# How To: Perform a Process Capability Analysis Using STATGRAPHICS Centurion

by

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### Introduction

For individuals concerned with the quality of the goods and services that they provide, comparing observed performance to established standards or specifications is an important activity. Determining one's capability to meet whatever promises have been made, whether to the customer or to upper management, requires collecting data and conducting a statistical analysis of it. Such an activity is referred to as a *Process Capability Analysis*, and programs like STATGRAPHICS Centurion provide important tools to facilitate this type of analysis.

Quality engineers have routinely divided data into two major categories:

- (1) *variable data*, usually consisting of measurements made on a continuous scale. Variables such as strength, weight, length, and concentration are typical examples.
- (2) *attribute data*, usually consisting of a non-quantitative appraisal. Examples are PASS/FAIL evaluations and counts of customer complaints.

Since the analysis of these two types of data is very different, this *How To* guide will restrict the discussion to variable data. A future guide will deal with the equally important topic of attribute capability analysis.

### Sample Data

As an example, we will consider the following data, which represent consecutive measurements of the resistivity of 100 silicon wafers. This data is similar to an example presented by Douglas Montgomery in <u>Introduction to Statistical Quality Control</u>, fifth edition (Wiley, 2005), which is an excellent text on SPC techniques. The data are shown below:

227.0	233.2	263.0	292.3	139.7	220.3	151.6	230.6	231.4	270.2
264.6	209.2	268.8	411.7	245.7	185.6	199.8	182.8	130.1	303.8
190.7	370.5	242.1	362.1	141.7	226.2	167.9	249.1	215.3	178.1
241.2	174.9	240.1	187.8	240.1	319.6	222.8	332.0	257.4	192.4
280.9	268.1	431.6	231.2	169.3	167.4	196.2	331.2	282.8	188.4
280.9	268.1	431.6	231.2	169.3	167.4	196.2	331.2	282.8	188.4
148.4	360.5	150.0	353.2	350.2	288.7	251.3	156.8	222.4	59.7
439.6	201.7	155.7	281.9	513.2	169.5	198.6	205.6	257.8	164.6
162.7	168.4	233.7	304.2	318.0	209.3	229.0	246.3	236.4	232.4
165.0	193.8	390.4	260.3	199.6	280.8	155.5	205.6	214.4	236.1

Figure 1: Sample Resistivity Data

The target resistivity for the wafers is 225, with an allowable range of 100 to 500.

### Step 1: Plot the Data

When beginning to analyze a new set of data, it is always a good idea to plot it. Before blindly applying any statistical procedure, we must be sure that it makes sense to do so. In particular, most capability analysis procedures assume (at least by default) that the data are:

- 1. Stable over time, without major changes in the mean level or amount of variability.
- 2. Free from outliers.
- 3. Independent from sample to sample.

### **Procedure: Run Chart**

A good STATGRAPHICS Centurion procedure for plotting time-ordered data is the *Run Chart*, located under:

- If using the Classic menu: *Plot Time Sequence Plots Run Charts.*
- If using the Six Sigma menu: Measure Time Sequence Plots Run Charts.

There are two run charts: one for individuals data such as that above, where each observation is taken at a different time (perhaps once every 15 minutes), and one for data taken in groups (perhaps 5 measurements at the end of each shift). After selecting the proper menu item, a data input dialog box will be displayed:

Run Chart	
resistivity	Observations:
	(Date/Time/Labels:)
	(Select.)
Sort column names	
OK Cancel	Delete Transform Help

Figure 2: Data Input Dialog Box for Run Chart Procedure

Double-click on *resistivity* to enter it into the *Observations* field and press *OK* to display the following chart:



Figure 3: Run Chart for Resistivity Measurements

The run chart shows the observations plotted in time order. A solid line is drawn at the median of the sample.

Important questions to ask of this data are: Does it appear to be stable throughout the sampling period? Has the level remained constant? Has the variability changed? To help answer this, try

double-clicking on the run chart to enlarge it and then press the *Smooth/Rotat*e button in the analysis toolbar. On the subsequent dialog box, ask for a robust LOWESS smoother to be added to the chart:

Scatterplot Smoothing Options	
Туре	ОК
C None	Cancel
C Running Means	
C Running Lines	Help
C Locally Weighted Regression	
Robust Lowess	
Smoothing Fraction:	

Figure 4: Smooth/Rotate Dialog Box

LOWESS stands for *Locally Weighted Scatterplot Smoothing* and is a technique that can be applied to any X-Y scatterplot to help visualize the relationship between the variables plotted on each axis. In this case, it shows that the level of the series has changed very little during the data collection period, perhaps rising slightly near the middle of the period:



Figure 5: Run Chart with LOWESS Smoother

The Run Chart procedure also displays in its Analysis Summary the results of two runs tests:

- 1. The *runs above and below median* test, which counts the number of groups of consecutive points that are all above the median or all below.
- 2. The *runs up and down* test, which counts the number of groups of consecutive points that are all going up or all going down.

<b>Run Chart (Grouped Da</b> Data variable: resistivity (resistivi 100 values ranging from 59.7 to 5 Median = 231.2	ta) - resist ity of silicon v i13.2	<mark>ivity</mark> vafers)					
Test	Observed	Expected	Longest	P(>=)	$P(\leq=)$	7	
Runs above and below median	48	50.0	4	0.69417	0.380326		
Runs up and down	61	66.3333	4	0.918674	0.123665		
This procedure is used to examine sometimes be seen:	e data for tren	ds or other pa	atterns over	time. Four ty	pes of non-ra	andom patterns c	an
<ol> <li>Mixing - too many runs abo</li> <li>Clustering - too few runs ab</li> <li>Oscillation - too many runs</li> <li>Trending - too few runs up a</li> </ol>	ve or below the ove or below the or below the or below the output of the	ne median the median					
The P-values are used to determine are less than $0.025$ these are no s	e whether any	y apparent pa -random patt	tterns are st	atistically sig	nificant. Sinc	ce none of the P-	-valı

Figure 6: Run Chart Analysis Summary

If we suspect that the mean may have changed, we would expect to see:

- Less runs above and below the median than expected.
- Less runs up and down than expected.

In fact, both observed counts are less than expected. However, the differences from expected behavior are not statistically significant, since the P values in the rightmost column are greater than or equal to 0.05. Therefore, there is no evidence to indicate any serious change in level over the sampling period.

Several other observations are worthy of note:

- 1. With respect to the amount of variability, there also does not appear to have been much change.
- 2. With respect to the general distribution of the observations, there is a noticeable lack of symmetry. Observations tend to deviate farther above the median than below it. This indicates the possible presence of skewness in the distribution, which means that the assumption of a normal distribution may not be tenable.
- 3. There are several points that may be potential outliers: one on the low side and several on the high side. These points could have a big impact on the calculated capability of the process.

### **Procedure: Descriptive Time Series Methods**

There is one assumption we have not looked at yet: the assumption of independence between consecutive samples. This is an extremely important assumption, since indices such as  $C_{pk}$  are usually calculated from a moving range or a within-group standard deviation. Correlation between consecutive observations can lead to a badly underestimated process sigma and thus to an overly optimistic estimate of the process capability. With today's automated data measurement systems, the short time intervals between samples makes this a real concern.

The best way to look for correlation between consecutive measurements is to calculate the autocorrelation function of the data. To generate this plot:

- If using the Classic menu: select *Describe Time Series Descriptive Methods*.
- If using the Six Sigma menu: select *Forecast Descriptive Time Series Methods*.

Complete the data input dialog box as shown below:

Descriptive Methods	
resistivity	Data: resistivity Sampling Interval Once Every:          Year(s) (4-digit)       Hour(s)         Year(s) (4-digit)       Hour(s)         Quarter(s)       Minute(s)         Month(s)       Second(s)         Day(s)       Other
	(Trading Days Adjustment:)
🔽 Sort column names	
ОК	Cancel Delete Transform Help

Figure 7: Data Input Dialog Box for Descriptive Time Series Methods

The estimated autocorrelations will be plotted at various lags:



Figure 8: Estimated Autocorrelation Function

The autocorrelation function examines values of the data separated by k time periods and computes their correlation, on a scale of -1 to 1. It does this for different values of k and plots the correlation coefficients. Any correlations beyond the 95% probability limits (shown as horizontal lines) would be statistically significant.

In this case, there are no correlations large enough to suggest any lack of independence between consecutive measurements. If there were, we would need to deal with that correlation in one of two ways:

- 1. Build a time series model to represent the dynamics of the process.
- 2. Increase the time interval between samples to eliminate the correlation.

Dealing with autocorrelated measurements will be the subject of a later How To guide.

## Step 2: Deal with Any Non-Normality in the Data

The apparent skewness in the data is troubling, since most statistical procedures assume that the data follow a normal distribution. If normality is not tenable, we must either:

- 1. Fit a different distribution to the data and adapt our statistical procedures to that distribution.
- 2. Find a transformation of the data such that normality is a reasonable assumption in the transformed metric.

### **Procedure: Distribution Fitting**

The first step here is to perform a formal test for normality, since we don't want to complicate the analysis unless we really need to. In STATGRAPHICS Centurion, a formal test for normality may be conducted by selecting:

- If using the Classic menu: Describe Distribution Fitting Fitting Uncensored Data.
- If using the Six Sigma menu: Analyze Variable Data Distribution Fitting Fitting Uncensored Data.

The data input dialog box is shown below:

Distribution Fitting (Uncensor	red Data) 🛛 🔀
resistivity	Data: resistivity (Select:)
Sort column names	
OK Cancel	Delete Transform Help

Figure 9: Data Input Dialog Box for Distribution Fitting

Part of the standard default output is the Shapiro-Wilks test:

Tests for Normality	for resistivit	y
Test	Statistic	P-Value
Shapiro-Wilks W	0.941697	0.000372934

#### The StatAdvisor

This pane shows the results of several tests run to determine whether resistivity can be adequately modeled by a normal distribution. The Shapiro-Wilks test is based upon comparing the quantiles of the fitted normal distribution to the quantiles of the data.

Since the smallest P-value amongst the tests performed is less than 0.05, we can reject the idea that resistivity comes from a normal distribution with 95% confidence.

Figure 10: Tests for Normality Output from Distribution Fitting Procedure

A P-Value below 0.05, as in the above table, rejects the hypothesis that the data come from a normal distribution.

To select an alternative distribution, press the Tabular Options button  $\square$  on the analysis toolbar and select *Comparison of Alternative Distributions*. This option will fit a wide variety of distributions and order them according to a goodness-of-fit criterion, such as the Anderson-Darling A<sup>2</sup> statistic:

Comparison of Alternative Distributions					
Distribution	Est. Parameters	KS D	A^2		
Loglogistic	2	0.0477508	0.289605		
Largest Extreme Value	2	0.0563468	0.377902		
Lognormal	2	0.0553747	0.422962		
Inverse Gaussian	2	0.0557318	0.472093		
Birnbaum-Saunders	2	0.0562796	0.477558		
Gamma	2	0.0631831	0.62267		
Laplace	2	0.0806926	1.02854		
Logistic	2	0.0746663	1.07395		
Normal	2	0.103212	1.77192		
Weibull	2	0.104577	2.00503		
Smallest Extreme Value	2	0.17388	5.49967		
Exponential	1	0.4202	22.9211		
Pareto	1	0.58171	40.9609		
Uniform	2	0.308049			

#### The StatAdvisor

This table compares the goodness-of-fit when various distributions are fit to resistivity. You can select other distributions using Pane Options.

According to the Anderson-Darling  $A^2$  statistic, the best fitting distribution is the loglogistic distribution. To fit this distribution, press the alternate mouse button and select Analysis Options.

Figure 11: Comparison of Alternative Distributions Output from Distribution Fitting Procedure

According to both the Kolmogorov-Smirnov D statistic and the Anderson-Darling  $A^2$  statistic, the loglogistic distribution seems to fit the data best.

Using *Analysis Options*, you can specify up to 5 distributions to plot at the same time. The plot below shows the five best-fitting distributions:



Figure 12: Five Fitted Distributions

The best-fitting loglogistic distribution is the one with the highest peak.

### **Procedure: Power Transformations**

The second method for dealing with non-normal data is to seek a transformation of the data that normalizes it. The most common transformations used in statistics are power transformations of the form

Y<sup>p</sup>

in which the data are raised to the p-th power. This covers common transformations such as:

- a square root, for p = 0.5
- a reciprocal, for p = -1
- a logarithm, for p=0

Although the last is not obvious, it can be shown mathematically that as p approaches 0, the effect on the distribution of the data is the same as taking logs.

STATGRAPHICS Centurion contains a special procedure for helping determine a good transformation to apply to a given set of data. To run it:

- If using the Classic menu: select *Describe Numeric Data Power Transformations*.
- If using the Six Sigma menu: select *Analyze Variable Data Distribution Fitting Power Transformations*.

The data input dialog box is shown below:

Power Transformations	
resistivity	Data: resistivity (Select:)
Sort column names	
OK Cancel	Delete Transform Help

Figure 13: Data Input Dialog Box for Power Transformations

Using the methods of Box and Cox, the procedure will select an optimal transformation of the form

 $Y' = (Y + \lambda_2)^{\lambda_1}$ 

Usually, the shift parameter  $\lambda_2$  is set equal to 0:



Figure 14: Power Transformations Analysis Summary

The above table indicates that the optimal power transformation for this data is to raise it to the 0.237 power. However, the 95% confidence for the power extends from -0.154 to 0.662, covering both the logarithm and the square root.

An interesting plot is available in the *Power Transformations* procedure by pressing the *Graphics Options* button for the analysis toolbar and selecting *Skewness and Kurtosis Plot*:



Figure 15: Plot of Standardized Skewness and Kurtosis

This plot shows the standardized skewness and standardized kurtosis values for the data after transforming it according to different powers. If a power transformation successfully normalizes the data, both the skewness and kurtosis should fall within the two horizontal lines. At the optimal power of 0.237, shown as the middle vertical line, skewness is essentially 0. However, the kurtosis is right on the boundary of being unacceptable. In this case, the Box-Cox procedure has not done a very good job in normalizing the data.

Further insight can be gained by selecting *Normal Probability Plot* from the *Graphics Options* menu (within the *Power Transformations* procedure). This option creates a normal probability plot for the transformed data, using the derived optimum power:



Figure 16: Normal Probability Plot for Transformed Data

If the transformation effectively normalized the data, the transformed values should fall approximately along a straight line. In this case, some obvious curvature may be seen, as well as an apparent outlier. It is that aberrant data value that we will focus on next.

# Step 3: Identify and Deal with any Outliers in the Data

It is not uncommon to observe a data value that does not appear to belong with the rest. Ideally, the analyst would have the opportunity to go back to the source of the data and identify an assignable cause for the unusual value that could then be corrected. In such a case, one would be fully justified in removing such an observation and performing the capability analysis on the remainder.

Sometimes, follow-up is impossible, so that we must make the best decision we can about whether to include the observation in the analysis. Obviously, erroneously removing an outlier that represents a repeating event would lead to an overly optimistic estimate of the process capability. On the other hand, keeping an observation that was incorrectly recorded could lead to a seriously pessimistic estimate of capability. In such cases, some statistical help can be useful in quantifying the likelihood that the suspect observation actually belongs with the rest.

### **Procedure: Outlier Identification**

Our next step in the analysis will be to take the transformed data and pass it through the *Outlier Identification* procedure. To run this procedure:

- If using the Classic menu: select *Describe Numeric Data Outlier Identification*.
- If using the Six Sigma menu: select *Analyze Variable Data Outlier Identification*.

The data input dialog box is shown below:

Outlier Identification	
resistivity	Data: resistivity^0.237 (Select:)
Sort column names	
OK Cancel	Delete Transform Help

Figure 17: Data Input Dialog Box for Outlier Identification

Notice that we have used STATGRAPHICS' "on-the-fly" transformation feature, so that we do not have to change the original datasheet.

The procedure creates a helpful *Outlier Plot* that shows each point together with the sample mean plus and minus 1, 2, 3 and 4 standard deviations:



Figure 18: Outlier Plot from Outlier Identification

All of the points are within 3 standard deviations of the mean, except for the suspect point, which is almost 4 standard deviations low. The *Analysis Summary* table lists the 5 largest and 5 smallest values, together with the result of Grubbs' test:

Outil	iei iueniin				
Data va	ariable: resist	ivity^0.237			
100 va	lues ranging f	from 2.63576 to 4.38872			
Numbe	er of values cu	urrently excluded: 0			
Locati	ion estimates	+			
Sampl	le mean	3.63584			
Sampl	le median	3.633			
Trimn	ned mean	3.62134			
Winsc	orized mean	3.62781			
Frimm	ning: 15.0%				
Scale e	estimates				
Sampl	le std. deviatio	on 0.27253			
	10 (715	0 254737			
MAD	/0.6/45	0.234737			
MAD/ Sbi	/0.6/45	0.261106			
MAD/ Sbi Winsc	orized sigma	0.234737 0.261106 0.276897			
MAD/ Sbi Winsc	orized sigma	0.234737 0.261106 0.276897			
MAD/ Sbi Winsc Sorted	orized sigma	0.234737 0.261106 0.276897	Studentized Values	Modified	٦
MAD/ Sbi Winsc Sorted	Values	0.234737 0.261106 0.276897 Studentized Values Without Deletion	Studentized Values With Deletion	Modified M4D 7-Score	]
MAD/ Sbi Winsc Sorted Row 70	I Value Value	0.234737 0.261106 0.276897 Studentized Values Without Deletion	Studentized Values With Deletion	Modified MAD Z-Score	
MAD/ Sbi Winsc Sorted Row 70 19	IV.0.6743           prized sigma           IValues           Value           2.63576           3.17018	0.234737           0.261106           0.276897           Studentized Values           Without Deletion           -3.66962           -1 70866	Studentized Values With Deletion -3.97077 -1.74335	Modified MAD Z-Score -3.91478 -1.81685	-
MAD/ Sbi Winsc Sorted Row 70 19 5	I Values 2.63576 3.17018 2.2412	0.234737           0.261106           0.276897           Studentized Values           Without Deletion           -3.66962           -1.70866           -1.51073	<i>Studentized Values</i> <i>With Deletion</i> -3.97077 -1.74335 -1.53625	Modified MAD Z-Score -3.91478 -1.81685 -1.60509	
MAD/ Sbi Winsc Sorted Row 70 19 5 25	I Values 2.63576 3.17018 3.22412 3.235	0.234737           0.261106           0.276897           Studentized Values           Without Deletion           -3.66962           -1.70866           -1.51073           -1 4708	<i>Studentized Values</i> <i>With Deletion</i> -3.97077 -1.74335 -1.53625 -1.49473	Modified           MAD Z-Score           -3.91478           -1.81685           -1.60509           -1.56238	
MAD/ Sbi Winsc Sorted Row 70 19 5 25 61	Vol.6743           prized sigma           I Values           2.63576           3.17018           3.22412           3.235           3.27062	0.234737           0.261106           0.276897           Studentized Values           Without Deletion           -3.66962           -1.70866           -1.51073           -1.4708           -1 34012	Studentized Values           With Deletion           -3.97077           -1.74335           -1.53625           -1.49473           -1.35931	Modified           MAD Z-Score           -3.91478           -1.81685           -1.60509           -1.56238           -1.42257	
MAD/ Sbi Winsc Sorted Row 70 19 5 25 61	Vol.6743           prized sigma           I Values           2.63576           3.17018           3.22412           3.235           3.27062	Studentized Values           Without Deletion           -3.66962           -1.70866           -1.51073           -1.34012	Studentized Values           With Deletion           -3.97077           -1.74335           -1.53625           -1.49473           -1.35931	Modified           MAD Z-Score           -3.91478           -1.81685           -1.60509           -1.56238           -1.42257	
MAD/ Sbi Winsc Sorted Row 70 19 5 25 61  14	Vol.6743           prized sigma           I Values           2.63576           3.17018           3.22412           3.235           3.27062           4.16539	Studentized Values           Without Deletion           -3.66962           -1.70866           -1.51073           -1.34012           -1.94309	Studentized Values           With Deletion           -3.97077           -1.74335           -1.53625           -1.49473           -1.35931	Modified           MAD Z-Score           -3.91478           -1.81685           -1.60509           -1.56238           -1.42257           2.08997	
MAD/ Sbi Winsc Sorted Row 70 19 5 25 61  14 53	Vol.6743           prized sigma           I Values           2.63576           3.17018           3.22412           3.235           3.27062           4.16539           4.21225	0.234737         0.261106         0.276897         Studentized Values         Without Deletion         -3.66962         -1.70866         -1.51073         -1.4708         -1.34012         0.241504	Studentized Values           With Deletion           -3.97077           -1.74335           -1.53625           -1.49473           -1.35931           1.99151           2.17582	Modified           MAD Z-Score           -3.91478           -1.81685           -1.60509           -1.56238           -1.42257           2.08997           2.27393	
MAD/ Sbi Winsc Sorted Row 70 19 5 25 61  14 53 43	Vol.6743           prized sigma           I Values           2.63576           3.17018           3.22412           3.235           3.27062           4.16539           4.21225           4.21225	0.234737         0.261106         0.276897         Studentized Values         Without Deletion         -3.66962         -1.70866         -1.51073         -1.4708         -1.34012         0.211504         2.11504	Studentized Values           With Deletion           -3.97077           -1.74335           -1.53625           -1.49473           -1.35931           1.99151           2.17582           2.17582	Modified           MAD Z-Score           -3.91478           -1.81685           -1.60509           -1.56238           -1.42257           2.08997           2.27393           2.27393	
MAD/ Sbi Winsc Sorted Row 70 19 5 25 61  14 53 43 71	Vol.6743           prized sigma           I Values           2.63576           3.17018           3.22412           3.235           3.27062           4.16539           4.21225           4.23063	0.234737           0.261106           0.276897           Studentized Values           Without Deletion           -3.66962           -1.70866           -1.51073           -1.4708           -1.34012           1.94309           2.11504           2.18246	Studentized Values           With Deletion           -3.97077           -1.74335           -1.53625           -1.49473           -1.35931           1.99151           2.17582           2.17582           2.24866	Modified           MAD Z-Score           -3.91478           -1.81685           -1.60509           -1.56238           -1.42257           2.08997           2.27393           2.27393           2.34607	
MAD/ Sbi Winsc Sorted Row 70 19 5 25 61  14 53 43 71 75	Vol.6743           prized sigma           I Values           2.63576           3.17018           3.22412           3.235           3.27062           4.16539           4.21225           4.21225           4.23063           4.38872	Studentized Values           0.261106           0.276897           Studentized Values           Without Deletion           -3.66962           -1.70866           -1.51073           -1.4708           -1.34012           1.94309           2.11504           2.18246           2.76256	Studentized Values         With Deletion         -3.97077         -1.74335         -1.53625         -1.49473         -1.35931         1.99151         2.17582         2.17582         2.24866         2.89118	Modified           MAD Z-Score           -3.91478           -1.81685           -1.60509           -1.56238           -1.42257           2.08997           2.27393           2.27393           2.34607           2.96669	

Figure 19: Outlier Identification Analysis Summary

Grubbs' test takes the most extreme data value and expresses it in terms of the number of standard deviations away from the mean. In this case, the most extreme point is 3.67 standard deviations below the mean. It then computes a P-Value to determine how significant the outlier is. A P-Value of .05 or below indicates that the point is a significant outlier at the 5% significance level. It this case, the outlier is nearly significant at the 1% level. We would therefore conclude that it is very unlikely that the suspect data value comes from the same population as the rest.

To tentatively remove the point from the calculations, we can return to the outlier plot, click on

the suspect point, and press the *Exclude* button  $\checkmark$  on the analysis toolbar. The mean and standard deviation will then be recalculated using the remaining 99 observations, and the plot will be automatically redrawn:



Figure 20: Outlier Plot after Removal of Suspect Data Value

At the same time, Grubbs' test will be rerun for the most extreme data value in the remaining sample:

		Studentized Values	Studentized Values	Modified
Row	Value	Without Deletion	With Deletion	MAD Z-Score
0 X	2.63576	-3.97077		
9	3.17018	-1.87011	-1.91435	-1.81936
	3.22412	-1.65807	-1.69055	-1.60731
5	3.235	-1.6153	-1.64572	-1.56454
1	3.27062	-1.47531	-1.49966	-1.42453
•				
3	4.11328	1.83697	1.87919	1.888
4	4.16539	2.04182	2.09767	2.09286
3	4.21225	2.22602	2.29665	2.27708
-3	4.21225	2.22602	2.29665	2.27708
'1	4.23063	2.29824	2.37539	2.34931
'5	4.38872	2.91967	3.07247	2.97078

P-Value = 0.286059

Figure 21: Grubbs' Test for Remaining 99 Data Values

The P-Value for the most extreme point is now well above 0.05, indicating that there are no outliers remaining.

## Step 4: Rerunning the Earlier Procedures

Having determined that an outlier is present in the data, we should now redo the earlier analyses without the outlier. This is extremely easy to do in STATGRAPHICS Centurion, since you can

activate any earlier window and press the Input button input data selection. For example, to determine the best-fitting distribution, return to the *Distribution Fitting* window and modify the data input dialog box as shown below:

Distribution Fitting (Uncensored Data)							
resistivity	Data: resistivity (Select:) resistivity > 100						
Sort column names							
OK Cancel	Delete Transform Help						

Figure 22: Modified Data Input Dialog Box

By entering *resistivity* > 100 in the *Select* field, we will analyze only the 99 data values that we want.

The resulting comparison of distributions now shows:

Comparison of Alternativ	ve Distributions		
Distribution	Est. Parameters	KS D	$A^2$
Largest Extreme Value	2	0.0628601	0.317665
Loglogistic	2	0.0536372	0.394557
Lognormal	2	0.0532022	0.404248
Inverse Gaussian	2	0.0543899	0.413534
Birnbaum-Saunders	2	0.0543173	0.425763
Gamma	2	0.0701443	0.761322
Laplace	2	0.0873128	1.23229
Logistic	2	0.0783686	1.24552
Normal	2	0.10881	2.00441
Weibull	2	0.108898	2.26449
Smallest Extreme Value	2	0.179501	5.74851
Exponential	1	0.427645	23.4344
Pareto	1	0.590797	40.85
Uniform	2	0.399389	

#### The StatAdvisor

This table compares the goodness-of-fit when various distributions are fit to resistivity. You can select other distributions using Pane Options.

According to the Anderson-Darling  $A^2$  statistic, the best fitting distribution is the largest extreme value distribution. To fit this distribution, press the alternate mouse button and select Analysis Options.

Figure 23: Comparison of Distributions after Removing Outlier

The Anderson-Darling statistic suggests that the largest extreme value distribution would now be best, although the loglogistic distribution is extremely close. A plot of the fitted distributions shows how close the top two choices are:



Figure 24: Fitted Distributions after Removing Outlier

Making the same changes to the *Power Transformations* procedure creates the following plot:



Figure 25: Skewness and Kurtosis Plot after Removing Outlier

The optimal power has moved to -0.53, or essentially a reciprocal square root. At that power, both the standardized skewness and standardized kurtosis are well within the expected range. Note also that the normal probability plot is now more like that expected for data from a normal distribution:



Figure 26: Normal Probability Plot after Removing Outlier

# Step 5: Calculating Process Capability

We are now ready to calculate the capability of our process. Two procedures are available for doing so: the *Process Capability Analysis* procedure, which has many options, and the *Capability Assessment SnapStat*, which has limited options but produces a single page of preformatted output. In this case, we'll use the former, which you can access by:

- If using the Classic menu: select SPC Capability Analysis Variables Individuals.
- If using the Six Sigma menu: select *Analyze Capability Analysis Variables Individuals.*

A similar procedure is available for grouped data. The data input dialog box should be completed as shown below:

Process Capability Analysis (Individuals)						
resistivity	Data:					
	USL:					
	Nominal:					
	LSL:					
Sort column names	(Select:) resistivity > 100					
OK Cancel	Delete Transform Help					

Figure 27: Data Input Dialog Box for Process Capability Analysis

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Notice the following:

- We have entered the specification limits and the target (nominal) values. At least one of the USL and LSL fields must have an entry.
- We have used the *Select* field to exclude the identified outlier through a Boolean expression that will select only data values that are greater than 100.

When the analysis window first appears, a histogram will be drawn with statistics based on a normal distribution:



Figure 28: Capability Analysis Based on Normal Distribution

Assuming a normal distribution, which we know is not appropriate, yields an estimate of 29,360 wafers outside the specification limits, for a Sigma Quality Level of 3.39. If we select *Analysis Options*, we can change the assumed distribution or specify a transformation:

Distribution			
			ОК
C Birnbaum-Saunders	Generalized Logistic	C Lognormal (3-parameter)	Cancel
C Cauchy	C Half Normal (2-parameter)	C Maxwell (2-parameter)	
C Exponential	C Inverse Gaussian	C Normal	нер
C Exponential (2-parameter)	C Laplace	C Pareto	
C Exponential Power	Eargest Extreme Value	Pareto (2-parameter)	Parameters
C Folded Normal	C Logistic	C Rayleigh (2-parameter)	
C Gamma	C Loglogistic	C Smallest Extreme Value	
🔘 Gamma (3-parameter)	C Loglogistic (3-parameter)	C Weibull	
🕤 Generalized Gamma	C Lognormal	🔘 Weibull (3-parameter)	
Include C Long-term and short-term C Long-term only (labeled P) C Long-term only (labeled C) C Short-term only	Data Transformation None Logarithm Power: -0.5 Roy-Cox (optimized)	Lower Threshold: 0.0	Sigma Limits:  6.0

Figure 29: Capability Analysis Options Dialog Box

Based upon what we know, we could either:

- 1. Select the Largest Extreme Value radio button.
- 2. Select the *Power* radio button in the *Data Transformation* section and enter the transformation we wish to use.

Taking the first approach yields:



Figure 30: Fitted Largest Extreme Value Distribution

The estimated defects per million is now only 6,902, much less than when a normal distribution was assumed. The Sigma Quality Level is 4.01.

One final note concerning the capability indices is in order. A very commonly used index for process capability is  $C_{pk}$ , defined for data from a normal distribution by

$$C_{PK} = \min\left(\frac{USL - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right)$$

where  $\hat{\mu}$  is the estimated process mean and  $\hat{\sigma}$  is the estimated process standard deviation. This is essentially a ratio of the distance to the nearer specification limit divided by the distance from the mean to the point on the normal curve leaving only 0.135% in the tail.

When a normal distribution is not appropriate, STATGRAPHICS Centurion gives you two options for how to compute the indices (selected using the *Edit – Preferences* dialog box):

- 1. Use Corresponding Z-Scores: With this method, the location of the sample mean and the specification limits are converted to standardized normal Z-scores. The capability index is then calculated from those Z-scores. This insures that a given value of  $C_{pk}$  corresponds to the same percentage beyond the specification limit as when the data follow a normal distribution. Thus rules such as desiring  $C_{pk}$  to be at least 1.33 still give the same assurance regarding DPM (defects per million).
- 2. If *Use Distance between Percentiles* is selected, then the sample mean and specification limits are replaced by corresponding percentiles of the fitted distribution. The interpretation of C<sub>pk</sub> as a ratio of two distances is maintained, but a C<sub>pk</sub> of 1.33 will not correspond to the same DPM as for a normal distribution.

By default, STATGRAPHICS Centurion uses the first option, which maintains the expected relationship between the capability indices, DPM, and the Sigma Quality Level. This latter quantity is often used in Six Sigma projects as a summary of how well the process is performing, with an SQL of 6 representing "world class" quality or 3.4 defects per million.

# Conclusion

This document has discussed some of the difficulties that can arise in practice when performing a process capability analysis. Non-normality and outliers are common problems, and failure to deal with them properly can give a very misleading picture of how capable a process is.

It should be emphasized that the question of what distribution to use for a particular variable and whether and how to transform it should *not* be done every time a new sample of data is analyzed. Rather, a protocol should be established for how to handle a specific variable, based on some initial detailed study of a large amount of data. Then, whenever that variable is analyzed, the same protocol should be applied. Otherwise, the random variability in each sample of data will be magnified by affecting not only the capability estimates but also the manner in which they are obtained. In short, study your process closely, establish a protocol for handling data obtained from it, and then stick by that protocol.

Note: The author welcomes comments about this guide. Please address your responses to <u>neil@statgraphics.com</u>.