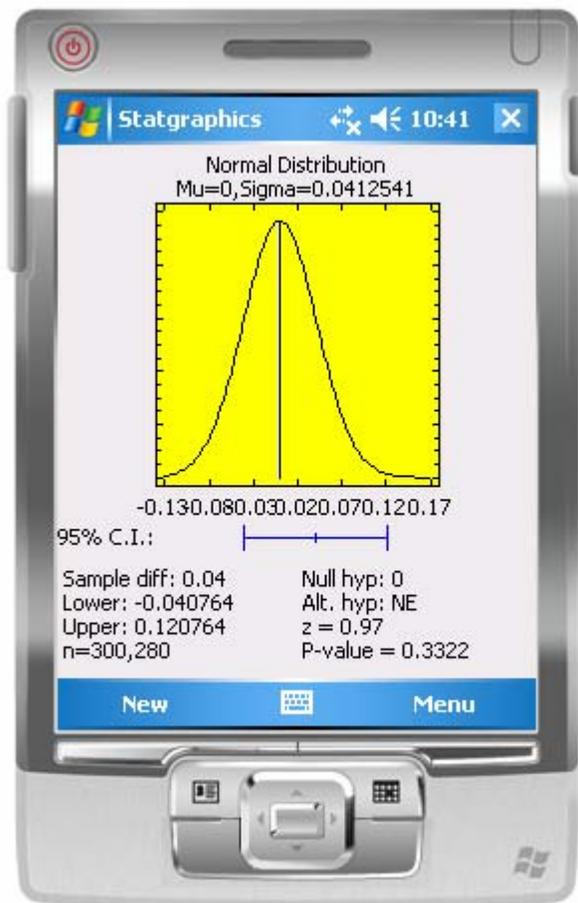
For the sample data, the 95% confidence interval for the population proportion runs from 0.403 to 0.518. The hypothesis that the population proportion equals 0.5 is not rejected at the 5% significance level since $P \geq 0.05$.

Test to Compare Two Proportions

To compare the values of two proportions, enter:

1. *Sample 1 and sample 2 proportions*: the sample proportions p_1 and p_2 .
2. *Size*: the sample sizes n_1 and n_2 .
3. *Null hyp*: the hypothesized value of the difference between the population proportions $\Delta = \theta_1 - \theta_2$.

The sample below tests the hypothesis $\Delta = 0$ based on $p_1 = 0.46$, $p_2 = 0.42$, $n_1 = 300$, and $n_2 = 280$.



The output includes:

1. A plot of an approximating normal distribution for the difference between the proportions with a vertical line drawn at the value specified by the null hypothesis.
2. A $100(1-\alpha)\%$ two-sided confidence interval or one-sided confidence bound for Δ . The two-sided interval is

$$(p_1 - p_2) \pm z_{\alpha/2} \sqrt{p_1(1-p_1)/n_1 + p_2(1-p_2)/n_2} \quad (26)$$

3. Value of the test statistic for the null hypothesis. If the hypothesized $\Delta = 0$, the test statistic is

$$z = \hat{\Delta} / \sqrt{\hat{\theta}(1-\hat{\theta})/n_1 + \hat{\theta}(1-\hat{\theta})/n_2} \quad (27)$$

where

$$\hat{\theta} = (n_1 p_1 + n_2 p_2) / (n_1 + n_2) \quad (28)$$

If the hypothesized $\Delta \neq 0$, the test statistic is

$$z = (p_1 - p_2 - \Delta_0) / \sqrt{p_1(1-p_1)/n_1 + p_2(1-p_2)/n_2} \quad (29)$$

4. A P-value for the hypothesis test. If $P < \alpha$, then the null hypothesis is rejected in favor of the alternative hypothesis. P is calculated using a standard normal distribution.

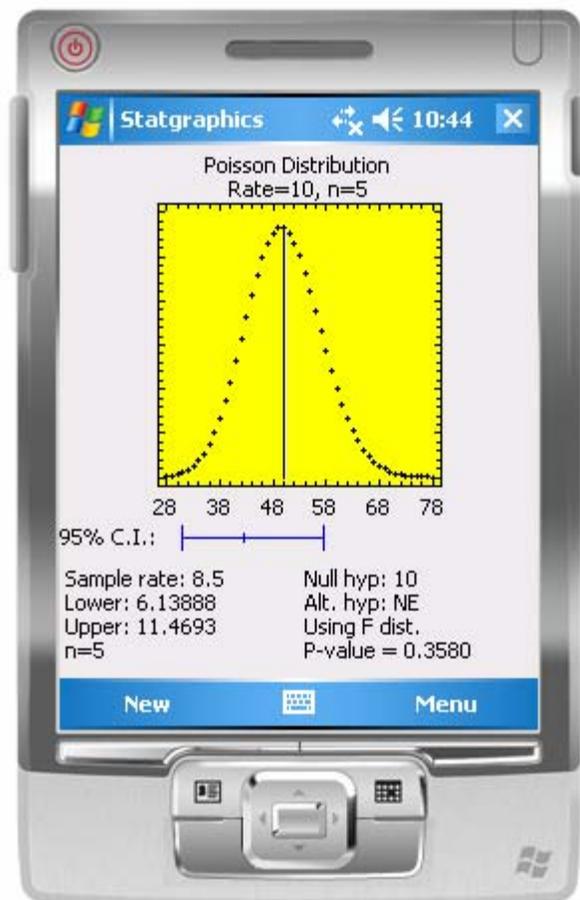
For the sample data, the 95% confidence interval for the difference between the proportions runs from -0.04 to 0.12. The hypothesis that the difference between the proportions equals 0 is not rejected at the 5% significance level since $P \geq 0.05$.

Test of a Single Rate

To test the value of a single rate, enter:

1. *Sample 1 rate*: the sample rate r .
2. *Size*: the sample size n . Note that n does not have to be an integer in this case, since it represents the size of the sampling interval or region over which the rate was measured (often time or space).
3. *Null hyp*: the hypothesized value of the population proportion λ .

The sample below tests the hypothesis $\lambda = 10$ based on $r = 8.5$ and $n = 5$.



The output includes:

1. A plot of a Poisson distribution with a vertical line drawn at the value of the mean specified by the null hypothesis.
2. A $100(1-\alpha)\%$ two-sided confidence interval or one-sided confidence bound for λ . The two-sided interval is

$$\left[\frac{\chi_{1-\alpha/2, 2nr}^2}{2n}, \frac{\chi_{\alpha/2, 2(nr+1)}^2}{2n} \right] \quad (30)$$

3. For large samples, the value of the test statistic for the null hypothesis. The test statistic is

$$z = \frac{r - \lambda_0}{\sqrt{\frac{\lambda_0}{n}}} \quad (31)$$

4. A P-value for the hypothesis test. If $P < \alpha$, then the null hypothesis is rejected in favor of the alternative hypothesis. For large samples, P is calculated using a standard normal distribution. For small samples, P is calculated directly from the cumulative Poisson distribution. For a two-sided test:

$$P = \min\{2F_p(nr | n, \lambda_0), 2(1 - F_p(nr - 1 | n, \lambda_0))\} \quad (32)$$

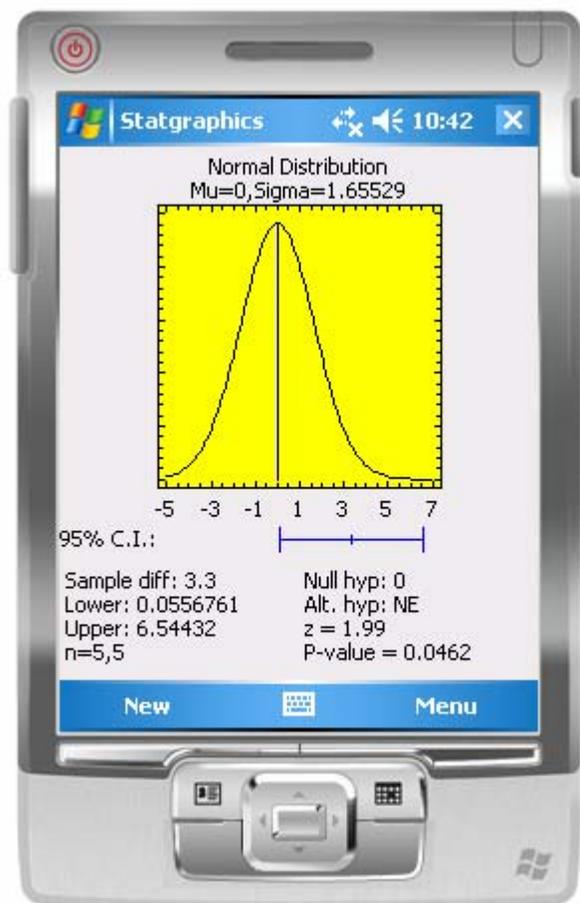
For the sample data, the 95% confidence interval for the population rate runs from 6.14 to 11.47. The hypothesis that the rate equals 10 is not rejected at the 5% significance level since $P \geq 0.05$.

Test to Compare Two Rates

To compare the values of two rates, enter:

4. *Sample 1 and sample 2 rates*: the sample rates r_1 and r_2 .
5. *Size*: the sample sizes n_1 and n_2 .
6. *Null hyp*: the hypothesized value of the difference between the population rates $\Delta = \lambda_1 - \lambda_2$.

The sample below tests the hypothesis $\Delta = 0$ based on $r_1 = 8.5$, $r_2 = 5.2$, $n_1 = 5$, and $n_2 = 5$.



The output includes:

1. A plot of an approximating normal distribution for the difference between the rates with a vertical line drawn at the value specified by the null hypothesis.
2. A $100(1-\alpha)\%$ two-sided confidence interval or one-sided confidence bound for Δ . The two-sided interval is

$$(r_1 - r_2) \pm z_{\alpha/2} \sqrt{\hat{\lambda}_1 / n_1 + \hat{\lambda}_2 / n_2} \quad (33)$$

3. Value of the test statistic for the null hypothesis. If the hypothesized $\Delta = 0$, the test statistic is

$$z = (r_1 - r_2) / \sqrt{\hat{\lambda} / n_1 + \hat{\lambda} / n_2} \quad (34)$$

where

$$\hat{\lambda} = (n_1 r_1 + n_2 r_2) / (n_1 + n_2) \quad (35)$$

If the hypothesized $\Delta \neq 0$, the test statistic is

$$z = (\hat{\Delta} - \Delta_0) / \sqrt{r_1 / n_1 + r_2 / n_2} \quad (36)$$

4. A P-value for the hypothesis test. If $P < \alpha$, then the null hypothesis is rejected in favor of the alternative hypothesis. P is calculated using a standard normal distribution.

For the sample data, the 95% confidence interval for the difference between the two rates runs from -0.05 to 6.54. The hypothesis that the difference between the rates equals 0 is rejected at the 5% significance level since $P < 0.05$.